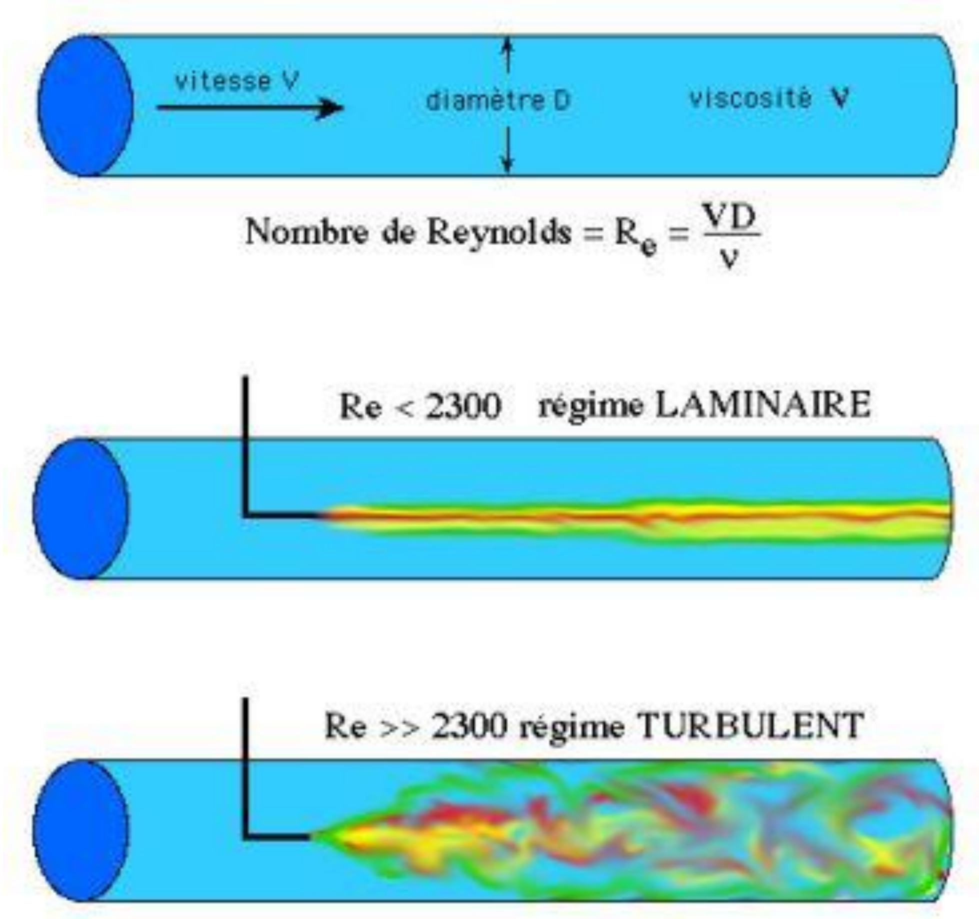
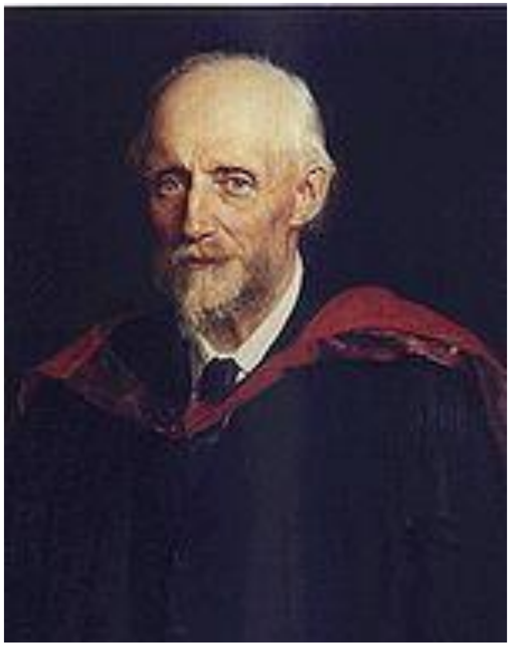
J. Szantyr – Lecture No. 4 – Principles of the Turbulent Flow Theory

The phenomenon of two markedly different types of flow, namely laminar and turbulent, was discovered by Osborne Reynolds (1842 – 1912) in 1883 in his well known experiment concerning the flow in a pipe. He established that the laminar flow occurs up to Re=2300. Above that value the flow becomes unstable and an intensive mixing of fluid in form of vortex, turbulent motion takes place.



