ABSTRACT:
The paper discusses the determination of a set of constants for the high-cycle accumulation model (HCA) recently proposed by the authors (Niemunis et al., 2005). The HCA model may be used to predict permanent strains or excess pore water pressures in a non-cohesive soil due to a cyclic loading with relatively small amplitudes and a large number of cycles. The necessary laboratory tests and their analysis are explained in detail in this paper.

1 INTRODUCTION
The HCA model recently proposed by the authors (Niemunis et al., 2005) predicts permanent deformations or excess pore water pressures in non-cohesive soils due to many cycles ($N > 10^3$) with relatively small amplitudes ($\varepsilon_{\text{ampl}} < 10^{-3}$, so-called high- or polycyclic loading). The HCA model is based on an extensive laboratory testing program (Wichtmann, 2005) on a medium coarse sand. The model can be applied to foundations subjected to traffic loading, to offshore and onshore wind power plants, to machine foundations, to problems related to mechanical compaction of granular soils, etc.

In the HCA model a single material constant (critical friction angle $\varphi_c$) is used in the equations for the direction of accumulation $\mathbf{m} = \hat{\varepsilon}^\text{acc} / \| \hat{\varepsilon}^\text{acc} \|$. Eight material constants ($C_{e}, C_{p}, C_{Y}, C_{N1}, C_{N2}, C_{N3}, C_{\pi1}, C_{\pi2}$) are required for the intensity of accumulation $\dot{\varepsilon}^\text{acc} = \| \dot{\varepsilon}^\text{acc} \|$. Furthermore, two material constants (e.g. Poisson’s ratio $\nu$, Young’s modulus $E$) are needed for the isotropic elastic stiffness $E$ coupling the predicted trends of stress and strain.

Although the test results and the constants for a medium coarse sand have already been presented by Wichtmann (2005) there is a need for a more detailed description of the procedure for the determination of the material constants. It is the objective of the present paper. The paper gives recommendations for laboratory tests and discusses the evaluation of the test results.

2 HIGH-CYCLE ACCUMULATION MODEL
The differences between the “implicit” and the “explicit” calculation strategy have been explained by Niemunis et al. (2005). The main constitutive equation of the HCA model for explicit calculations reads

$$\mathbf{\sigma} = E : (\dot{\varepsilon} - \dot{\varepsilon}^\text{acc} - \dot{\varepsilon}^\text{pl})$$

with the Jaumann stress rate $\mathbf{\dot{\sigma}}$ of the effective stress $\mathbf{\sigma}$, the strain rate $\dot{\varepsilon}$, the prescribed strain accumulation rate $\dot{\varepsilon}^\text{acc}$, the plastic strain rate $\dot{\varepsilon}^\text{pl}$ and the (barotropic) elastic stiffness $E$. In the high-cyclic context “rate” means the derivative with respect to the number of cycles $N$, i.e. $\dot{\mathbf{\varepsilon}} = \partial \mathbf{\varepsilon} / \partial N$. The accumulation rate is calculated as the product of the scalar intensity of accumulation $\dot{\varepsilon}^\text{acc}$ and the direction of accumulation $\mathbf{m}$ (a unit tensor):

$$\dot{\varepsilon}^\text{acc} = \dot{\varepsilon}^\text{acc} \mathbf{m} = f_{\text{ampl}} f_{\text{e}} f_{\text{p}} f_{\text{Y}} f_{\pi} \mathbf{m}$$

The flow rule of the modified Cam clay (MCC) model has been experimentally found to approximate $\mathbf{m}$ well. A multiplicative approach is used for the intensity of strain accumulation $\dot{\varepsilon}^\text{acc}$. Each function (see Table 1) considers separately the influence of a different parameter (strain amplitude $\varepsilon_{\text{ampl}}$, cyclic preloading, void ratio $e$, average mean pressure $p^\text{av}$, average stress ratio $\eta^\text{av} = q^\text{av} / p^\text{av}$ or $Y$, polarization changes). In the function $f_{\pi}$, $\alpha$ is the angle between the current polarization and the so-called “back polarization”. For the definition of a multiaxial amplitude and its polarization from a 6D strain path see (Niemunis et al., 2005).

