Simplified calibration procedure for a high-cycle accumulation model based on cyclic triaxial tests on 22 sands

T. Wichtmann, A. Niemunis & Th. Triantafyllidis
Institute of Soil Mechanics and Rock Mechanics, Karlsruhe Institute of Technology

ABSTRACT: A simplified calibration procedure for the material constants used in the authors' high-cycle accumulation model has been improved based on data from more than 350 drained cyclic triaxial tests performed on 22 clean quartz sands with different grain size distribution curves. The simplified method allows the estimation of a set of parameters from characteristics of the grain size distribution curve (mean grain size, coefficient of uniformity) and index quantities (minimum and maximum void ratio).

1 INTRODUCTION

The high-cycle accumulation (HCA) model proposed by Niemunis et al. (2005) predicts the accumulation of permanent deformations or the build-up of excess pore water pressure due to a cyclic loading with many cycles \((N > 10^3)\) of small to intermediate strain amplitudes \((\varepsilon_{\text{ampl}} < 10^{-3})\). The model can be used for example for the prediction of permanent deformations of offshore wind power plant (OWPP) foundations (Wichtmann et al., 2010b).

The determination of the material constants of the HCA model (Wichtmann et al., 2010a) is quite laborious. Drained cyclic triaxial tests with different amplitudes, densities and average stresses are necessary. Regarding the large number of OWPPs in a wind park and the layered soil, an experimental determination of the constants for each OWPP foundation and each soil type would be tedious. Therefore, a simplified calibration procedure has already been proposed by Wichtmann et al. (2009) based on cyclic triaxial tests on eight quartz sands with different grain size distribution curves. Correlations of the HCA model constants with index properties (mean grain size \(d_{50}\), coefficient of uniformity \(C_u\), minimum void ratio \(e_{\text{min}}\)) have been developed for that purpose. However, some of the correlations showed a significant amount of scatter.

Therefore, 14 more grain size distribution curves with linear shape (in the semi-logarithmic scale) and with different mean grain sizes and coefficients of uniformity were tested in order to improve the correlations and to adapt them to a wider range of \(d_{50}\)- and \(C_u\)-values. The present paper reports on this effort.

2 TESTED MATERIALS AND TESTING PROCEDURES

The 14 tested grain size distribution curves are shown in Figure 1. They were mixed from a natural quartz sand with subangular grain shape. The sands and gravels L1 to L7 (Figure 1a) have mean grain sizes in the range \(0.1 \text{ mm} \leq d_{50} \leq 3.5 \text{ mm}\) and the same coefficient of uniformity \(C_u = d_{60}/d_{10} = 1.5\). The materials L4 and L10 to L16 (Figure 1b) have the same mean grain size \(d_{50} = 0.6 \text{ mm}\) while \(C_u\) varies between 1.5 and 8.
The samples with a diameter of 10 cm and a height of 20 cm were prepared by dry air pluviation and afterwards saturated with de-aired water. They were consolidated for one hour at the average stress. Due to large deformations the first irregular cycle was applied with a low loading frequency of 0.01 Hz while \( f = 1 \) Hz was used for the subsequent 100,000 regular cycles. The only exception was the fine sand L1 with 2,000 or 10,000 regular cycles were tested with frequencies of 0.01 or 0.1 Hz, respectively. For each material several tests with different amplitudes, initial densities, average mean pressures \( p^a \) and average stress ratios \( \eta^a = q^a/p^a \) were performed (\( p = (\sigma_1 + 2\sigma_3)/3, q = \sigma_1 - \sigma_3 \)).

Since the HCA model predicts the accumulation due to the regular cycles only, the irregular cycle is not discussed in the following. The direction of accumulation \( m = \dot{\varepsilon}^{\text{acc}}/\|\dot{\varepsilon}^{\text{acc}}\| \) (flow rule) used in the HCA model could be confirmed for all tested sands and is also not further addressed here.

