ABSTRACT: Isochoric response of the hypoplastic constitutive model is improved by means of small modifications in the linear stiffness tensor and in the barotropy function. Comparisons with the Rowe theory of dilatancy and with the experimental results by Pradhan et al. are presented. The essential modification consists in increasing of shear stiffness. Moreover two anisotropic effects have been implemented: one in the linear part and one in the definition of barotropy factor. The modified hypoplastic model is presented using isomorphic variables $P, Q$.

1 INTRODUCTION
Hypoplastic constitutive models originate from a formalism alternative to elastoplasticity. First models were formulated independently in Karlsruhe (Kolymbas 1978) and in Grenoble (Chambon 1982) in late seventies. Numerous versions, improvements and applications have made evident the attractiveness of this approach. The 'Karlsruhe'-hypoplastic models for sand are based on a single equation (25), or similar, expressing the stress rate $\dot{T}$ as a nonlinear function of stretching $\mathbf{D}$, stress $\mathbf{T}$ and of the void ratio $e$. Readers familiar with the endochronic theory may discover that hypoplasticity is closely related with the early Valanis models (Valanis 1971a; Valanis 1971b) with a single kernel and with an expression for stress written directly in the rate form. Equations of a contemporary hypoplastic model (Gudehus 1996; Wolfersdorf 1997) are given in Appendix 2. An extensive discussion of this model is given in the monography (Niemunis 2003). Notation is explained in Appendix 1.

The reference model presented in Appendix 2 has no inherent anisotropy. All material constants are independent of the orientation of the coordinate system. The only tensorial state variable is stress $\mathbf{T}$ which induces incremental orthotropy with the planes of symmetry given by the eigenvectors of $\mathbf{T}$. Choosing the initial stress to be isotropic, $\mathbf{T}_0 \sim \mathbf{I}$, the total hypoplastic response can be considered isotropic. Otherwise it is orthotropic with symmetry planes given by eigenvectors of $\mathbf{T}_0$.

In elastoplasticity the back-stress anisotropy is usually implemented by means of nested loading sub-surfaces, e.g. (Mróz and Norris 1982; Daflia and Hermann 1982; Hashiguchi 1988). In hypoplasticity we may superpose several equations similar to (25) for partial stresses as proposed by Valanis (Valanis and Lee 1984; Niemunis 2003) or alternatively we may use the so-called intergranular strain tensor $\mathbf{h}$, (Niemunis and Herle 1997). This strain-like tensor $\mathbf{h}$ is pointing in the direction of the recent deformation, which is used to increase the tangential stiffness after rapid reverses of the strain path. Tensors $\mathbf{h}$ and $\mathbf{T}$ are not necessarily coaxial so the hypoplastic model with the intergranular strain has no symmetry group. Relatively small changes of strain suffice to erase this 'short range' $\mathbf{h}$-anisotropy and therefore one associates it with the orientation of grain contacts whereas the textural anisotropy results from the orientation of nonspherical grains (Oda 1972b). The analysis of directional data leads to anisotropy tensors of high order (Kanatani 1984).

Measurements of the back-stress anisotropy require tests with bender elements, e.g. (Bellotti, Jamiolkowski, Lo Presti, and O'Neill 1996) or recording of acoustic emission from individual grain breaking or slippage (Oda and Iwashita 1999) pp.270-276. Measurements of textural anisotropy are easier. It has been widely reported in the literature (Arthur and Mencies 1972; Oda 1972a; Tatsuoka, Toki, Miura, Kato, Okamoto, Yamada, Yasuda, and Tanizawa 1986), that the peak strength of natural sand deposits is higher if compression is perpendicular to the bedding plane. This effect cannot be attributed either to the initial stress or to the back-stress anisotropy. A proper distinction should be made between anisotropic effects due
to back-stresses, due to stress and due to the texture (Mróz and Niemunis 1987). Here an implementation of the textural anisotropy to hypoplasticity is attempted. For simplicity the intergranular strain disregarded.