3 DETERMINATION OF A SET OF CONSTANTS
The determination of the constants is demonstrated for a quartz sand with sub-angular grain shape. The grain size distribution curve is almost linear in the
the reposed angle. For this purpose a cone of sand is deposited by slow centric lifting of a funnel. The critical friction angle is needed in the equations described by Wichtmann (2005).

The axial deformation and the volume change might significantly oscillate during a cycle or it might even accumulate. In the case of fine sands with a low permeability formations (Wichtmann, 2005) as long as inertia forces are negligible. For the determination of the constants of the HCA model, a loading frequency of 1 Hz may be suitable for most sands.

In the following some remarks on suitable test conditions and test analysis are given:

- It is distinguished between the first "irregular" cycle in general should be applied slower (e.g. with \( N = 100 \) seconds) than the subsequent ones.
- For sand the loading frequency does not significantly influence the residual and resilient deformations (Wichtmann, 2005) as long as inertia forces are negligible. For the determination of the constants of the HCA model, a loading frequency of 1 Hz may be suitable for most sands. In the case of fine sands with a low permeability a lower frequency might become necessary in order to assure drained conditions. Otherwise the pore water pressure \( a \) might significantly oscillate during a cycle or it might even accumulate with \( N \). Due to the large deformations the first cycle in general should be applied slower (e.g. with a period \( T = 100 \) seconds) than the subsequent ones.
- The axial deformation and the volume change should be measured. It is sufficient to record the data at several time instances, e.g. from the irregular cycle, from the first 25 regular cycles and from 4 or 5 cycles at \( N = 50, 100, 1,000, 2,000, \)

<table>
<thead>
<tr>
<th>Influencing parameter</th>
<th>Function</th>
<th>Material constants</th>
<th>Reference quantities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strain amplitude</td>
<td>( f_{\text{ampl}} = \min \left{ \left( \frac{\varepsilon_{\text{ampl}}}{\varepsilon_{\text{ref}}} \right)^2 ; 100 \right} )</td>
<td>( \varepsilon_{\text{ampl}} )</td>
<td>( \varepsilon_{\text{ref}} = 10^{-4} )</td>
</tr>
<tr>
<td>Cyclic preloading</td>
<td>( \hat{f}_N = \hat{f}_N^A + \hat{f}_N^B )</td>
<td>( C_{N1}, C_{N2}, C_{N3} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \hat{f}<em>N^A = C</em>{N1} N_{N2} \exp \left[ g^A \left/ \left( C_{N1} f_{\text{ampl}} \right) \right. \right] )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \hat{f}<em>N^B = C</em>{N1} C_{N3} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Void ratio</td>
<td>( f_e = \frac{(C_e - e)^2}{1 + e_{\text{ref}}} )</td>
<td>( C_e )</td>
<td>( e_{\text{ref}} = e_{\text{max}} )</td>
</tr>
<tr>
<td>Average mean pressure</td>
<td>( f_p = \exp \left[ -C_p \left( \frac{p}{p_{\text{ref}}} - 1 \right) \right] )</td>
<td>( C_p )</td>
<td>( p_{\text{ref}} = p_{\text{atm}} = 100 ) kPa</td>
</tr>
<tr>
<td>Average stress ratio</td>
<td>( f_Y = \exp \left( C_Y Y^{\alpha^Y} \right) )</td>
<td>( C_Y )</td>
<td></td>
</tr>
<tr>
<td>Polarization changes</td>
<td>( f_{\alpha} = 1 + C_{\alpha 1} (1 - \cos \alpha) )</td>
<td>( C_{\alpha 1} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \hat{\alpha} = -C_{\alpha 2} \alpha (\varepsilon_{\text{ampl}})^2 )</td>
<td>( C_{\alpha 2} )</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Summary of the functions, material constants and reference quantities of the high-cycle model

semi-logarithmic scale. The mean grain size is \( d_{50} = 0.6 \) mm and the coefficient of uniformity is \( C_u = 2.5 \).

3.1 Index tests
The maximum void ratio \( e_{\text{max}} \) is used as a reference quantity in the function \( f_e \) of the HCA model (Table 1). The minimum and maximum void ratios \( e_{\text{min}} \) and \( e_{\text{max}} \) may be determined according to the well-known procedures described in standard codes (e.g. DIN 18126 or ASTM-D-4253/ASTM-D-4254). For the present sand \( e_{\text{max}} = 0.856 \) and \( e_{\text{min}} = 0.495 \) have been obtained following the procedure of DIN 18126. For quartz sand the specific density may be assumed as \( \rho_s = 2.65 \) g/cm\(^3\).