3 TEST RESULTS AND DETERMINATION OF HCA MODEL CONSTANTS

The increase of the intensity of accumulation \( \dot{\varepsilon}^{\text{acc}} = \|\dot{\varepsilon}^{\text{acc}}\| = \partial \varepsilon^{\text{acc}}/\partial N \) (with \( \varepsilon = \sqrt{\varepsilon_1^2 + 2\varepsilon_2^2} \)) with increasing stress or strain amplitude becomes clear from Figures 2 and 3. Figure 2 shows the accumulated permanent strain \( \varepsilon^{\text{acc}} \) as a function of the number of cycles \( N \) in the tests with different deviatoric stress amplitudes \( q^\text{ampl} \). In Figure 3 the permanent strain after different numbers of cycles is plotted versus the strain amplitude. Since in the stress-controlled tests the strain amplitude decreased slightly with \( N \), a mean value \( \bar{\varepsilon}^{\text{ampl}} = 1/N \int \varepsilon^{\text{ampl}}(N) dN \) over \( N \) is used on the abscissa. On the ordinate the data is divided by the void ratio function of the HCA model in order to purify it from the influence of void ratio:

\[
f_e = \frac{(C_e - e)^2}{1 + e} \frac{1 + e_{\max}}{(C_e - e_{\max})^2}
\]

with the maximum void ratio \( e_{\max} \) and the material constant \( C_e \). The bar over \( f_e \) in Figure 3 denotes that the void ratio function has been calculated with a mean value \( \bar{e} = 1/N \int e(N) dN \) of void ratio. An overproportional increase of the intensity of strain accumulation with the strain amplitude can be concluded from Figure 3. The amplitude function

\[
f^{\text{ampl}} = (\varepsilon^{\text{ampl}}/10^{-4})C^{\text{ampl}}
\]

of the HCA model has been fitted to the data shown in Figure 3 (solid curves) delivering \( C^{\text{ampl}} \). The \( C^{\text{ampl}} \) values given in column 3 of Table 1 are mean values over 100,000 cycles.

![Figure 2](image1.png)  
**Figure 2.** Accumulation curves \( \varepsilon^{\text{acc}}(N) \) in tests with different stress amplitudes \( q^{\text{ampl}} \) (all tests: \( p^a = 200 \) kPa, \( \eta^a = 0.75 \)), thick solid curves = recalculation with HCA model.

![Figure 3](image2.png)  
**Figure 3.** Accumulated strain \( \varepsilon^{\text{acc}}/\bar{e} \) as a function of strain amplitude \( \varepsilon^{\text{ampl}} \) (all tests: \( p^a = 200 \) kPa, \( \eta^a = 0.75 \)).

![Figure 4](image3.png)  
**Figure 4.** Accumulated strain \( \varepsilon^{\text{acc}} \) as a function of a) mean grain size \( d_{50} \) and b) coefficient of uniformity \( C_u \).

The dependence of \( \varepsilon^{\text{acc}} \) on the grain size distribution curve is inspected in Figure 4, where the residual strain after 10,000 cycles is plotted versus \( d_{50} \) or...
$C_u$, respectively. In accordance with Wichtmann et al. (2009) the intensity of accumulation increases with decreasing mean grain size and with increasing coefficient of uniformity.

Figures 5 and 6 demonstrate the increase of the strain amplitude with increasing pressure. Figure 5 compares the curves $\varepsilon^{acc}(N)$ for different initial relative densities $I_{D0}$, Figure 6 presents the residual strain after different $N$-values as a function of void ratio $\bar{e}$. In order to purify the data from the influence of slightly different strain amplitudes, $\varepsilon^{acc}$ has been divided by the amplitude function of the HCA model. The bar over $f_{ampl}$ in Figure 6 denotes that the amplitude function has been calculated with a mean value $\bar{e}_{ampl}$ of the strain amplitude. The parameter $C_p$ (column 4 of Table 1) was obtained from a curve-fitting of the function $f_e$ to the data in Figure 6. Since $f_{ampl}$ is necessary to purify the data in Figure 6 and $f_e$ is used on the ordinate in Figure 3, the determination of $C_{ampl}$ and $C_p$ has to be done by iteration.