Inherent anisotropy has already been implemented to hypoplasticity (Wu 1998) to modify the nonlinear part of (25). Tensor \( \mathbf{N} \) has been replaced by \( \mathbf{A} : \mathbf{N} \) wherein \( \mathbf{A} \) is the fourth order tensor of transverse isotropy. Originally (Boehler and Sawczuk 1977) the idea was to modify stress. Expression \( \mathbf{A} : \mathbf{T} \) was used to replace \( \mathbf{T} \) in the stress rate function \( \dot{\mathbf{T}}(\mathbf{T}) \). If \( x_3 \) coincide with the privileged direction perpendicular to the bedding plane then \( A_{1111} = A_{2222} = 2A_{1232} = \gamma, \ A_{3333} = \alpha \) and \( 2A_{1313} = 2A_{2323} = \beta \). Other non-zero components are obtained from the major and minor symmetries and \( \alpha, \beta, \gamma \) are material constants.

The Wu model is attractive due to its simplicity but it does not provide a desirable independent control over the dilatancy, strength and stiffness. Moreover, this model is based on an old hypoplastic equation which does not distinguish between the peak and the residual response. The idea was to modify stress. Expression \( \mathbf{A} : \mathbf{T} \) was used to replace \( \mathbf{T} \) in the stress rate function \( \dot{\mathbf{T}}(\mathbf{T}) \). If \( x_3 \) coincide with the privileged direction perpendicular to the bedding plane then \( A_{1111} = A_{2222} = 2A_{1232} = \gamma, \ A_{3333} = \alpha \) and \( 2A_{1313} = 2A_{2323} = \beta \). Other non-zero components are obtained from the major and minor symmetries and \( \alpha, \beta, \gamma \) are material constants.

The Wu model is attractive due to its simplicity but it does not provide a desirable independent control over the dilatancy, strength and stiffness. Moreover, this model is based on an old hypoplastic equation which does not distinguish between the peak and the residual response. The idea was to modify stress. Expression \( \mathbf{A} : \mathbf{T} \) was used to replace \( \mathbf{T} \) in the stress rate function \( \dot{\mathbf{T}}(\mathbf{T}) \). If \( x_3 \) coincide with the privileged direction perpendicular to the bedding plane then \( A_{1111} = A_{2222} = 2A_{1232} = \gamma, \ A_{3333} = \alpha \) and \( 2A_{1313} = 2A_{2323} = \beta \). Other non-zero components are obtained from the major and minor symmetries and \( \alpha, \beta, \gamma \) are material constants.

The expression in brackets varies between 0 and 1 so for \( n \to \infty \) an elastic-perfectly-plastic response is obtained.

For an axially symmetric case a helpful graphic interpretation of \( \mathbf{N} \) and \( \mathbf{L} \) has been proposed (Gudehus 1979) in the form of polar diagrams of stiffness termed the response envelopes. In the past numerous versions of hypoplastic models have been proposed and the shape and orientation of the response envelopes (plotted for different stresses) were used to judge about their quality. For the present purposes, it is convenient to modify the hypoplastic model referring to the traditional notions of stiffness, yield surface and flow rule. Unfortunately, \( \mathbf{L} \) and \( \mathbf{N} \) describe them collectively and it is rather difficult to modify (25) in order to improve just the flow rule without deteriorating others features. To circumvent this problem (25) is now brought to a more convenient form using the scalar \textit{degree of nonlinearity} \( 0 < Y < 1 \) and the unit 2-nd order tensor \( \mathbf{m} \) describing the direction of flow. Both \( Y = Y(\mathbf{T}) \) and \( \mathbf{m} = \mathbf{m}(\mathbf{T}) \) are functions of stress obliquity \( \dot{T} = \mathbf{T}/tr\mathbf{T} \) only. Modified equation (25) is proposed in the form

\[
\dot{T} = \mathbf{L} : (\mathbf{D} - f_d Y \mathbf{m} || \mathbf{D} ||) \quad (5)
\]

which is chosen to be the basic one, i.e., \( \mathbf{m} \) and \( Y \) are \textit{independent} functions. In (25) the analogous terms \( Y = ||\mathbf{B}|| \) and \( \mathbf{m} = -\mathbf{B} \) are coupled via \( \mathbf{B} = \mathbf{L}^{-1} : \mathbf{N} \). Elements of (5) can be conveniently molded as demonstrated in the following. Of course, incrementally linear response corresponds to \( f_d Y = 0 \) and \( f_d Y = 1 \) with \( \mathbf{D} \sim \mathbf{m} \) leads to \( \mathbf{T} = 0 \), i.e. to perfect plastic flow.