3.2 Tests on the critical friction angle \( \varphi_c \)
The critical friction angle is needed in the equations for the "cyclic flow rule" \( \mathbf{m} \). \( \varphi_c \) may be obtained from the reposed angle. For this purpose a cone of sand is deposited by slow centric lifting of a funnel. \( \varphi_c \) is the inclination of the cone. For the present sand \( \varphi_c = 33^\circ \) was obtained as the mean value of five tests.

3.3 Cyclic triaxial tests for \( C_e, C_p, C_Y, C_{N1}, C_{N2}, C_{N3} \)
The constants of the functions \( f_e, f_p, f_Y \) and \( \hat{f}_N \) may be obtained from load-controlled drained cyclic triaxial tests. The test device and the method of sample preparation used for the present study have been described by Wichtmann (2005).

In the following some remarks on suitable test conditions and test analysis are given:

- For sand the loading frequency does not significantly influence the residual and resilient deformations (Wichtmann, 2005) as long as inertia forces are negligible. For the determination of the constants of the HCA model, a loading frequency of 1 Hz may be suitable for most sands. In the case of fine sands with a low permeability a lower frequency might become necessary in order to assure drained conditions. Otherwise the pore water pressure \( a \) might significantly oscillate during a cycle or it might even accumulate with \( N \). Due to the large deformations the first cycle in general should be applied slower (e.g. with a period \( T = 100 \) seconds) than the subsequent ones.
- The axial deformation and the volume change should be measured. It is sufficient to record the data at several time instances, e.g. from the irregular cycle, from the first 25 regular cycles and from 4 or 5 cycles at \( N = 50, 100, 1,000, 2,000, \)

![Typical curve \( \varepsilon(t) \) in a cyclic triaxial test](image-url)
This test series is recommended in order to choose an appropriate amplitude ratio for the subsequent test series. The tests may also be used in order to confirm the function $f_{ampl}$ of the HCA model. The tests should be performed with medium dense specimens (e.g. $I_{D0} = (\epsilon_{\max} - \epsilon)/ (\epsilon_{\max} - \epsilon_{\min}) \approx 0.6$). We recommend to perform three tests in which at least $N = 10^5$ cycles are applied at a constant average stress. For the present study $p^{av} = 200$ kPa and $\eta^{av} = 0.75$ (i.e. $K_0 = \sigma_3^{av}/\sigma_1^{av} = 0.5$) have been chosen. For uniform sands ($C_u \leq 3$) the amplitude ratio may be varied within the range $0.2 \leq \zeta = q^{ampl}/p^{av} \leq 0.4$. For sands with higher $C_u$-values lower values ($0.1 \leq \zeta \leq 0.3$) are recommended since the residual deformations increase with increasing $C_u$. In the present test series the chosen amplitude ratios $\zeta = 0.2, 0.3$ and 0.4 correspond to stress amplitudes $q^{ampl} = 40, 60$ and 80 kPa. The regular cycles were applied with a loading frequency of 1 Hz.

The accumulated strain $\varepsilon^{acc}$ is plotted versus the number of cycles $N$ in Figure 3. The stress ratio for the subsequent test series on $f_e$, $f_f$ and $f_Y$ should be chosen large enough so that the residual deformations can be accurately measured. However, $\zeta$ should be chosen small enough in order to avoid excessive accumulation in tests on loose soil ($f_e$) or in tests with large stress ratios ($f_Y$). From the tests with different amplitudes, the amplitude ratio $\zeta$ of the test which leads to a moderate accumulation, say $1 \leq \varepsilon^{acc} \leq 3\%$ after $N = 10^5$ cycles, could be chosen for the subsequent test series. According to Figure 3 the stress ratio $\zeta = 0.3$ was used in the present study.

