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REMARKS ON AN INVERSE MODELLING OF WELDING PROCESSES

ABSTRACT

At the beginning of this work a short characteristic of the methodology of modelling rules of welding process is provided. The relation between both the intensive and extensive parameters are mainly discussed. Additionally, the theoretical bases of modelling of welding processes are presented. In further on the bases of modelling of inverse heat conduction problem is talked over. It bases on the strategy of solving inverse problems [2], it employs the hybrid an analytic – numerical method for analysis these questions. Finally, the appropriate algorithms in moving and stationary systems are established which can be directly applied to solving inverse problem.

Key words*: temperature distribution, welded joints, algorithms, inverse modelling.*

INTRODUCTION

The process of welding has dynamic character and is related with the local change of the internal energy E of welded system. The change of the internal energy E can be defined by general dependence:

$$E = \sum_{j=1}^{n} \varphi_j \psi_j \tag{1}$$

where:

φ_j – intensive parameters,

ψ_j – extensive parameters.

The intensive parameters (e.q. temperature T, pressure p, chemical potential μ, voltage U) and pseudo intensive ones (quotients of two extensive magnitudes – like mass density $m/V = \rho$ etc.) are field magnitudes, creating time – space field where in every space point a real physic magnitude is defined.

The extensive parameters may be transported and summed up in finish dimension areas and may be the scalar (mass, entropy), vectors (energy stream) and tensors (momentum stream).

Some examples of change of the internal energy E in welded joints through interaction of the parameters φ and $\Delta\psi$ are presented in Tab.1.

The selection of the proper intensive parameter φ_j related with the extensive parameter $\Delta\psi_j$ and energy E is possible to perform according to dependence:

$$\varphi_j = \frac{\partial E}{\partial \psi_j} \tag{2}$$

The knowledge of the run of thermo-dynamical process under welding indicates on the possibility of active modelling and control of welding process. Moreover, in calculating process there are material parameters, e.g. thermal conductivity λ, thermal diffusivity α, specific heat c, etc.

The transport process of the extensive magnitudes requires observations and estimation of the intensive parameters during welding and is realised by using such procedures as transient Lagrangian or steady state Eulerian formulations of thermal cycle.

We can define Eulerian (moving) frame with origin at the centre of the source and co-ordinates (x, y, z). For cartesian co-ordinate system (x_0, y_0, z_0) which remains stationary for all time t and the loading history, the Lagrangian co-ordinate reference is defined.

Table 1. *Characteristic of change of the internal energy E as result of interaction of parameters φ and $\Delta\psi$*

Kind of interaction between φ and $\Delta\psi$	**Change of internal energy E**
thermal	T ΔS [T – temperature (φ), ΔS – entropy ($\Delta\psi$)],
mechanical	p ΔV [p – pressure (φ), ΔV – volume ($\Delta\psi$)],
chemical	μ_i Δm_i [μ_i – chemical potential (φ),Δm_i – mass of i-component ($\Delta\psi$)].

THEORETICAL BASES OF MODELLING OF WELDING PROCESSES

Practically, there are a few groups of modelling tasks, in dependence on which mathematical modelling elements are known:

- the direct task, when a reaction of system to outside factors is investigated, e.g. the results of heat flow inside steel plate from the moving point heat source,
- the indirect task, inverses to the previous one, when we want to obtain information how the conditions must change in order to get the required reaction of system, e.g. define temperature distribution in welding steel plate,
- the inductive task, when relying on series of measurements for various boundary conditions, mathematical form describing the real process is to be found, e.g. estimation of temperature distribution during layer stitch welding.

The definition of theoretical structure of research object is performed with the use of:

- the physic model, describing the actual object,
- the mathematical model, being an equation or system of equations, describing processes together with the boundary conditions, characteristic for given phenomenon.

The indispensable conditions for similarity existing between the models and actual objects is describing the happening physic processes by:

- the same differential equations with appropriate boundary and initial conditions,
- the similarity of criterion verification.