### 3 HYPOPLASTIC MODEL IN \( P - Q \) SPACE

For axially symmetric case with \( T_y = T_3 \) and \( D_y = D_3 \) the isomorphic stress variables \( \mathbf{P} = -\frac{1}{\sqrt{3}}(T_1 + T_2 + T_3) \) and \( Q = -\sqrt{\frac{2}{3}}(T_1 - T_2) \) may be used, analogously to the popular Roscoe variables \( p = \frac{1}{3}(T_1 + T_2 + T_3) \) and \( q = -(T_1 - T_2) \). The strain rate components which are work-conjugated with \( \mathbf{P} \) and \( Q \) are \( D_P = -\frac{1}{\sqrt{3}}(D_2 + D_3 + D_3) \) and \( D_Q = -\frac{\sqrt{2}}{3}(D_1 - D_2) \) respectively\(^1\). In the present context such isomorphic variables are convenient because apart from preserving power

\[
\mathbf{T} : \mathbf{D} = PD_P + QD_Q \quad (= pD_v + qD_q) \quad (6)
\]

they fulfill conditions

\[
\mathbf{T} : \mathbf{T} = P^2 + Q^2 \quad \text{and} \quad \mathbf{D} : \mathbf{D} = D_P^2 + D_Q^2 \quad (7)
\]

\(^1\)\(D_v = -(D_2 + D_3 + D_3) \) and \( D_y = -\frac{2}{3}(D_1 - D_2) \) are components conjugated with \( p \) and \( q \). We have \( P = \sqrt{3}p, \quad Q = \sqrt{\frac{2}{3}}q, \quad D_P = \frac{1}{\sqrt{3}}D_v, \quad D_Q = \sqrt{\frac{2}{3}}D_q \).
The normal to the yield surface in stress space is not perpendicular to the contour of the yield surface in the \( p - q \) plane but it is in the \( P - Q \) plane. Coulomb yield condition \( (T_{\text{max}} - T_{\text{min}})/(T_{\text{max}} + T_{\text{min}}) \leq \sin \varphi \) corresponds to

\[
M_E = -\frac{2\sqrt{2}\sin \varphi}{3 + \sin \varphi} \leq \eta \leq \frac{2\sqrt{2}\sin \varphi}{3 - \sin \varphi} = M_C
\]  

wherein \( \eta = Q/P \). Expression (29) is simply

\[
F = \begin{cases} 
1 & \text{if } \eta > 0 \\
1 + \eta/\sqrt{2} & \text{if } \eta < 0 
\end{cases}
\]  

and the hypoplastic model for \( P - Q \) space has the form

\[
\left\{ \begin{array}{c}
P \\
Q
\end{array} \right\} = \frac{f_b f_e}{\frac{2}{3}(1 + \eta^2)} \left[ \frac{F^2 + \frac{1}{3}a^2}{\frac{2}{3}a^2} \eta \right] \left( \begin{array}{c}
D_P \\
D_Q
\end{array} \right) + \frac{f_b f_e f_d}{\frac{2}{3}(1 + \eta^2)} \frac{aF}{\sqrt{3}} \left( \begin{array}{c}
-1 \\
-2\eta
\end{array} \right) \sqrt{D_P^2 + D_Q^2}
\]  

wherein functions \( f_b(), f_e() \) and \( f_d() \) are given in Appendix 2.