### 3.3.2 Constant $C_e$ of function $f_e$

For the determination of $C_e$ at least three tests with different initial densities of the specimens are advised (e.g. $I_{D0} = 0.4, 0.6$ and 0.8). All tests should be performed with the same average stress $\sigma^{av}$ and with the same stress amplitude. It is recommended to apply at least $10^5$ cycles. If a test series with different stress amplitudes according to Section 3.3.1 has been con-
duced the test with the chosen \( \zeta \)-value and \( I_{D0} \approx 0.6 \)
can be supplemented by two tests with a lower or a
higher initial density, respectively. As an example, be-
side the test with \( I_{D0} = 0.66, p^v = 200 \text{ kPa}, \eta^v = 0.75 \)
and \( q^{\text{ampl}} = 60 \text{ kPa} \) (Figure 3) two tests with \( I_{D0} = 0.52 \) and 0.82 have been performed. The accumu-
lation curves of the three tests are given in Figure 4.
Obviously the accumulation rate decreases with in-
creasing \( I_{D0} \).

For the determination of the constant \( C_e \) we need
a presentation of the test results as given in Figure 5.
It shows the accumulated strain after different num-
bers of cycles as a function of the void ratio. Since
the tests are performed stress-controlled the strain am-
plitude \( \varepsilon^{\text{ampl}} \) varies with \( N \). Furthermore, the looser
specimens have larger strain amplitudes. In order to
"eliminate" the influence of the strain amplitude, the
accumulated strain is divided by the amplitude func-
tion \( \bar{f}_{\text{ampl}} = (\varepsilon^{\text{ampl}} / \varepsilon_{\text{ref}})^2 \) with \( \varepsilon_{\text{ref}} = 10^{-4} \). For each
number of cycles \( N \) a mean value of the strain am-
plitude \( \varepsilon^{\text{ampl}} \) has to be calculated. Since amplitude data is
available at \( N \)-values in logarithmic distance (i.e. \( N = 1, 2, 5, 10 \ldots \)) the sum \( \bar{\varepsilon}^{\text{ampl}} = 1/N \sum \varepsilon^{\text{ampl}}(N) \)
is used, e.g. for \( N = 10 \): \( \varepsilon^{\text{ampl}} = 1/10 \) \{\( \varepsilon^{\text{ampl}}(N =
1) + \varepsilon^{\text{ampl}}(N = 2) + \frac{1}{2}[\varepsilon^{\text{ampl}}(N = 2) + \varepsilon^{\text{ampl}}(N = 5)] \cdot
3 + \frac{1}{2}[\varepsilon^{\text{ampl}}(N = 5) + \varepsilon^{\text{ampl}}(N = 10)] : 5 \}. In a simi-
lar way also an integral value of the void ratio \( \bar{e} \) is
calculated for each \( N \).

As an alternative the amplitudes or void ratios
at lower numbers of cycles could receive a higher
weighting by using \( \varepsilon^{\text{ampl}} = \frac{1}{N} \sum \varepsilon^{\text{ampl}}(N) \) where \( i \) is
the number of \( \varepsilon^{\text{ampl}}(N) \)-values considered for the
calculation, e.g. for \( N = 10 \): \( \varepsilon^{\text{ampl}} = \frac{1}{4}[\varepsilon^{\text{ampl}}(N = 1) +
\varepsilon^{\text{ampl}}(N = 2) + \varepsilon^{\text{ampl}}(N = 5) + \varepsilon^{\text{ampl}}(N = 10)] \). Hav-
ing \( \bar{f}_{\text{ampl}} \) and \( \bar{e} \) for a certain \( N \), one can plot \( \bar{\varepsilon}^{\text{acc}} / \bar{f}_{\text{ampl}} \)
versus \( \bar{e} \) as it has been done in Figure 5.

For each \( N \)-value the function

\[
f = k (C_e - \bar{e})^2 / (1 + \bar{e})
\]

has to be fitted to the data. The curve-fitting delivers
the constant \( C_e \). The constant \( k \) is not further used.
The constants \( C_e \) obtained for the data in Figure 5
are summarized in Table 2. \( N = 10 \) was considered
as the lowest \( N \)-value. A slight increase of \( C_e \) can
be observed up to \( N = 200 \). For larger \( N \)-values, \( C_e \)
is almost constant. The constant \( C_e \) may be simply
taken as the mean value of the values determined for
the different numbers of cycles. This results in \( C_e =
0.50 \). If the curve-fitting is restricted to a lower num-
ber of \( N \)-values (e.g. \( N = 20, 100, 10^3, 10^4 \) and \( 10^5 \))
the same mean value \( C_e = 0.50 \) is obtained in this ex-
ample. Thus, such a simplified procedure seems suffi-
cient.

