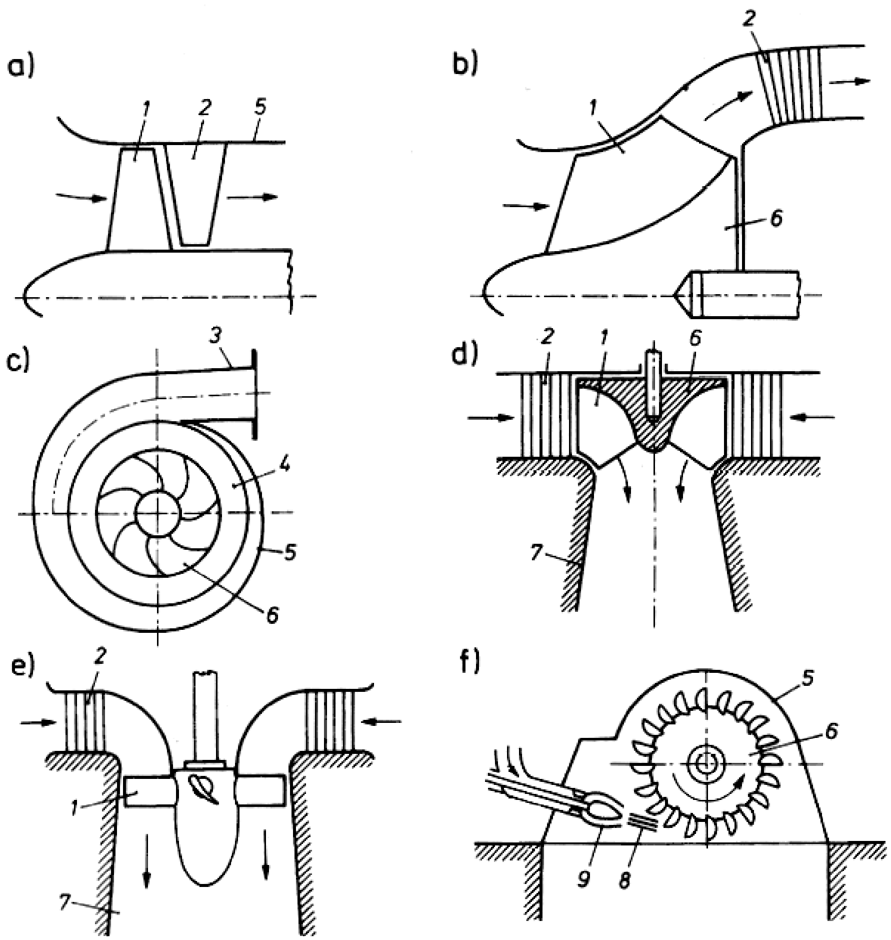
J. Szantyr – Lecture No. 2 – Principles of the Theory of Turbomachinery



a) Axial ventilator or pump

b) Diagonal (mixed flow) ventilator or pump

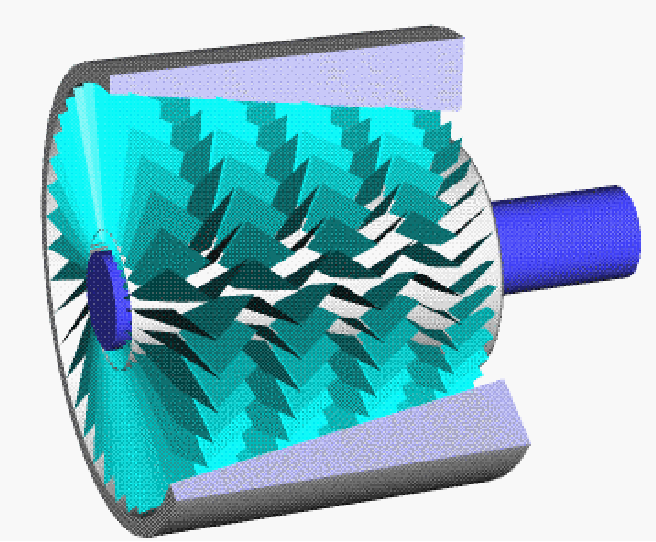
c) Centrifugal compressor or pump

d) Axial-radial water turbine (Francis turbine)