The similarity parameter enabling definition of the flow character on the basis of its main parameters is the Reynolds number:

$$\mathrm{Re} = \frac{u \cdot l}{\nu}$$



Osborne Reynolds
1842 - 1912

u – characteristic flow velocity

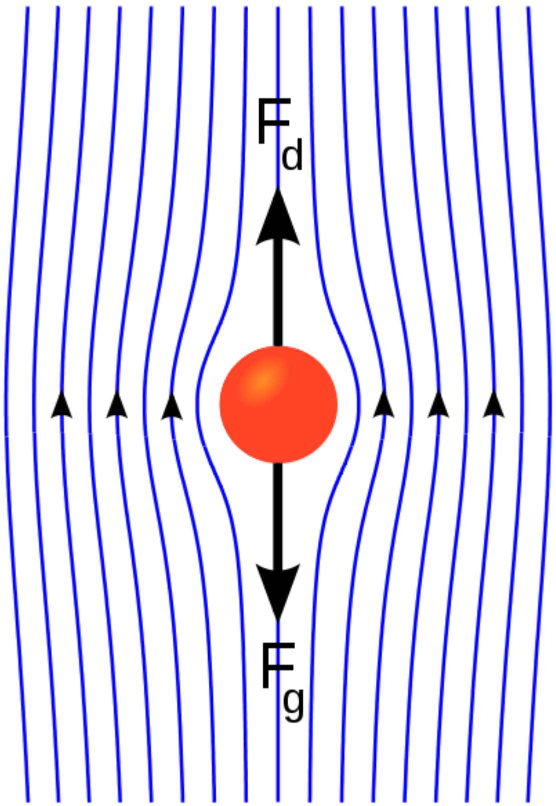
l – characteristic dimension of flow, in most cases parallel to the flow velocity *u* **(with exception of flow in a pipe, where *l*=diameter**)

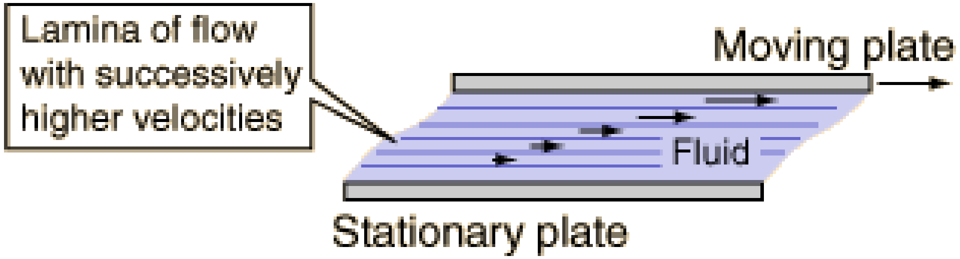
ν – kinematic viscosity coefficient

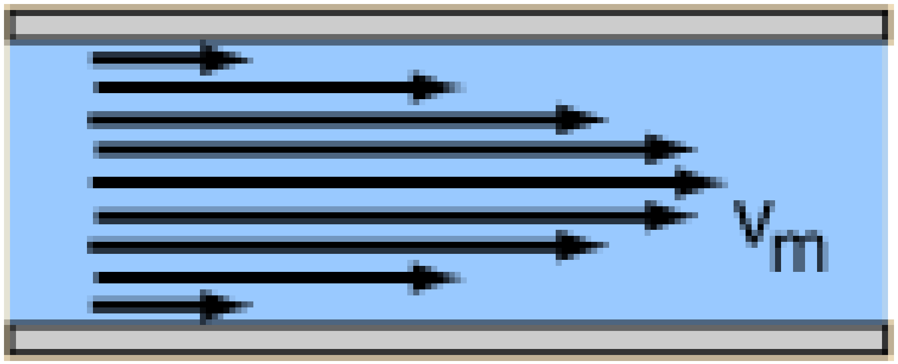
The non-dimensional Reynolds number describes the ratio of inertia forces to the viscous forces in a given flow. High Reynolds number points to the domination of inertia forces, while low Reynolds number means that the flow is dominated by viscous forces.