4 DILATANCY IN HYPOPLASTICITY

In hypoplasticity the dilatancy \( d \) is defined as the ratio of the total volumetric strain rate to the total deviatoric strain rate because the plastic portion of strain rate is not defined. The peak dilatancy is dictated by the direction of \( \mathbf{m}(\hat{T}) \), i.e.

\[
d = \frac{\sqrt{3}D_P}{D_Q} = \frac{\text{tr} \mathbf{m}}{||\mathbf{m}||} = \frac{\sqrt{3}m_P}{m_Q}
\]  

which is a stress function independent on the void ratio \( e \). This independence is in accordance with the postulate of Rowe dilatancy theory (Rowe 1962) and does not contradict the fact that the peak dilatancy is higher for dense sands and lower for loose ones. Lower void ratios allow for higher stress ratios for which the direction \( \mathbf{m} \) is more dilatative. For a given stress ratio \( \hat{T} \), however, the dilatancy is independent\(^2\) of the void ratio.

In the hypoplastic model the peak friction angle is made dependent on density by means of barotropy factor \( f_d \) defined in (31). The critical surface is given by the condition \( Y(\hat{T}) = 1 \) which coincides with \( \text{tr} \mathbf{m} = 0 \). The yield surface, i.e. the condition of perfect flow \( \mathbf{T} = 0 \) and \( \mathbf{D} \neq 0 \), corresponds to the condition

\[
f_dY = 1
\]  

\(^2\)In some recently proposed models, e.g. (Li and Dafalias 2000), this assumption is left out.

which coincides with the critical yield surface for \( f_d(e, p) = 1 \), i.e., for \( e = e_c \). The flow rule is dilative (\( \text{tr} \mathbf{m} > 0 \)) above the critical surface, \( Y > 1 \), and contractive (\( \text{tr} \mathbf{m} < 0 \)) below this surface, \( Y < 1 \), see Fig. 1. Therefore shearing of loose sand \( (e > e_c, f_d > 1) \) is accompanied by volumetric hardening (contractancy) and shearing of dense sand \( (e < e_c, f_d < 1) \) - by softening (dilatancy), until the critical state void ratio

\[
e = e_c(P)
\]  

is reached. This asymptotic behaviour with limit (13) pertains also to undrained tests for which instead of dilatancy (contractancy) an increase (decrease) of effective mean stress occurs during shearing. Perfect flow corresponds to \( Y = 1 \) and \( f_d = 1 \). As shown in Section 6 the notions 'loose' and 'dense' in the sense \( e \geq e_c(P) \) are oversimplified.

The stress response to isochoric shearing given by most hypoplastic models is inaccurate, especially in the vicinity of hydrostatic axis. The predicted decrease of effective mean stress \( P/\sqrt{3} \) (or build up of pore pressure) is much too high, see Fig. 2.

In the past the overly contractive hypoplastic dilatancy rule was blamed for this deficiency and several remedies to this problem has been already proposed. The problem was to reduce the influence of the nonlinear term \( \mathbf{N} \) in case of deviatoric deformation and to preserve \( \mathbf{N} \) in case of isotropic deformation in order to keep isotropic extension stiffer than isotropic compression. One proposition was to use

\[
\hat{T} = f_e f_b \hat{\mathbf{L}} : \left\{ \mathbf{D} - f_dY\left[ (1 - w) \mathbf{m}||\mathbf{D}|| + w \mathbf{m}||\mathbf{D}|| \right] \right\}
\]  

In the nonlinear term we have interpolation between the original term \( \mathbf{m}||\mathbf{D}|| \) and \( \mathbf{m}||\mathbf{D}|| \) with \( \mathbf{m} : \mathbf{D}^* = 0 \) on the \( P \)-axis. The weighting function \( w \in (0, 1) \) vanishes as the stress approaches the yield surface \( Y = 1 \). Another way to weaken \( \mathbf{N} \) upon deviatoric path consists in replacing \( \mathbf{m}||\mathbf{D}|| \) in (14) by the following...
The overlooked aspect in the above propositions is the fact that the dilatancy angle measured in the strain space was actually well(!) predicted by the original hypoplastic model by v.Wolffersdorff. At the beginning (point *) of isochoric shearing pore pressure generation is too high. The stress path should be steeper there. All markers are placed after every 1% deformation.