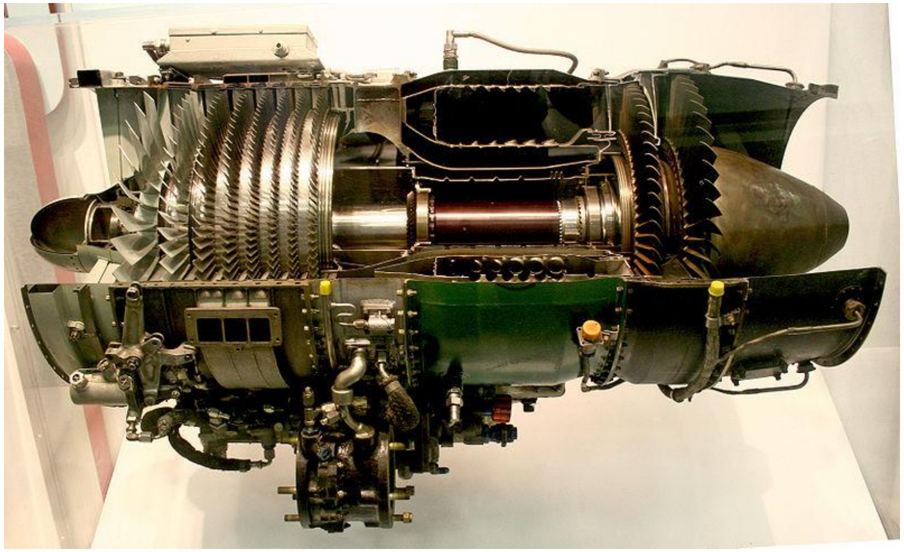
e) Axial water turbine (Kaplan turbine)

f) Impulse water turbine (Pelton turbine)