The accumulation curves $\varepsilon^{acc}(N)$ in the tests with different average mean pressures $\bar{p}$ and with a constant average stress ratio (here $\eta^{av} = 0.75$) coincide approximately if the tests are performed with the same amplitude-pressure ratio $\zeta = q_{ampl}/\bar{p}$ (Figure 7). The increase of the strain amplitude with increasing pressure for $\zeta = constant$ has been considered in Figure 8 where the residual strain has been divided by $f_{ampl}$ and $f_e$ and plotted versus $\bar{p}$. The decrease of the intensity of accumulation with increasing average mean pressure is obvious in Figure 8. It becomes less pronounced with increasing mean grain size. The data for some sands (e.g. L15, Figure 8) indicate almost constant accumulation rates for larger pressures. Tests with $\bar{p} > 300$ kPa are planned for the future. The HCA model parameter $C_p$ (column 5 of Table 1) was obtained from a curve-fitting of the function $f_p$ to the data in Figure 8:

$$f_p = \exp[-C_p (\bar{p}/100 \text{ kPa} - 1)] \quad (3)$$

For all tested materials the increase of the strain accumulation rate with increasing average stress ratio was confirmed. Figures 9 and 10 compare the accumulation curves $\varepsilon^{acc}(N)$ or show the residual strain as a function of the normalized average stress ratio $\bar{Y}^{av}$, where $\bar{Y}$ and $\eta$ are interrelated via

$$\bar{Y} = \frac{27(3 + \eta)/(3 + 2\eta)(3 - \eta) - 9}{(9 - \sin^2 \varphi_c)/(1 - \sin^2 \varphi_c) - 9} \quad (4)$$

with critical friction angle $\varphi_c$. $\bar{Y}^{av}$ is zero for isotropic stress conditions and equal to one on the critical state line. The HCA model parameter $C_Y$ (column 6 of Table 1) was obtained from a curve-fitting of the function $f_Y$ to the data in Figure 10:

$$f_Y = \exp(C_Y \bar{Y}^{av}) \quad (5)$$

The shape of the curves $\varepsilon^{acc}(N)$ can be judged from Figure 11 where the residual strain has been divided by the functions $f_{ampl}, f_e, f_p$ and $f_Y$ of the HCA model (calculated with the constants given in columns 3 to 9 of Table 1), that means it was purified from the influences of amplitude, void ratio and average stress. For uniform sands the residual strain increases almost proportional to $\ln(N)$ up to at least $N = 10^4$. At large numbers of cycles $N > 10^4$, the residual strain grew faster than proportional to $\ln(N)$ for some of the sands L1 to L7 (see e.g. L2 in Figure 11). The curves
Figure 7. Accumulation curves $\varepsilon^{\text{acc}}(N)$ in tests with different average mean pressures $p^{\text{av}}$ (all tests: $\eta^{\text{av}} = 0.75$, $\zeta = q^{\text{ampl}} / p^{\text{av}}$), thick solid curves = recalculation with HCA model.

Figure 8. Accumulated strain $\varepsilon^{\text{acc}} / (f_{\text{ampl}} f_e)$ as a function of average mean pressure $p^{\text{av}}$ (all tests: $\eta^{\text{av}} = 0.75$, $\zeta = q^{\text{ampl}} / p^{\text{av}}$).

Figure 9. Accumulation curves $\varepsilon^{\text{acc}}(N)$ in tests with different stress ratios $Y^{\text{av}}$ (all tests: $p^{\text{av}} = 200$ kPa), thick solid curves = recalculation with HCA model.

Figure 10. Accumulated strain $\varepsilon^{\text{acc}} / (f_{\text{ampl}} f_e)$ as a function of normalized average stress ratio $Y^{\text{av}}$ (all tests: $p^{\text{av}} = 200$ kPa).

Figure 11. Curves $\varepsilon^{\text{acc}}(N) / (f_{\text{ampl}} f_e f_p f_Y)$, fitting of function $f_N$.
The parameters given in columns 10 to 16 of Table 1 differ from those calibrated "by hand" due to simplifications of the "by hand" method (for example mean values \( \bar{e}_{\text{ampl}} \) and \( \bar{e} \) are used in the diagrams, parameters determined for different \( N \)-values are averaged).