### 3.3.3 Confirmation of \( f_{\text{ampl}} \)

Having determined the constant \( C_e = 0.50 \), the func-
tion \( f_{\text{ampl}} \), i.e. the linear proportionality between
the accumulation rate \( \bar{\varepsilon}^{\text{acc}} \) and the square of the strain am-
plitude \( (\varepsilon^{\text{ampl}})^2 \) may be confirmed. The data from the
direct tests with different amplitudes (Section 3.3.1) is
used. As described in Section 3.3.2 integral values of
void ratio \( \bar{e} \) and strain amplitude \( \varepsilon^{\text{ampl}} \) are calculated.

![Figure 3. Accumulation curves \( \varepsilon^{\text{acc}}(N) \) in tests with different stress amplitudes \( q^{\text{ampl}} \)](image3.png)

![Figure 4. Accumulation curves in tests with different initial relative densities \( I_{D0} \)](image4.png)

![Figure 5. Determination of constant \( C_e \) by fitting of function \( f_e \)](image5.png)
for different \( N \)-values. Setting \( \bar{e} \) into \( f_e \) (Table 1) delivers the quantity \( f_{\bar{e}} \). For a given \( N \) the residual strain \( \varepsilon_{\text{acc}} \) has to be divided by the corresponding \( \bar{e} \). In Figure 6, the values \( \varepsilon_{\text{acc}} / f_{\bar{e}} \) are plotted versus the square of the strain amplitude \( (\varepsilon_{\text{amp}})^2 \). Straight lines through the origin confirm the function \( f_{\text{amp}} \).

### Table 2

<table>
<thead>
<tr>
<th>( N )</th>
<th>( C_c )</th>
<th>( C_p )</th>
<th>( C_Y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.43</td>
<td>0.42</td>
<td>1.4</td>
</tr>
<tr>
<td>20</td>
<td>0.45</td>
<td>0.41</td>
<td>1.6</td>
</tr>
<tr>
<td>50</td>
<td>0.47</td>
<td>0.38</td>
<td>1.9</td>
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<tr>
<td>100</td>
<td>0.48</td>
<td>0.36</td>
<td>2.1</td>
</tr>
<tr>
<td>200</td>
<td>0.50</td>
<td>0.33</td>
<td>2.3</td>
</tr>
<tr>
<td>500</td>
<td>0.51</td>
<td>0.33</td>
<td>2.5</td>
</tr>
<tr>
<td>1,000</td>
<td>0.51</td>
<td>0.36</td>
<td>2.7</td>
</tr>
<tr>
<td>2,000</td>
<td>0.52</td>
<td>0.40</td>
<td>2.8</td>
</tr>
<tr>
<td>5,000</td>
<td>0.52</td>
<td>0.43</td>
<td>2.8</td>
</tr>
<tr>
<td>10,000</td>
<td>0.52</td>
<td>0.47</td>
<td>2.8</td>
</tr>
<tr>
<td>20,000</td>
<td>0.52</td>
<td>0.49</td>
<td>2.9</td>
</tr>
<tr>
<td>50,000</td>
<td>0.52</td>
<td>0.49</td>
<td>2.9</td>
</tr>
<tr>
<td>100,000</td>
<td>0.52</td>
<td>0.51</td>
<td>2.7</td>
</tr>
</tbody>
</table>

Mean: 0.50, 0.41, 2.4  
Mean*: 0.50, 0.42, 2.4

<table>
<thead>
<tr>
<th>( N )</th>
<th>( \varepsilon_{\text{acc}} / f_{\bar{e}} ) [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>50</td>
<td>20</td>
</tr>
<tr>
<td>100</td>
<td>40</td>
</tr>
<tr>
<td>500</td>
<td>80</td>
</tr>
<tr>
<td>1,000</td>
<td>100</td>
</tr>
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</tr>
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<td>5,000</td>
<td>400</td>
</tr>
<tr>
<td>10,000</td>
<td>800</td>
</tr>
</tbody>
</table>

Figure 6. Confirmation of function \( f_{\text{amp}} \)