The direct task with indispensable conditions which have been described over are referred to as direct thermal problems. The main purpose of direct problems is to find the results, e.g. the temperature distribution in welded joints from the known differential equation, boundary and initial conditions, but the similarity of criterion verification are not used principally.

The another class of problems arises in indirect task when some parameters or conditions are either unknown or not fully specified. The unknown quantity is to be determined with the help of an extra condition are fulfil. Such a problem is termed an inverse problem and can be regarded as discovering the cause from a know result.

BASES OF MODELLING OF INVERSE HEAT CONDUCTION PROBLEM

The inverse heat conduction problem is much more difficult to solve than the direct problem because it is mathematically ill – posed problem [1, 2]. The mathematical ill-posed nature of inverse problems results from their physical nature. The ill-posed nature makes that various fine methods based on procedures used for direct problems are inapplicable to a wide range of inverse problems but the algorithm of the inverse heat conduction problem solution is based on the corresponding direct problem.

Currently predominate solutions of the heat transfer in welded joints base on the Fourier-Kirchhoff (F-K) equation:

$$\operatorname{div}(\lambda \operatorname{grad} T) - c_p \rho \frac{\partial T}{\partial t} = -q_v \tag{3}$$

where:

- T – temperature, °C or K,
- λ – thermal conductivity, W cm^{-1}K^{-1},
- c_p – specific heat, J kg^{-1}K^{-1},
- ρ – mass density, kg cm^{-3},
- t – time, sec,
- q_v – power input in volume, W cm^{-3}.

In this situation heat transport in welded joints is mainly progressed by the thermal conductions. A useful parameter for describing conditions of heat transport is the Peclet number [3]:

$$P_e = \frac{v \cdot l}{\alpha} \qquad \alpha = \lambda \cdot \rho^{-1} c_p^{-1} \tag{4}$$

where:

- v – constant velocity of transformation, cm s^{-1}
- l – characteristic length scale of the process, cm,
- α – thermal diffusivity, m s^{-2}.

The heat can be transported in liquid molten region – weld pool by both convection and conduction process. In the solid region heat is transported relative to material of the work piece by conduction process only [4]. The conduction becomes significant and dominate under welding process in weld pool as the Peclet number is much less than unity: $P_e < 1$. The only difference between conduction and convection lies in the type of particle movement [2]. When the particles demonstrate only atomic and molecular activity around ground level, energy is transported by conduction. The convective heat transfer requires the bulk motion of the medium in weld pool and involves the transport of energy due to gradient of temperature but also due to enthalpy transport, viscous dissipation, compression, etc. [2]. If the Peclet number is greater than 1 it is necessary to insert the additional term:

$$c_p\rho\left[\left(w_x - v\right)\frac{\partial T}{\partial x} + w_y\frac{\partial T}{\partial y} + w_z\frac{\partial T}{\partial z}\right] \tag{5}$$

where:

w_x
w_y – are the components of convective velocity w of the fluid in weld pool,
w_z
v – velocity of heat source in direction x.

to equation (3).

Besides, motion of the medium is described by the continuity equation and the Navier – Stokes equation and that simultaneously complicates solution of the modified eq. (3).

The main purpose of inverse heat conduction problem solution is to obtain the parameters of the welding process which agree satisfactorily with the observed or desired characteristic of the welded joints. Furthermore, the structures dimensions, material properties and boundary conditions are assumed to be known.

The general strategy of solving the inverse problems, suggested by Nowak [2], can be summarised by the following steps:

- to make the mathematical description of the boundary problem completion assuming arbitrary values as required by direct problem but not specified in inverse problem input date. In another words, to make the considered problem well – posed,
- to solve direct problem,
- to calculate values of measured quantity at measuring sensor locations,
- to compare calculated and measured values and to modify assumed input date to ensure the best matching of these quantities.