The prediction of the hypoplastic models does not depend on void ratio \( e \). In this picture \( \frac{Q}{P} = \frac{3Q}{\sqrt{2}P} \) and \( \frac{\dot{v}}{\dot{\gamma}} \approx \frac{DP}{\sqrt{2}DQ} = -\sqrt{\frac{2}{a}} \)

stress \( \eta = 0 \). In particular we ask what Poisson ratio would be equivalent to the ratio of shear stiffness and volumetric stiffness upon unloading calculated with the hypoplastic model for \( f_d = 1 \). \( \eta = 0 \) and \( \varphi = 30^\circ \). From this comparison follows that

\[
\frac{P}{Q} = 1 + \frac{1}{3}a^2 + \frac{a}{\sqrt{3}} = \frac{(1 + \nu)}{(1 - 2\nu)}
\]

which means that the quasi elastic response in hypoplasticity corresponds to the Poisson ratio

\[
\nu \approx 0.38
\]

This value seems to be too large. Recent experimental data (Koseki, Kawakami, Nagayama, and Sato 2000) reveal much lower typical values of \( \nu = 0.15 \) for Toyoura sand. For Hostun sand even lower value of \( \nu = 0.1 \) has been assume (Gajo and Wood 1999).

Let us modify the hypoplastic model increasing the shear stiffness component \( L_{QQ} = F^2 + \frac{1}{3}a^2\eta^2 \) of the hypoplastic model keeping \( m \) unchanged. The reference hypoplastic equation (10) rewritten in terms of \( L \) and \( m \) takes the form

\[
\begin{bmatrix}
\dot{P} \\
\dot{Q}
\end{bmatrix} = f_s \begin{bmatrix}
F^2 + \frac{1}{3}a^2 & \frac{1}{2}(a^2\eta + \frac{1}{\eta}) \\
\frac{1}{2}(a^2\eta + \frac{1}{\eta}) & F^2 + \frac{1}{3}a^2\eta^2 + \frac{1}{1 - \eta^2}
\end{bmatrix} \begin{bmatrix}
\frac{m_P}{m_Q} \\
\frac{m_D}{m_Q}
\end{bmatrix} \sqrt{D_c^2 + D_Q^2}
\]

Figure 2: Undrained paths for different initial \( e/e_c \) values predicted by hypoplastic model by v.Wolffersdorff. At the beginning (point *) of isochoric shearing pore pressure generation is too high. The stress path should be steeper there. All markers are placed after every 1% deformation.

Figure 3: Dilatancy diagram compared with experimental results (Pradhan, Tatsuoka, and Sato 1989) for dense sand. The experimental results for loose sands are very similar. The prediction of the hypoplastic models does not depend on void ratio \( e \). In this picture \( \frac{Q}{P} = \frac{3Q}{\sqrt{2}P} \) and \( \frac{\dot{v}}{\dot{\gamma}} \approx \frac{DP}{\sqrt{2}DQ} = -\sqrt{\frac{2}{a}} \)

stress \( \eta = 0 \). In particular we ask what Poisson ratio would be equivalent to the ratio of shear stiffness and volumetric stiffness upon unloading calculated with the hypoplastic model for \( f_d = 1 \). \( \eta = 0 \) and \( \varphi = 30^\circ \). From this comparison follows that

\[
\frac{P}{Q} = 1 + \frac{1}{3}a^2 + \frac{a}{\sqrt{3}} = \frac{(1 + \nu)}{(1 - 2\nu)}
\]

which means that the quasi elastic response in hypoplasticity corresponds to the Poisson ratio

\[
\nu \approx 0.38
\]

This value seems to be too large. Recent experimental data (Koseki, Kawakami, Nagayama, and Sato 2000) reveal much lower typical values of \( \nu = 0.15 \) for Toyoura sand. For Hostun sand even lower value of \( \nu = 0.1 \) has been assume (Gajo and Wood 1999).