### 3.3.4 Constant \( C_p \) of function \( f_p \)

For the determination of \( C_p \) tests with different average mean pressures \( p^{\text{av}} \) are necessary. It is recommended to supplement the test with \( I_{D0} \approx 0.60 \) and \( p^{\text{av}} = 200 \text{ kPa} \) from the test series on the \( e \)-influence by additional tests with lower (e.g. \( p^{\text{av}} = 50 \) or 100 kPa) and higher (e.g. \( p^{\text{av}} = 300 \text{ kPa} \)) pressures. The average stress ratio \( \eta^{\text{av}} \) and the amplitude ratio \( \zeta = q^{\text{amp}} / p^{\text{av}} \) should be kept constant within the test series. Figure 7 presents the accumulation curves \( \varepsilon_{\text{acc}}(N) \) in four tests with \( \eta^{\text{av}} = 0.75 \) and \( \zeta = 0.3 \). The accumulation rates are quite similar in the four tests. However, the increase of the strain amplitude with \( p^{\text{av}} \) due to the under-linear increase of the secant stiffness with \( p^{\text{av}} \) has to be considered.

For each \( N \) the accumulated strain \( \varepsilon_{\text{acc}} \) is divided by the functions \( f_{\text{amp}} \) and \( f_{\bar{e}} \). They are calculated with \( \bar{e}^{\text{amp}} \) and \( \bar{e} \). The values \( \varepsilon_{\text{acc}} / (f_{\bar{e}} f_{\text{amp}}) \) are plotted versus \( p^{\text{av}} \) (Figure 8). The function

\[
f = k \exp[-C_p \ (p^{\text{av}}/100 - 1)]
\]

is fitted to the data resulting in the constant \( C_p \). The \( C_p \) values obtained from Figure 8 are summarized in Table 2. The mean value is \( C_p = 0.41 \). The simplified procedure delivers a similar constant \( C_p = 0.42 \).

### 3.3.5 Constant \( C_Y \) of function \( f_Y \)

The constant \( C_Y \) may be determined from tests with different average stress ratios \( \eta^{\text{av}} \) while keeping \( p^{\text{av}}, q^{\text{amp}} \) and \( I_{D0} \) constant. The test with \( p^{\text{av}} = 200 \text{ kPa} \), \( \eta^{\text{av}} = 0.75 \) and \( I_{D0} \approx 0.6 \) from the series on \( C_e \) may be supplemented by tests with lower (e.g. \( \eta^{\text{av}} = 0.5 \)) and higher (e.g. \( \eta^{\text{av}} = 1.0 \) or 1.25) average stress ratios. Figure 9 presents the accumulation curves in tests
with \( \eta^{av} = 0.5, 0.75, 1.0 \) and 1.25. The accumulation rate increases with increasing average stress ratio.

As a measure of the stress ratio we use \( Y = (Y - 9)/(Y_c - 9) \) with \( Y = 27(3 + \eta)/(3 + 2\eta) \) and \( Y_c = (9 - \sin^2 \varphi_c)/(1 - \sin^2 \varphi_c) \) with the critical friction angle \( \varphi_c \). In order to determine \( C_Y \) the residual strain after different \( N \)-values is divided by \( f_c \) and \( f_{ampl} \) (since the strain amplitude \( \varepsilon_{ampl} \) decreases with increasing \( \eta^{av} \)) and plotted versus the stress ratio \( Y^{av} \) of the test (Figure 10). The function

\[
    f = k \exp(C_Y Y^{av})
\]

is fitted to the data. The obtained constants \( C_Y \) are summarized in Table 2. An increase of \( C_Y \) during the first 500 cycles is obvious while the values stay almost constant at larger \( N \). A mean value \( C_Y = 2.4 \) has been determined.

If the boundary value problems calculated with the HCA model involve very large numbers of cycles \( N \geq 10^6 \) a long-term test should be performed and the function \( f_N \) should be fitted to the data of this test.

### 3.4 Constants \( C_{\pi 1} \) and \( C_{\pi 2} \) of function \( f_{\pi} \)

BVPs with a constant polarization of the cycles may be calculated using \( f_{\pi} = 1 \) as a first approximation. If the polarization varies, the constants \( C_{\pi 1} \) and \( C_{\pi 2} \) of function \( f_{\pi} \) have to be determined. Tests with a sudden change of the direction of the cycles are necessary. A suitable test is the cyclic multiaxial simple shear test (Wichtmann, 2005)).