Multistage axial compressor

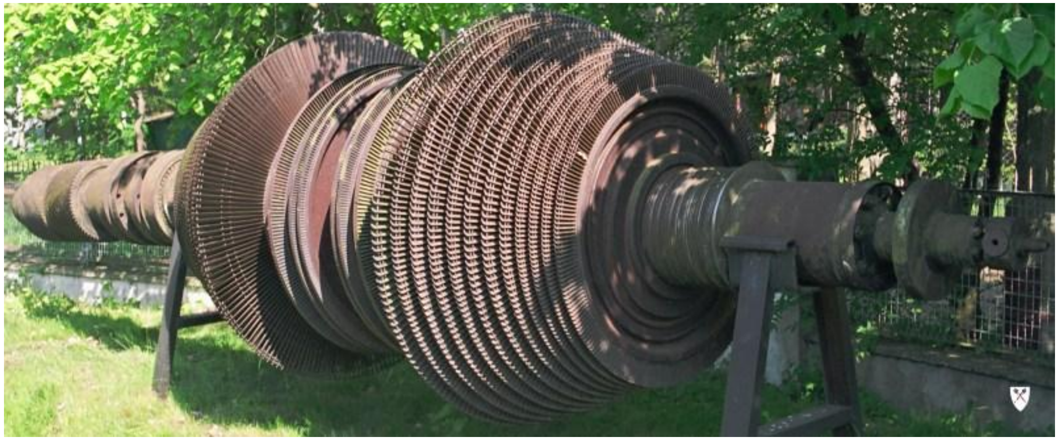


Compressor and gas turbine in a turbojet engine



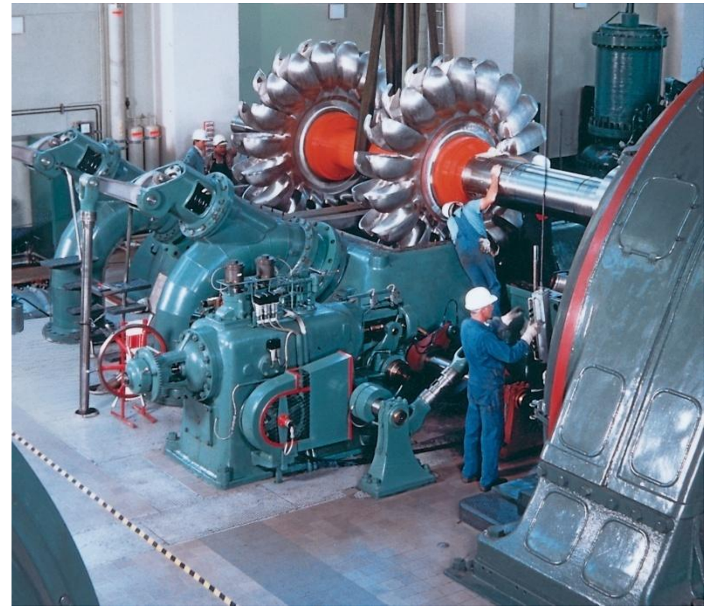
Rotor of the Francis water turbine

Rotor of a steam turbine



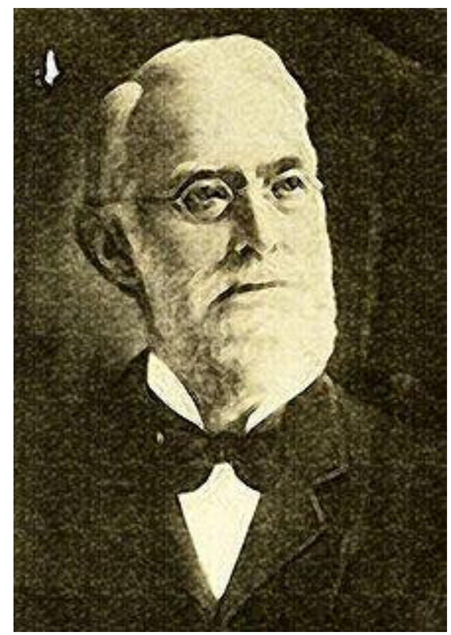
Rotor of a Banki-Michell water turbine

Rotor of a Pelton water turbine



Rotor of a diagonal (mixed flow) pump

Lester Pelton
1829 - 1908



Types of turbomachinery depending on their specific speed

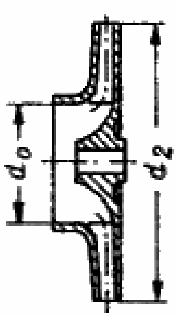
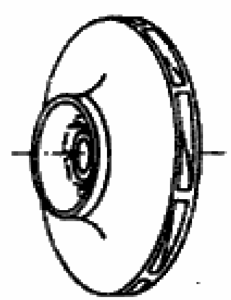
Specific speed of a turbo machine is the rotational speed of a geometrically similar machine having unit hydraulic head H and unit capacity Q at its best efficiency point

$$n_{sQ} = \frac{n}{\sqrt{H}}\sqrt{\frac{Q}{\sqrt{H}}}$$

Non-dimensional specific speed

$$n_{sf} = n \cdot \frac{Q^{1/2}}{(g \cdot H)^{\frac{3}{4}}}$$

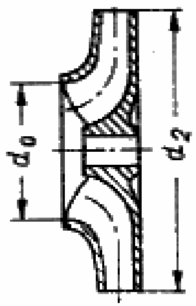
Specific speed univocally determines the type of the machine rotor. The value of specific speed increases with with increasing capacity and rotational speed and falls with increasing head.