The mutual relation of inertia and viscous forces, expressed by the Reynolds number, strongly influences the character of flow. At low Reynolds numbers, i.e. with relatively high viscous forces the flow has an orderly character – the elements of fluid move along parallel paths without mixing. Such a flow is called **laminar** or layered. Above certain value of a Reynolds number (known as the **critical number**), the flow loses its inherent stability and the regions of stochastic velocity fluctuations appear. If the Reynolds number increases further up to the so called **transition number**, the regions of stochastic velocity fluctuations cover the entire flow domain. Such a flow is called **fully turbulent**. The critical and transition values of the Reynolds number are not universal, they are different for different flows, e.g. for the flow in a pipe and the flow along a flat plate.

Laminar flow – an orderly motion of fluid along parallel paths, fluid elements do not mix with each other, a purely viscous mechanism of transport of momentum and energy controls the flow

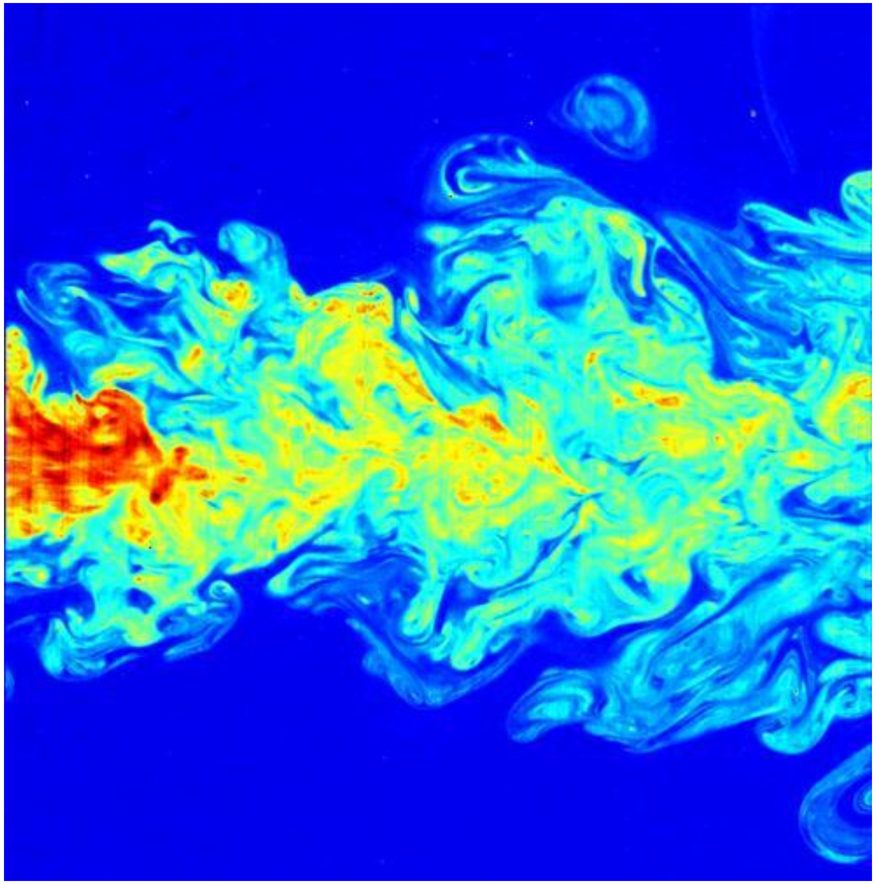






Increase of the Reynolds number (mostly due to increasing flow velocity) leads to the loss of stability of the laminar flow and its conversion into a turbulent flow.

Turbulent flow – chaotic motion of fluid of a stochastic character, unsteady even with steady boundary conditions, fluid elements mix vigorously, leading to the much more intensive exchange of mass, momentum and energy.



