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QUANTITATIVE EVALUATION OF THE FRACTURING PROCESS OF NATURAL COMPOSITE ON EXAMPLE OF THE CORTICAL PART OF BONE

ABSTRACT

Modelling of local failures of the natural composite – bone is based on the solution of a number of problems involving the interaction of macrocrack with voids, crazes and micro cracks in heterogeneous material. Base upon the concept of microcrack precursor nucleation, a micromechanics model is used to study the formation of near-tip microcracking and consequent toughening effects. ultimately it causes a redistribution of stresses in the near-tip stress field of the macrocrack and this can affect the stress intensity factor (SIF) CTOD and fracture toughness..

***Key words**: natural composite, stress state, constraint effect.*

INTRODUCTION

The fracture mechanics methodology is based on the assumption that all materials, also natural composites, contain cracks from which failure starts. The estimation of the remaining life or structural components requires the knowledge of the redistribution of stresses caused by the introduction of cracks in conjunction with a crack growth condition. The general solution of the stress and displacement fields in the vicinity of the crack tip in anisotropic composites was first derived by Sih et al. [1]. The realistic characterisation of the failure of fibber composites is necessary to consider the heterogeneous nature of the materials. The number of local failure modes such as matrix cracking and interface debonding precede fracture. They absorb a large amount of the energy supplied to composite and delay the formation of the large crack leading to instability.

STRESS STATE IN THE VICINITY OF A MACROCRACK

The damage distribution and damage zones just ahead of macrocrack affect the crack propagation and conventional fracture parameters such as CTOD and SIF. This is because these parameters are dependent on the effective length and effective direction of the microcracks and their interactions, on the stability of void growth and on the coalescence process of the microcracks. The experimental evidence showed that instead of continuous separation, microcracks and voids are created and enlarged to link with the macrocrack. Kachanov et al. [2] described these defects (voids, microcracks) via a statistical distribution as it is illustrated in *Fig. 1*. Furthermore, it should also be referred that Kachanov established that the zone of "short range" interactions plays more dominant role compared to the zone of the "long range" interactions.

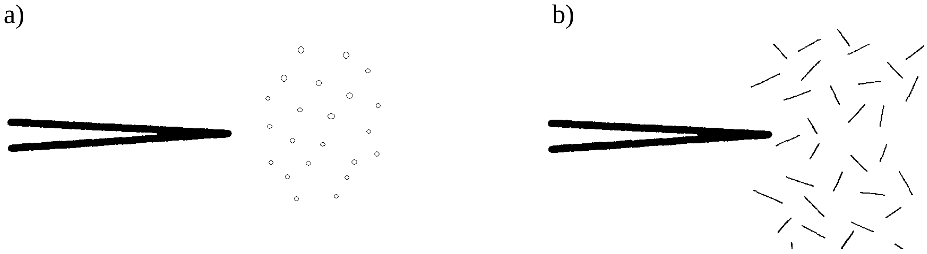

Fig. 1. Statistical distribution of: a) voids, b) microcracks in the vicinity of the macrocrack

Let's consider influence of the state of stress at the macrocrack with an infinitesimal path deflection at its tip – *Fig. 2*.

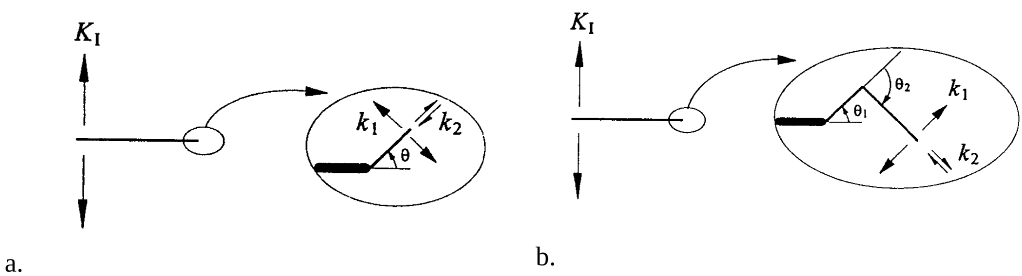

Fig. 2. Macrocrack with microcrack path defections: a) simple kink with kink angle Θ, b) double kink with kink angle Θ_1 and Θ_2