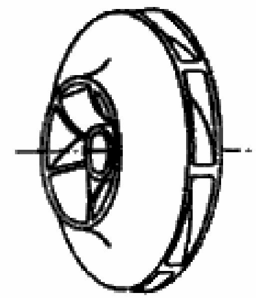
n_{sQ}	10–30	Wirnik odśrodkowy o pojedynczej krzywiźnie łopatek  	pompy odśrodkowe jedno- i wielostopniowe do dużych wysokości podnoszenia, np. wysokociśnieniowe zasilające
n_{sf}	30–90		
d_2/d_0	3,5–2		

Single and multistage centrifugal pumps with rotors of single blade curvature for high hydraulic heads, e.g. high pressure feeding pumps

n_{sQ}	30–50
n_{sf}	90–150
d_2/d_0	2–1,5

Wirnik odśrodkowy o przestrzennej krzywiźnie łopatek





Single stage centrifugal pumps with one or two-sided inlets (two-sided rotors) for high hydraulic heads

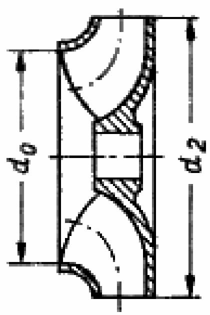
pompy odśrodkowe jednostopniowe z jedno- i obustronnym wlotem (wirniki dwustronne), pompy wielostopniowe na wyższe wysokości podnoszenia,

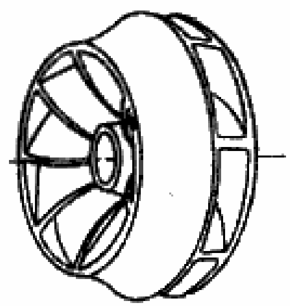
maszyny odwracalne promieniowe

Radial reversible machines

n_{sQ}	50–80
n_{sf}	150–240
d_2/d_0	1,5–1,3

Wirnik helikoidalny zamknięty





Single stage helicoidal pumps with one or two-sided inlet for low head and high capacity

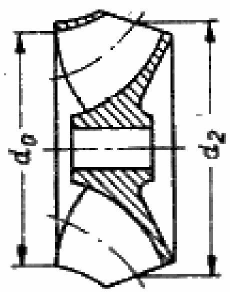
pompy helikoidalne jednostopniowe z jedno- i dwustronnym wlotem (wirniki dwustronne) na niewielkie wysokości podnoszenia i duże wydajności;

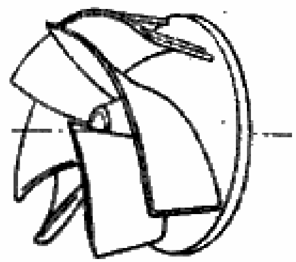
maszyny odwracalne helikoidalne

Reversible machines with helicoidal blades

n_{sQ}	80–150
n_{sf}	240–450
d_2/d_0	1,2–1,1

Wirnik helikoidalny lub diagonalny zamknięty, a przy wyższych wartościach wyróżnika szybkobieżności n_{sQ} otwarty





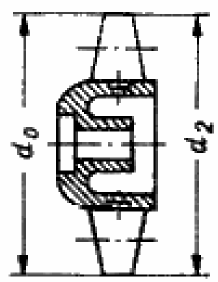
Single or multistage diagonal pumps, mainly with vertical axes

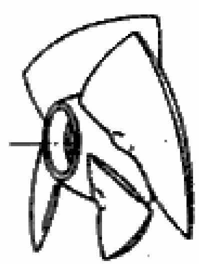
pompy helikoidalne jw.,
pompy diagonalne jedno- i kilkustopniowe przeważnie pionowe,
maszyny odwracalne o łopatkach nastawialnych (Deriaza)

Reversible machines with controllable blades

n_{sQ}	135–320
n_{sf}	405–960
d_2/d_0	1

Wirnik śmigłowy





Single or multistage axial pumps for very high capacity and low hydraulic head

pompy śmigłowe jednostopniowe (wyjątkowo dwu- lub trzystopniowe) przeważnie pionowe na bardzo duże wydajności i małe wysokości podnoszenia,
(maszyny odwracalne śmigłowe (Kaplana))