APPLICATION OF AN INVERSE – PROBLEM APPROACH TO WELDING PROCESSES

The direct heat conduction problem is solved by using an analytic method for plate with optional thickness by using the Fourier transformation method [5]. The cylindrical – involution – normal (C-I-N), three dimensional heat source (HS) model, is used in this study:

$$q_v = \frac{Q}{\pi\left(1-\exp\left(\left(-K_z \cdot s\right)\right)\right)} \cdot k \cdot K_z \cdot \exp\left(-k\left(x^2+y^2\right) - K_z \cdot z\right) \cdot \left(1 - u\left(z-s\right)\right) \tag{6}$$

where:

Q – net power received by the weldment, W,
k – a factor designating the HS concentration , cm^{-2},
K_z – involution factor of HS, cm^{-1},
s – HS penetration depth, cm,
$u(z\text{-}s)$ – Heaviside's function.

An analytic solutions for the temperature distribution in the welded joints are established [5, 6]. This is possible because the partial differential equation (3) is linear, the boundary condition for plate with optional thickness:

$$\frac{\partial T}{\partial x_0} = 0 \, ; \, x_0 \to \infty \, , x_0 \to -\infty \tag{7a}$$

$$\frac{\partial T}{\partial y_0} = 0 \, ; \, y_0 \to \infty \, , y_0 \to -\infty \tag{7b}$$

$$\lambda \frac{\partial T}{\partial z_0} = \alpha_0 T \, ; \, z_0 = 0 \tag{7c}$$

$$\lambda \frac{\partial T}{\partial z_0} = \alpha_1 T \, ; \, z_0 = g \tag{7d}$$

and the initial condition

$$T(x_0 \, , y_0 \, , z_0 \, , t = 0) = T_0 = 0 \tag{7e}$$

where:

- α_0 – the surface coefficient of conductance at $z_0 = 0$, W $cm^{-2}K^{-1}$,
- α_1 – the surface coefficient of conductance at $z_0 = g$, W $cm^{-2}K^{-1}$,
- g – the thickness plate, cm,

are consistent, the problem is well posed and the unique solution exists.

The obtained temperature field solution has an algebraic form and must be discretised in order to make computer calculations possible. For this purpose we will use calculations in Mathcad programme [7]. An account will concern of the pulsed power welding (PPW). In this account we will follow PPW analytical scheme for the time dependence of heat input q(t) – proposed by Karkhin el.al [8] – Fig 1.

In the case when pulses have idealised trapezium waveform, the function q(t) can be described by 5 parameters: high pulse (peak) power q_p, high pulse time (peak duration) t_p, low pulse (background) power q_b, low pulse time (background duration) time t_b, and slope-up and slope-down pulse time t_s.

The heat input is a function of time q(t) and this way will be included in Mathcad procedures, which are very useful for modelling and simulation of welding thermal process.

If parameters λ, c_p , ρ, α there are functions of temperature T, eq. (3) is nonlinear and it makes pure analytical calculation impossible. The nonlinear form of eq. (3) can be solved only by approximate manner , because it is mathematically complicated problem, and a pure nonlinear solution of eq. (3) does not exist.

Presently, the finite element method (FEM) has the best capability for nonlinear analysis of thermal cycle welding in approximate way. The another methodology and created hybride an analytic-numerical method is used in this solution which lead to similar results as FEM [9]. The basis of this method is a linear interpolation procedure. In calculating procedures there is no local fitting of the properties λ and α as in a pure numerical calculation. The idea in this solution in fact was to use the same general solution obtained for the linear equation of heat conduction, but use it repeatedly in respect of temperature change in every point. These solutions are valid only if the properties of λ and α are constant at any time t and of course these properties are constant but within given impulse affects lasting. In other words, in every time step we solve the equations as linear problem with $\lambda(T)$, $\alpha(T)$ and generate the results for temporary situation. Then we analyse the local temperatures after an impulse generation and change $\lambda(T)$, $\alpha(T)$ (and many other physical parameters) in respect of actual temperature. Providing that the time step "Δt" is very small, we are allowed to assume that this thermal phenomena are being analysed at non-linear principles. So, there is a specific and logic assumption performed in order to numerically approximate well the pure analytic solution. The details of this method are presented in [9]. Finally, the following computing expressions for heat flow solutions are obtained from pulsed C-I-N heat source model:

Stationary co-ordinates system

$$T(x_0, y_0, z_0, t) = \sum_{j=1}^{n} \mathrm{if}\left\{ t < (j-1)\cdot\Delta t, 0, \frac{q(t)\cdot k\cdot K_z}{\pi\cdot c_\gamma\cdot(1-\exp(-K\cdot s))}\cdot \right.$$
$$\frac{1}{4\cdot\alpha\cdot k\cdot(t-(j-1)+1)}\cdot\exp\left[\frac{-k\left((x_0-(j-1)v\cdot\Delta t)^2+y_0^2\right)}{4\cdot\alpha\cdot k\cdot(t-(j-1)\Delta t)+1}\right]\cdot \tag{8}$$
$$\left.\cdot\sum_{j=1}^{last} B_i\cdot C_i\cdot D_i\cdot\exp\left[-\alpha\cdot r_i^2\cdot(t-(j-1)\Delta t)\right]\right\}$$

Moving co-ordinates system

$$T(x, y, z, t) = \sum_{j=1}^{n} \mathrm{if}\left\{ t < (j-1)\cdot\Delta t, 0, \frac{q(t)\cdot k\cdot K_z}{\pi\cdot c_\gamma\cdot(1-\exp(-K\cdot s))}\cdot \right.$$
$$\frac{1}{4\cdot\alpha\cdot k\cdot(t-(j-1)+1)}\cdot\exp\left[\frac{-k\left((x-(j-1)v\cdot\Delta t)^2+y_0^2\right)}{4\cdot\alpha\cdot k\cdot(t-(j-1)\Delta t)+1}\right]\cdot \tag{9}$$
$$\left.\cdot\sum_{j=1}^{last} B_i\cdot C_i\cdot D_i\cdot\exp\left[-\alpha\cdot r_i^2\cdot(t-(j-1)\Delta t)\right]\right\}$$

where:

$$B_i = \cos(r_i\cdot z_0) + \frac{\alpha_0}{\lambda\cdot r_i}\cdot\sin(r_i\cdot z_0) \tag{10}$$

$$C_i = \frac{2\cdot r_i^2}{\left(\frac{\alpha_0^2}{\lambda^2}+r_i^2\right)\cdot\left(g+\frac{\alpha_1\cdot\lambda}{\alpha_1^2+r_i^2\cdot\lambda^2}\right)+\frac{\alpha_0}{\lambda}} \tag{11}$$

$$D_i = \exp(-K_z\cdot s)\cdot$$
$$\cdot\frac{\left(-K_z\cdot\cos(r_i\cdot s)\cdot\lambda\cdot r_i + r_i^2\cdot\sin(r_i\cdot s)\cdot\lambda-\alpha_0\cdot r_i\cdot\cos(r_i\cdot s)-\alpha_0\cdot K_z\cdot\sin(r_i\cdot s)\right)}{\left(K_z^2+r_i^2\right)\cdot\lambda\cdot r_i}+ \tag{12}$$
$$+\frac{K_z\cdot\lambda+\alpha_0}{\left(K_z^2+r_i^2\right)\cdot\lambda}$$

$$E_i = \int_0^g \left(\cos(r_i \cdot z) + \frac{\alpha_0}{\lambda \cdot r_i} \cdot \sin(r_i \cdot z) \right) \cdot \mathrm{approx}(z, c, \mathrm{nlast})\,dz \tag{13}$$

and

$r_1, r_2, r_3 \; . \; . \; . \; r_i$ are roots of:

$$\mathrm{ctg}(r_i \cdot g) = \frac{\lambda^2 \cdot r_i^{\,2} - \alpha_0 \cdot \alpha_1}{\lambda \cdot r_i \cdot (\alpha_0 + \alpha_1)} \tag{14}$$

$c_\gamma = c_p\, \rho$ is a volumetric specific heat, J K^{-1} cm^{-3}.