### 4 RE-CALCULATION OF ELEMENT TESTS

The parameters given in columns 10 to 16 of Table 1 were used for recalculations of the element tests with the HCA model. The predicted curves have been added as solid lines in Figures 2, 5, 7 and 9. The parameters determined "by hand" (columns 3 to 9 of Table 1) deliver quite similar curves. In most cases the deviation between the experimental and the calculated data is small, confirming the good prediction of the HCA model. For some sands slightly too low accumulation rates are predicted for small pressures (Figure 7) which is due to deficits of the function \( f_p \). This will be inspected in more detail in future.

### 5 SIMPLIFIED CALIBRATION PROCEDURE

In Figure 12 the HCA model parameters are plotted versus mean grain size \( d_{50} \), coefficient of uniformity \( C_u \) or minimum void ratio \( \varepsilon_{\text{min}} \), respectively. The data from the tests described by Wichmann et al. (2009) were re-analyzed with \( C_{\text{ampl}} \neq 2.0 \) and are included in Figure 12. The correlations defined by Equations (7) to (13) are given in Figure 12 as solid lines and may be used for a simplified estimation of a set of parameters.

The parameter \( C_{\text{ampl}} \) does not correlate with \( d_{50} \) or \( C_u \) (Figure 12a,b). For \( C_e \), both, a correlation with \( d_{50} \) and \( C_u \) (Figure 12c,d) and with minimum void ratio \( \varepsilon_{\text{min}} \) (Figure 12e) could be established. The values of \( C_p \) and \( C_Y \) plotted in Figure 12f-i were obtained calculating \( C_{\text{ampl}} \) and \( C_e \) from Equations (7) and (8). Similarly, the data for \( C_{N1} \), \( C_{N2} \) and \( C_{N3} \) in Figure 12j-o have been analyzed with \( C_{\text{ampl}} \), \( C_e \), \( C_p \) and \( C_Y \) calculated from Equations (7) to (10). Beside the calibration methods discussed in Section 3, the parameters \( C_{\text{ampl}}, C_e, C_p \) and \( C_Y \) were also estimated from the rate data (see Wichmann et al., 2010a). \( C_{N1}, C_{N2} \) and \( C_{N3} \) were determined both, from the data of all curves \( \varepsilon_{\text{acc}}(N) \) and from the curves of the three tests with different amplitudes only. The poor correlation between \( C_{N3} \) and \( d_{50} \) can possibly be improved by means of data from tests with larger numbers of cycles \( (N > 10^5) \).

\[
C_{\text{ampl}} = 1.70 \tag{7}
\]

\[
C_e = 0.95 \cdot \varepsilon_{\text{min}} \tag{8}
\]

\[
C_p = 0.41 \cdot [1 - 0.34 \cdot (d_{50} - 0.6)] \tag{9}
\]

\[
C_Y = 2.60 \cdot [1 + 0.12 \ln(d_{50}/0.6)] \tag{10}
\]

\[
C_{N1} = 4.5 \cdot 10^{-4} \cdot [1 - 0.306 \ln(d_{50}/0.6)] \cdot [1 + 3.15 \cdot (C_u - 1.5)] \tag{11}
\]

\[
C_{N2} = 0.31 \cdot \exp[0.39 \cdot (d_{50} - 0.6)] \cdot \exp[12.3(\exp(-0.77C_u) - 0.315)] \tag{12}
\]

\[
C_{N3} = 3.0 \cdot 10^{-5} \cdot \exp[-0.84 \cdot (d_{50} - 0.6)] \cdot [1 + 7.85 \cdot (C_u - 1.5)]^{0.34} \tag{13}
\]
6 SUMMARY, CONCLUSIONS AND OUTLOOK

Based on the data from approx. 350 drained cyclic triaxial tests performed on 22 quartz sands with different grain size distribution curves a simplified procedure for the determination of the parameters of the authors’ high-cycle accumulation (HCA) model has been developed. Correlations of the HCA model parameters with mean grain size $d_{50}$, coefficient of uniformity $C_u$ or minimum void ratio $e_{\text{min}}$, respectively, have been formulated. In future the simplified calibration procedure will be extended to granular materials with fines content.

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