Axial reversible machines (e.g. Kaplan turbine

The objective of turbomachinery theory is to supply formulae for computation of the pressure variation in the flow through the machine and of the power associated with this flow. This theory also supplies data for design of the rotor blading optimum from the point of view of the machine efficiency. **One-dimensional** theory considers a simplified model of a steady flow of an incompressible fluid through a rotor having infinite number of very thin blades. This flow is axi-symmetrical with the velocity field described by the following relation:

$$\vec{v} = \vec{w} + \vec{\Omega} \times \vec{r} = \vec{w} + \vec{u}$$

where:

where:	$\vec{v}$	- absolute velocity
	$\vec{w}$	- relative velocity
	$\vec{u}$	- convective velocity
	$\vec{\Omega}$	- angular velocity of the rotor

The power of a hydraulic machine may be determined on the basis of the energy supplied to (in a pump) or extracted from (in a turbine) the unit mass of the flowing fluid:

$$N = \rho \cdot g \cdot Q \cdot H$$

where:

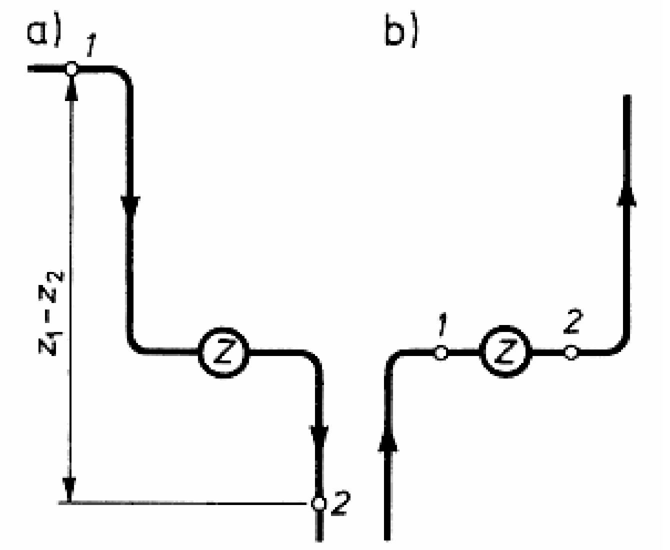
ρ	-fluid density,
g	-acceleration of gravity,
Q	-capacity (volumetric intensity of flow)
H	-hydraulic head.

This power is equal to the shaft power: $$N = M \cdot \Omega$$

where M is the torque on the shaft, Ω is the angular velocity

The hydraulic head H may be connected to the flow parameters by means of the Bernoulli equation:

$$\frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + H + h_{str}$$

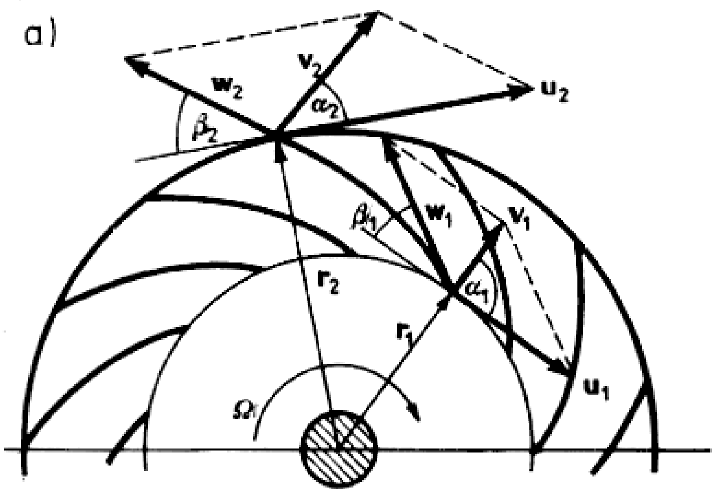


what leads to the relation:

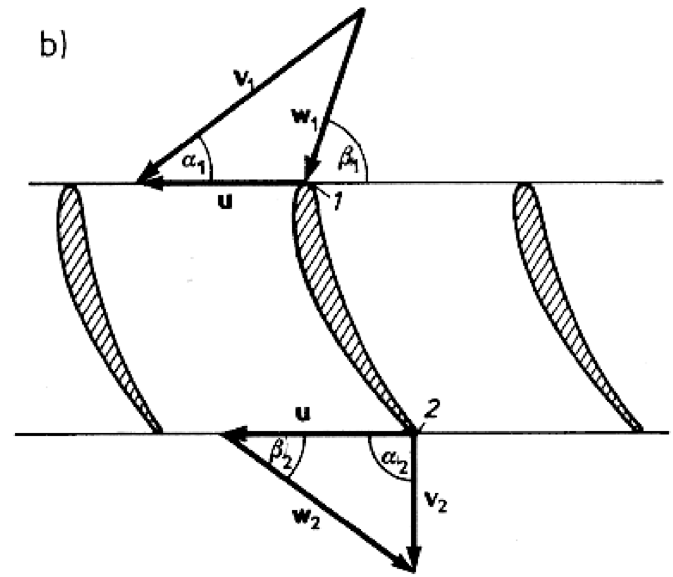
$$H = \frac{v_1^2 - v_2^2}{2g} + \frac{p_1 - p_2}{\rho g} + z_1 - z_2 - h_{str}$$

where index *1* denotes the section before the machine, and index *2* – the section behind the machine. In case of a pump the above relation is dominated by the pressure term, in case of a turbine – by the geometric elevation term. The term describing losses in the machine reduces the hydraulic head for a turbine (i.e. power of a real turbine is smaller than that of an ideal one), and increases head for a pump (i.e. the real pump requires more power than the ideal one).

Euler formula for the hydraulic machines



A radial rotor



An axial rotor

v – absolute velocity, u – convective velocity, w – relative velocity, 1 – inlet section, 2 – outlet section

The liquid flowing through the rotor experiences a change of the moment of momentum. This is described by the following conservation equation:

$$\frac{D}{Dt}\int_V \left(\vec{r} \times \rho\vec{v}\right) dV = \int_V \left(\vec{r} \times \vec{F}\right) \rho dV + \int_S \left(\vec{r} \times \vec{\tau}_n\right) dS$$

rate of change of m. of m.=moment of the mass forces+moment of the surface forces

Taking into account the velocity distributions in sections 1 and 2, together with the steadiness of flow and incompressibility of the liquid, the left hand side of the equation may be modified to the following form:

$$\frac{D}{Dt}\int_V \left(\vec{r}\times\rho\vec{v}\right)dV = \int_V \left(\vec{r}\times\vec{v}\right)\rho dQ = \left[\left(\vec{r}\times\vec{v}^{\infty}\right)_2 - \left(\vec{r}\times\vec{v}^{\infty}\right)_1\right]\rho Q$$

On the right hand side of the equation, the structure of the gravity field forces and field of the inertia forces due to rotation, leads to the conclusion that the moment of the mass forces is always zero in an arbitrary spatial orientation of the rotor:

$$\int_V \left(\vec{r}\times\vec{F}\right)\rho dV = 0$$

In turn, the surface forces represent the torque on the shaft:

$$\int_S \left(\vec{r}\times\vec{\tau}_n\right)dS = -\vec{M}^{\infty}$$

Hence we get: $$\vec{M}^{\infty} = \rho Q\left[\left(\vec{r}\times\vec{v}^{\infty}\right)_1 - \left(\vec{r}\times\vec{v}^{\infty}\right)_2\right]$$

The vector multiples in the above may be transformed into:

$$\vec{r}\times\vec{v}^{\infty} = rv^{\infty}\sin\left(\frac{\pi}{2}-\alpha\right) = rv^{\infty}\cos\alpha = rv_u^{\infty}$$

what leads to: $$M^{\infty} = \rho Q\left(r_1 v_{1u} - r_2 v_{2u}\right)$$

using the relation: $$\rho g Q H^{\infty} = M^{\infty}\Omega$$

Leonhard Euler
1700 - 1783

we finally obtain the Euler formula:

$$H^{\infty} = \frac{\Omega}{g}\left(r_1 v_{1u}^{\infty} - r_2 v_{2u}^{\infty}\right)$$

The term in parantheses is positive for a turbine and negative for a pump.