For accounts of distribution of the temperature fields in welded joints there is the necessity of physical parameters such as λ(T), α(T). The discrete values of λ(T), α(T) are known and shown in Tab. 2 the matrices containing and corresponding λ(T), α(T) values are defined experimentally.

Table 2. *λ(T), c_p(T), ρ(T), and α(T) values in several temperatures for low carbon steel–0,1%C*

T °C	λ(T) $\lambda Wcm^{-1}K^{-1}$
0	0,6285
100	0,5866
200	0,5447
300	0,5028
400	0,4609
500	0,4190
600	0,3771
700	0,3477
800	0,3268
900	0,3226
1000	0,3268
1100	0,3310
1200	0,3352
1300	0,3352
1400	0,3352
1500	0,3352

T °C	ρ(T).c_p (T) $JK^{-1}cm^{-3}$
0	3,307
100	3,666
200	4,190
300	4,570
400	4,950
500	5,303
600	6,082
700	6,955
768	9,809
800	6,536
900	5,866
901	5,204
1200	5,406
1300	5,406
1400	5,406
1500	5,406

T °C	α (T) cm^2s^{-1}
0	0,190
100	0,160
200	0,130
300	0,110
400	0,093
500	0,079
600	0,062
700	0,050
768	0,034
800	0,042
900	0,055
901	0,062
1200	0,062
1300	0,062
1400	0,062
1500	0,062

Than with use of linear interpolation procedure, continuous functions were created and built in inside calculation sheet in Mathcad programme as follows:

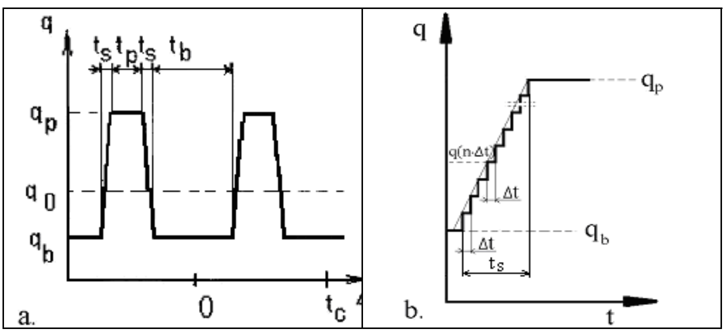

Fig. 1. Schematic diagram of pulsed power: a. course of function q(t) and her characteristic dimensions, b. details of course of function q(t) for t_s

- sub-procedure λ(T)

A =

⟨1⟩	⟨2⟩
0	0,6285
100	0,5866
200	0,5447
300	0,5028
400	0,4609
500	0,419
600	0,3771
700	0,3487
800	0,3268
900	0,3226
1000	0,3268
1100	0,331
1200	0,3352
1300	0,3352
1400	0,3352
1500	0,3352

$$\lambda(T) := \left| \begin{array}{l} i \leftarrow \mathrm{floor}\left(\frac{T}{100}\right) \\ i \leftarrow 14 \text{ if } T \geq 1500 \\ \left[\frac{\left(A_{i+2,2} - A_{i+1,2}\right)}{\left(A_{i+2,1} - A_{i+1,1}\right)} \cdot \left(t - A_{i+1,1}\right) + A_{i+1,2} \right] \end{array} \right. \tag{15}$$

$$T := 1 \ldots 1500$$

- sub-procedure α(T)