Let us modify the hypoplastic model increasing the shear stiffness component \( L_{QQ} = F^2 + \frac{1}{3}a^2\eta^2 \) of the hypoplastic model keeping \( m \) unchanged. The reference hypoplastic equation (10) rewritten in terms of \( L \) and \( m \) takes the form

\[
\begin{bmatrix}
\dot{P} \\
\dot{Q}
\end{bmatrix} = f_s \begin{bmatrix}
F^2 + \frac{1}{3}a^2 & \frac{1}{2}(a^2\eta + \frac{1}{\eta}) \\
\frac{1}{2}(a^2\eta + \frac{1}{\eta}) & F^2 + \frac{1}{3}a^2\eta^2 + \frac{1}{1 - \eta^2}
\end{bmatrix} \begin{bmatrix}
\frac{m_P}{m_Q} \\
\frac{m_D}{m_Q}
\end{bmatrix} \sqrt{D_c^2 + D_Q^2}
\]
wherein \( f_s = \frac{f_y f_c}{(1+\eta^2)} \) and

\[
Y = \frac{a \sqrt{(3F^2 - a^2 \eta^2)^2 + (a^2 + 6F^2)^2 \eta^2}}{\sqrt{3(a^2 + 3F^2 + a^2 \eta^2)}} \tag{21}
\]

\[
m_p = \frac{3F^2 - a^2 \eta^2}{\sqrt{(3F^2 - a^2 \eta^2)^2 + (a^2 + 6F^2)^2 \eta^2}} \tag{22}
\]

\[
m_q = \frac{(a^2 + 6F^2) \eta}{\sqrt{(3F^2 - a^2 \eta^2)^2 + (a^2 + 6F^2)^2 \eta^2}} \tag{23}
\]

All new terms are boxed. The shear stiffness (originally \( L_{QQ} = F^2 + \frac{1}{3} a^2 \)) is increased by \( \frac{1}{6} a^2 (1-\eta^2) \). The modification in the off-diagonal terms is discussed in the next section. It turns out that the change in \( L_{QQ} \) alone does not affect the dilatancy \( d \) presented in Fig. 3 but significantly improves the undrained stress path reducing the accumulation of pore pressure in the calculation as shown in Fig. 4. In 3-D case the correction of \( L_{QQ} \) should be written out in a form of an isotropic stress function.

6 ANISOTROPIC MODIFICATIONS

The critical state condition \( Y(T) = 1 \) is approximately isotropic and does not depend on the void ratio \( e \). The anisotropic modifications are proposed in the barotropy function \( f_d \) and in \( L \) only:

- the off-diagonal terms of \( L_{2 \times 2} \) are increased by \( \frac{1}{3} \), which is equivalent to \( \Delta \eta = 1/a^2 \approx \frac{1}{3} \). The effect of this modification is depicted in Fig. 3.

- the critical void ratio parameter \( \epsilon_{c0} \) is made dependent on \( \eta \) as follows

\[
\epsilon_{c0} = \bar{\epsilon}_{c0} + \left\{ \begin{array}{ll}
(\epsilon_{c0C} - \bar{\epsilon}_{c0}) \frac{\eta}{M_C}, & \text{for } \eta > 0 \\
(\epsilon_{c0E} - \bar{\epsilon}_{c0}) \frac{\eta}{M_E}, & \text{for } \eta < 0
\end{array} \right. \tag{24}
\]

with \( \bar{\epsilon}_{c0} = \frac{1}{2} (\epsilon_{c0E} + \epsilon_{c0C}) \) and with restriction \( \epsilon_{c0E} \leq \epsilon_{c0} \leq \epsilon_{c0C} \). Two critical void ratios are defined for compression \( \epsilon_{c0C} \) and for extension \( \epsilon_{c0E} \)

The first anisotropy improves the dilatancy curve produced by the hypoplastic model. The anisotropy within the barotropy function \( f_d(t) \) influences the undrained stress path as demonstrated qualitatively in Fig. 5. Writing a 3-D version of the above model the corrections to the off-diagonal terms in \( L \) in (20) and the modification of \( \epsilon_{c0} \) should be performed with respect to the spatial orientation of the normal to the bedding plane (and not of the major principal stress) because these effects of textural anisotropy. Here the coincidence with the sign of \( \eta \) is caused by the fact that a vertically cut sample is considered.