Based on the solution given by Williams [3], the stress state in a polar coordinate description – *Fig. 3* for case of a single kinked deflection, can be written as follows [4]:

$$\sigma_{\Theta} = C_{11}^{k}\frac{K_I}{\sqrt{2\pi r}} + C_{12}^{k}\frac{K_{II}}{\sqrt{2\pi r}} = \frac{k_1}{\sqrt{2\pi r}} \tag{1}$$

$$\tau_{r\Theta} = C_{21}^{k}\frac{K_I}{\sqrt{2\pi r}} + C_{22}^{k}\frac{K_{II}}{\sqrt{2\pi r}} = \frac{k_2}{\sqrt{2\pi r}} \tag{2}$$

where $C_{ij}^{k}(i = 1, 2\,;\, j = 1, 2)$: are the coefficients and defined by:

$$C_{11}^{k} = \cos^{3}\frac{\Theta}{2}, \qquad C_{12}^{k} = -3\sin\frac{\Theta}{2}\cos^{2}\frac{\Theta}{2}$$

$$C_{21}^{k} = \sin\frac{\Theta}{2}\cos^{2}\frac{\Theta}{2}, \qquad C_{22}^{k} = \cos\frac{\Theta}{2}\left(1-3\sin^{2}\frac{\Theta}{2}\right)$$

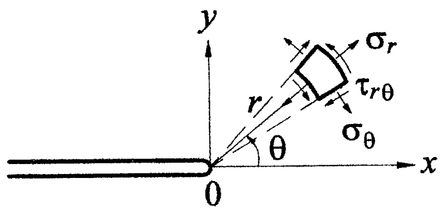

Fig. 3. Stress field of a crack in coordinate polar system

The equations (1) and (2) contain symmetrical and antisymetrical components, which match respectively to the Modes I and II cracks. Besides eqs. (1) and (2) can be re-expressed as:

$$k_1 = C_{11}^{k}(\Theta)K_I + C_{12}^{k}(\Theta)K_{II} \tag{3}$$

$$k_2 = C_{21}^{k}(\Theta)K_I + C_{22}^{k}(\Theta)K_{II} \tag{4}$$

In above eqs. (1) ÷ (4) K_I i K_{II} denote the SIFs for macrocracks, and likewise, k_1 and k_2 are the SIFs for the microcracks for symmetrical and antisymetrical loading of cracks, respective to the Mode I and Mode II. Beside, the angle Θ can be taken as the kink angle is shown in *Fig. 2a*.
For a double kinked microcrack with kink angle Θ_1 and Θ_2 – *Fig. 2b*, the corresponding k_1 and k_2 are as follows:

$$\begin{Bmatrix} k_1 \\ k_2 \end{Bmatrix} = \begin{bmatrix} C_{11}^{k}(\Theta_1) & C_{12}^{k}(\Theta_1) \\ C_{21}^{k}(\Theta_1) & C_{22}^{k}(\Theta_1) \end{bmatrix} \begin{bmatrix} C_{11}^{k}(\Theta_2) & C_{12}^{k}(\Theta_2) \\ C_{21}^{k}(\Theta_2) & C_{22}^{k}(\Theta_2) \end{bmatrix} \begin{Bmatrix} K_I \\ K_{II} \end{Bmatrix} \tag{5}$$

In this situation is entitled affirmation that the damage performance in the vicinity of the crack tip produces a shielding effect on the SIF for Mode I loading. In accordance with it the effective SIF at the tip of the deflected crack tip, K_{ef}, Mode I and Mode II contributions in terms of the strain energy realises rate, as:

$$K_{ef} = \left(k_1^2 + k_2^2\right)^{1/2} \tag{6}$$

It suggests that the value of the stress intensity at the crack tip is reduced locally due to such defection. Whereas this appears to be fairly significant contribution to the toughness. The crack deflection is promoted by feature in the microstructure that deviates the crack path from this plane, e.g., by crack deflection at hard particles or the crack path having an affinity for a specific dispersed phase. We think that there is main reason for acceptance of the assumption that fracture in bone is strain-controlled, i.e. ductile fracture akin to the process seen in structural steels at high temperature. Beside, also it confirms the marked anisotropy in the toughness of bone. Using a double – notched, four-point bend test – *Fig. 4.*, Ritchie and his co-workers [5] were able to examine the nature of a local fracture event in cortical bone at the onset of failure. The double-notched four point bend test was used to discern whether fracture is stress – or strain–controlled. Scanning electron micrographs revealed that is not only the fracture strain–based, but also confirmed the marked anisotropy in the toughness of bone (see [5, 6]).