C =

⟨1⟩	⟨2⟩
0	0,19
100	0,16
200	0,13
300	0,11
400	0,093
500	0,079
600	0,062
700	0,05
768	0,034
800	0,042
900	0,055
901	0,062
1200	0,062
1300	0,062
1400	0,062
1500	0,062

$$\alpha(T) := \left| \begin{array}{l} i \leftarrow \mathrm{floor}\left(\frac{T}{100}\right) \quad \text{if} \quad T < 768 \\ i \leftarrow 8 < \text{if } T \geq 768 \\ i \leftarrow 9 < \text{if } T \geq 800 \\ i \leftarrow 10 < \text{if } T \geq 900 \\ i \leftarrow 11 < \text{if } T \geq 901 \\ i \leftarrow 12 < \text{if } T \geq 1200 \\ \left[\frac{\left(C_{i+2,2} - C_{i+1,2}\right)}{\left(C_{i+2,1} - C_{i+1,1}\right)} \cdot \left(t - C_{i+1,1}\right) + C_{i+1,2} \right] \end{array} \right. \tag{16}$$

$$T := 0 \ldots 1500$$

On Fig. 2 discrete values of $\lambda(T)$, $\alpha(T)$ are presented and determined by continuous functions with used sub-procedures (15) and (16) for a. $\lambda(T)$, b. $\alpha(T)$.

We have high conformity of continuous functions and discrete value of $\lambda(T)$, $\alpha(T)$ from above-mentioned date on Fig. 2.

The final main-procedure "Temperature – T", in accordance with eqs. (8) or (9), summarises thermal fields from several heat energy impulses using a proper formula specific for appropriated H-S model. In Mathcad programme, the heat source pulses have trapezium waveform and are represented by the following sub-procedure:

$$q(t) = \left| \begin{array}{ll} q_b \text{ if } \mathrm{mod}\left(t-w, t_p+t_b+2\cdot t_s\right) \le 0 & \text{cond. 1} \\ \dfrac{q_p-q_b}{t_s}\mathrm{mod}\left(t-w, t_p+t_b+2\cdot t_s\right)+q_b \text{ if } 0 < & \text{cond. 2} \\ < \mathrm{mod}\left(t-w, t_p+t_b+2\cdot t_s\right) < t_s & \\ q_b \text{ if } \mathrm{mod}\left(t-w, t_p+t_b+2\cdot t_s\right) < t_s+t_p & \text{cond. 3} \\ \dfrac{q_p-q_b}{t_s}\mathrm{mod}\left(t-w, t_p+t_b+2\cdot t_s\right)+q_b \text{ if } t_s+t_p < & \\ < \mathrm{mod}\left(t-w, t_p+t_b+2\cdot t_s\right) < 2\cdot t_s+t_p & \text{cond. 4} \\ q_b \text{ if } 2\cdot t_s+t_p \le \mathrm{mod}\left(t-w, t_p+t_b+2\cdot t_s\right) < t_s+t_p+t_b & \\ & \text{cond. 5} \end{array} \right. \qquad (17)$$

where: w – dead pulse time

"mod(a,b)" is a function giving the rest of dividing "a" by "b" ($b \neq 0$).

Let's analyse the structure of (17) which may look a little complicated at the first look.