APPENDIX 1. NOTATION

A fixed orthogonal Cartesian coordinate system and the sign convention of general mechanics with tension positive are used. Vectors and second-order tensors are distinguished by bold typeface, for example \( {\bf N}, {\bf T}, {\bf v} \). Fourth order tensors are written in sans serif font (e.g. \( L \)). Multiplication with two dummy indices (double contraction) is denoted with a colon, e.g. \( A : B \). The fourth order unit tensor is denoted as \( (I)_{ijkl} = \frac{1}{2} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \). Wherein \( (I)_{ijkl} \) is an operator extracting the component \( (i, j, k, l) \) from the tensorial expression. The Euclidean norm is denoted as \( ||T|| = \sqrt{T : T} \). The deviatoric part of a tensor is denoted by an asterisk.
APPENDIX 2. REFERENCE MODEL

A recent hypoplastic constitutive model (Wolffersdorff 1996; Wolffersdorff 1997; Gudehus 1996)

$$\dot{T} = L : D + f_d N ||D||,$$

is taken as reference. The stress rate $\dot{T}$ is a function of stress rate $D$ and of stress $T$ and void ratio $e$ treated as state variables. The mathematical representation of $L(T, e)$ and $N(T, e)$ is:

$$L = \frac{f_f e}{\dot{T} : \dot{T}} a^2 \left( \frac{F'}{a} \right)^2 \left( 1 + \dot{T} \dot{T} \right),$$

$$N = \frac{f_f e}{\dot{T} : \dot{T}} a^2 \left( \frac{F'}{a} \right) \left( \dot{T} + \dot{T}^* \right),$$

$$a = \frac{\sqrt{3}(3 - \sin \varphi_c)}{2\sqrt{2}\sin \varphi_c},$$

$$F = \frac{1}{8} \tan^2 \psi + \frac{2 - \tan^2 \psi}{2 + 2\tan \psi \cos 3\theta} - \frac{1}{2\sqrt{2}} \tan \psi.$$  

The following stress invariants are used: $\tan \psi = \sqrt{3} ||\dot{T}^*||$, $\cos 3\theta = -3\sqrt{3} \det \dot{T}^* ||\dot{T}||^3$. The scalar functions $f_e(P, e)$, $f_d(P, e)$ and $f_b(P)$ are:

$$f_e = \left( \frac{e^2}{e} \right)^\beta,$$  

$$f_d = \left( \frac{e - \tilde{e}_d}{e - \tilde{e}_c} \right)^\alpha,$$  

$$f_b = \frac{(\frac{e_0}{\tilde{e}_0})^\beta}{\tilde{e}_d} + 1 + e_0 \left( \frac{\sqrt{3}P}{h_s} \right)^{1-n} - a^2 - a\sqrt{3} \left( \frac{e_{v0} - \tilde{e}_d}{\tilde{e}_d - \tilde{e}_0} \right)^\alpha .$$

in which the characteristic void ratios (Gudehus 1996) $e_i(P)$, $e_{c_i}(P)$, $e_{d_i}(P)$ are:

$$\left( \frac{e_{v0}}{e_{c0}} \right)^\beta h_s \left( \frac{\sqrt{3}P}{h_s} \right) = \exp \left( - \left( \frac{\sqrt{3}P}{h_s} \right)^n \right).$$

The above relations (Bauer 1996) need three material constants $e_{v0} > e_{c0} > e_{d0}$. 

REFERENCES


Mróz, Z. and V. Norris (1982). Elastoplastic and viscoplastic constitutive models for soil with application to cyclic loading. Soil mechanics-transient and