The increase of the accumulation rate after a change of the polarization is obvious in Figure 12 which presents accumulation curves \( \varepsilon_{acc}(N) \) in tests with and without a sudden 90°-change of the polarization. In Figure 13 the ratio of the accumulation rates \( \varepsilon_{acc} \) in the tests with and without a change of the polarization (= factor \( f_{\pi} \)) is plotted versus the number of cycles \( N - N_{cp} \) after the change of polarization. \( f_{\pi} \) decays during approx. 1,000 cycles. The curve \( f_{\pi}(N - N_{cp}) \) seems to be independent of the initial density. Thus, a pair of tests with and without a change of the polarization is sufficient in order to determine \( C_{\pi 1} \) and \( C_{\pi 2} \).
If the polarization is not changed, \( \cos \alpha = 1 \) and \( f_\pi = 1 \) hold. Directly after a change of the polarization by \( 90^\circ \), \( \cos \alpha = 0 \) and \( f_\pi = 1 + C_{\pi 1} \) are valid. The material constant \( C_{\pi 1} \) is the difference between \( f_\pi \) directly before and \( f_\pi \) directly after the polarization change. From the four tests \( C_{\pi 1} = 4.0 \) was determined as a mean value.

The rate of decay is governed by the second material constant \( C_{\pi 2} \). It can be determined from

\[
C_{\pi 2} = \frac{\ln(3/2)}{(\varepsilon_{\text{amp}})^2 (N - N_{\text{cp}})^{1/2}}
\]

with \((N - N_{\text{cp}})^{1/2}\) being the number of cycles for which the factor \( \varepsilon_{\pi} \) takes the value \( 1 + C_{\pi 1}/2 \) (Figure 13). The number of cycles \((N - N_{\text{cp}})^{1/2}\) can be seen as a kind of "half-life" of the polarization effect. For the four tests in Figure 13, \( C_{\pi 2} = 200 \) has been obtained.

A disadvantage of the simple shear test is the inhomogeneous field of stress and strain within the specimen. As an alternative a cyclic triaxial test with a sudden \( 90^\circ \) change of the polarization of 1-d cycles in the \( P-Q \) plane could be performed. \( P = \sqrt{3}p \) and \( Q = \sqrt{2/3}q \) are the isomorphic stress invariants.

3.5 Constants of elasticity \( E \)

For \( E \) in Equation (1) an isotropic (hypo)elastic stiffness with a barotropic Young’s modulus is set into approach. \( E \) can be developed by comparing the rate of cyclic relaxation and the rate of cyclic creep under the same conditions (\( \sigma^\pi, \varepsilon, N \)) and under the same cyclic load. Two elastic constants (e.g. Poisson’s ratio \( \nu \) and bulk modulus \( K(p) \)) have to be determined. Poisson’s ratio may be estimated as \( \nu = 0.2 \) to 0.3. The bulk modulus \( K = \frac{\hat{u}}{\varepsilon_{\text{acc}}} \) can be obtained from comparing the rate of pore water pressure \( \hat{u} \) in an undrained cyclic test and the rate of volumetric strain accumulation \( \varepsilon_{\text{acc}} \) in a drained cyclic test. A study of \( K \) with 15 pairs of drained and undrained cyclic triaxial tests on a medium coarse sand has been documented by Wichtmann et al. (2007). For the first approximation it is recommended to use \( K = A p_{\text{adm}}^{1-n} p^n \) with \( A = 540 \) and \( n = 0.30 \).

4 SUMMARY AND CONCLUSIONS

The paper presents a procedure for the determination of a set of material constants for the HCA model proposed by Niemunis et al. (2005). The critical friction angle \( \varphi_c \) is determined as the angle of repose. The constants \( C_{\pi 1}, C_{\pi 2}, C_Y, C_{N1}, C_{N2} \) and \( C_{N3} \) may be determined from at least nine drained stress-controlled cyclic triaxial tests. The constants \( C_{\pi 1} \) and \( C_{\pi 2} \) can be obtained from two cyclic (multiaxial simple shear or triaxial) tests, one with and one without a sudden change of the direction of the cycles. The constants of the elastic stiffness \( E \) may be determined by comparing drained and undrained cyclic tests. The paper discusses suitable test conditions and the analysis of the test results.

REFERENCES