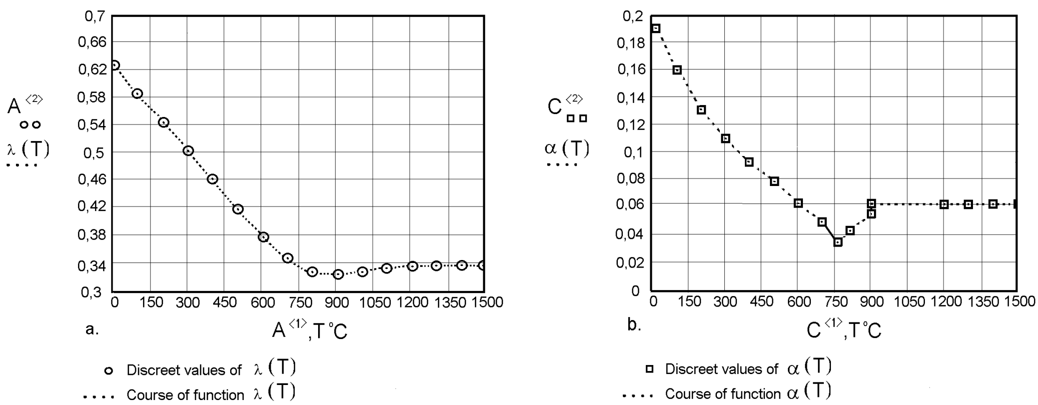

Fig. 2. Values of a. $\lambda(T)$ and b. $\alpha(T)$ in agreement with Tab. 1 and continuous functions for low carbon steel 0,1% C

There are five "if" conditions. The result of conditions no. 1, 3 and 5 is as seen below (peak and background values reflected) – Fig. 3.

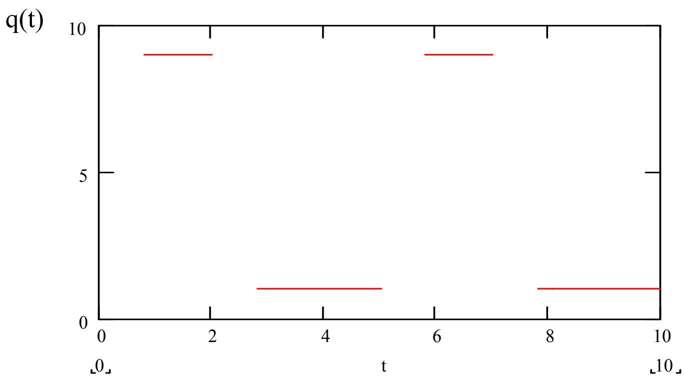

Fig. 3. Peak and background values of q(t) (conditions no. 1, 3, 5 assumed)

Whereas, the result of conditions no. 2 and 4 (slope and down) is seen below – Fig. 4.
Eqs. (8), (9) with appropriated sub-procedures (15), (16), (17) and date in Tab. 2 can be directly applied to solving an inverse problem.

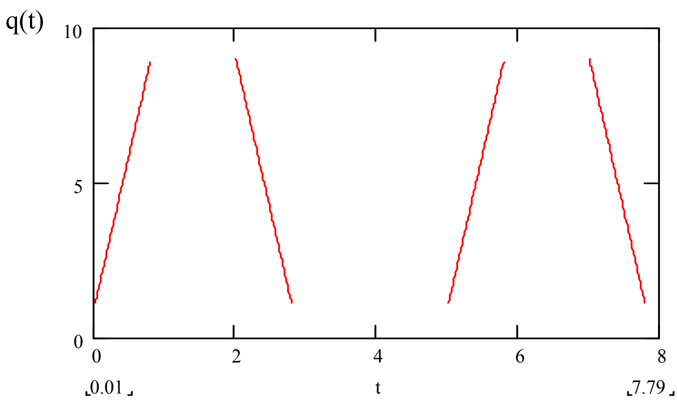

Fig. 4. Slope and down lines of q(t) characteristic (conditions no. 2 and 4 assumed)

CONCLUSIONS

In this work some extended consideration about analytic-numerical methods conforming has taken place. The temperature fields generated by three dimensional C-I-N heat source with pulsed power welding in both stationary and moving co-ordinates systems are established. This is possible through employment of hybrid analytical-numerical method and new methodology of non-linear calculation described in this paper. Finally, it bases on the strategy of solving inverse problems, the appropriate algorithms are established in moving and stationary systems, which can be directly applied to solving inverse problem with used Mathcad programme.

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