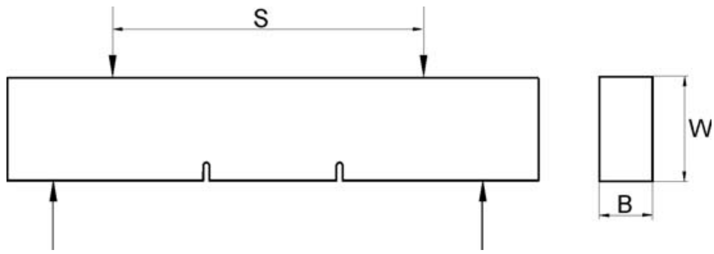

Fig. 4. The double-notched four-point bend test

For example the fracture toughness K_C values of bone is equal [5]:

- the transverse orientation $K_C = 5.33(\pm 0.41)\,\mathrm{MPa\,m^{1/2}}$;
- the antiplane longitudinal orientation $K_C = 2.21(\pm 0.18)\,\mathrm{MPa\,m^{1/2}}$;
- the in-plane longitudinal orientation $K_C = 3.53(\pm 0.13)\,\mathrm{MPa\,m^{1/2}}$.

This experimental investigation also confirmed the above results of theoretical analysis of stress state. From the physical point of view the heterogeneous nature of the cortical part of bone under Mode I loading evokes also the local change of mechanical properties caused by the constraint effect.

INFLUENCE OF THE CONSTRAINT EFFECT ON THE FRACTURE OF BONE

The theory of constrained materials in classical mechanics of deformable media is characterised by restriction of the class of possible motions. The constraint effect in the heterogeneous microstructure of bone and the level of constraint depend on the physical phenomena which occur at the interface of zone at different mechanical properties and their geometric configuration. Furthermore, the importance of “constraint” in the analysis of the notched or cracked bodies, also bones, has been recognised by many investigators.

For example, Ritchie et al. [5] indicated that the crack initiation and crack growth out the notch was not in the direction normal to the max tensile stress, but rather in the direction of the osteons, consistent with remark that the osteonal cement lines, which are the *interface between the osteonal system and the surrounding matrix.* The authors of this paper think that important influence in above mentioned phenomenon has also here the constraint effect.

The investigations of the near-tip region of macrocrack indicate complexity of above-mentioned question because they also revealed evidence of the uncracked – ligament bridging, as indicated in *Fig. 5,* which contributes to toughening mechanism in cortical bone.

Fig. 5. The evidence of uncracked – ligament bridging under fracture process of cortical bone

Constraint refers to the build-up of stress around a crack front due to the restraint against in-plane and out- of plane deformation. The analysis of failure in a structural component depends on two inputs, the deformation and fracture behaviours-both depend on constraint. Concern must be given to the effect of constraint on deformation behaviour, especially in the non-linear region of behaviour. The manner and extent to which constraint affects the failure behaviour of the structure depend on the type of the fracture and deformation. The magnitude of the load in the region of non-linear deformation is strongly influenced by constraint.

CONCLUSIONS

1. Realistic characterisation of the failure of the cortical bones must take into account the heterogeneous nature of the materials.
2. The microdefects interacting with the macrocrack and with each other, change the stress distribution.
3. The redistribution of stress in the near-tip stress field of the macro-crack caused that it can affect on the SIF and fracture toughness under Mode I loading.
4. The deformation process of bone and fracture parameters calibrations are influenced also by constraint effect. It is therefore important to determine the deformation behaviour and fracture parameters as a function of constraint.

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