Big whirls have little whirls,

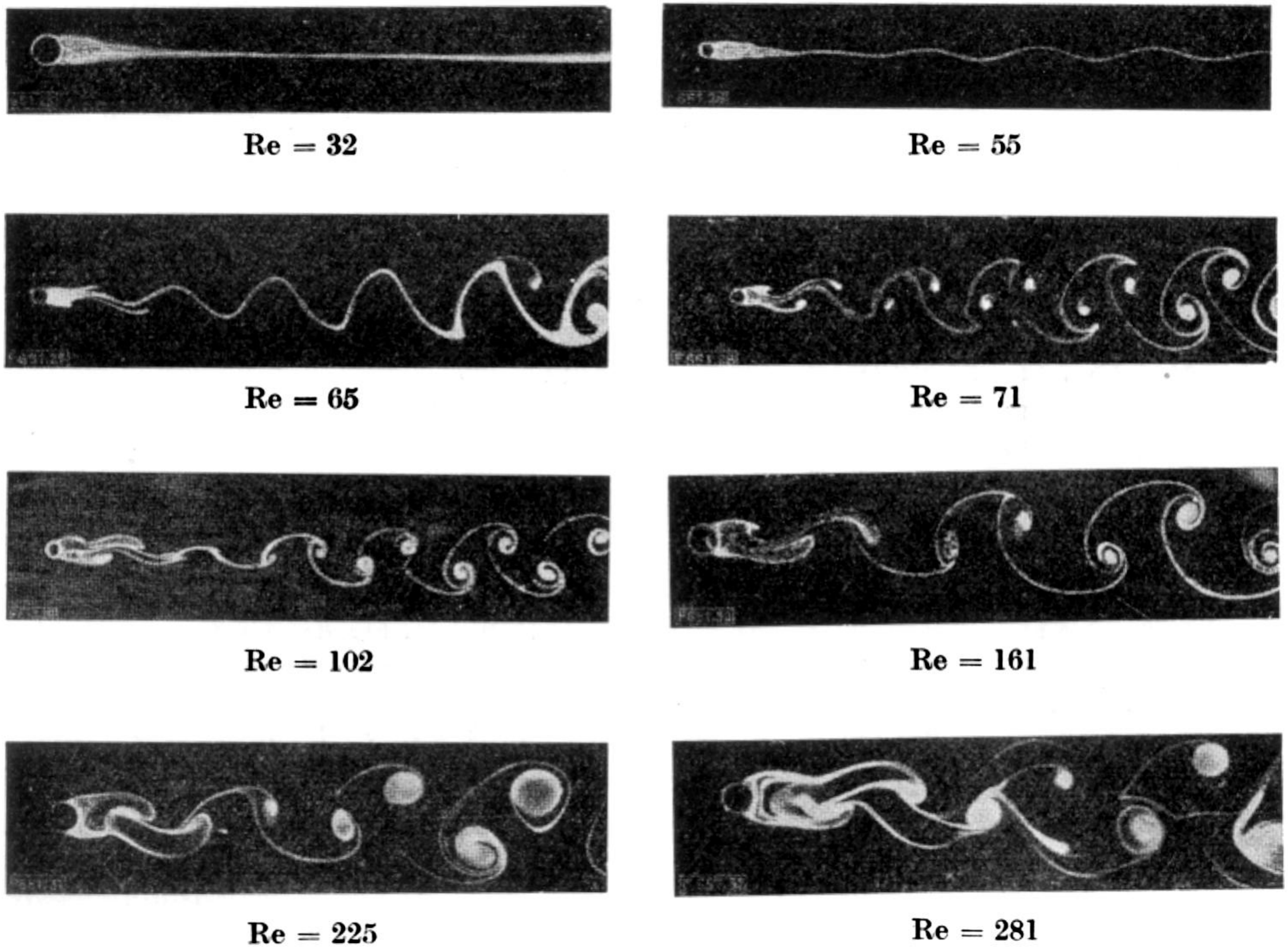
That feed on their velocity

And little whirls have lesser whirls

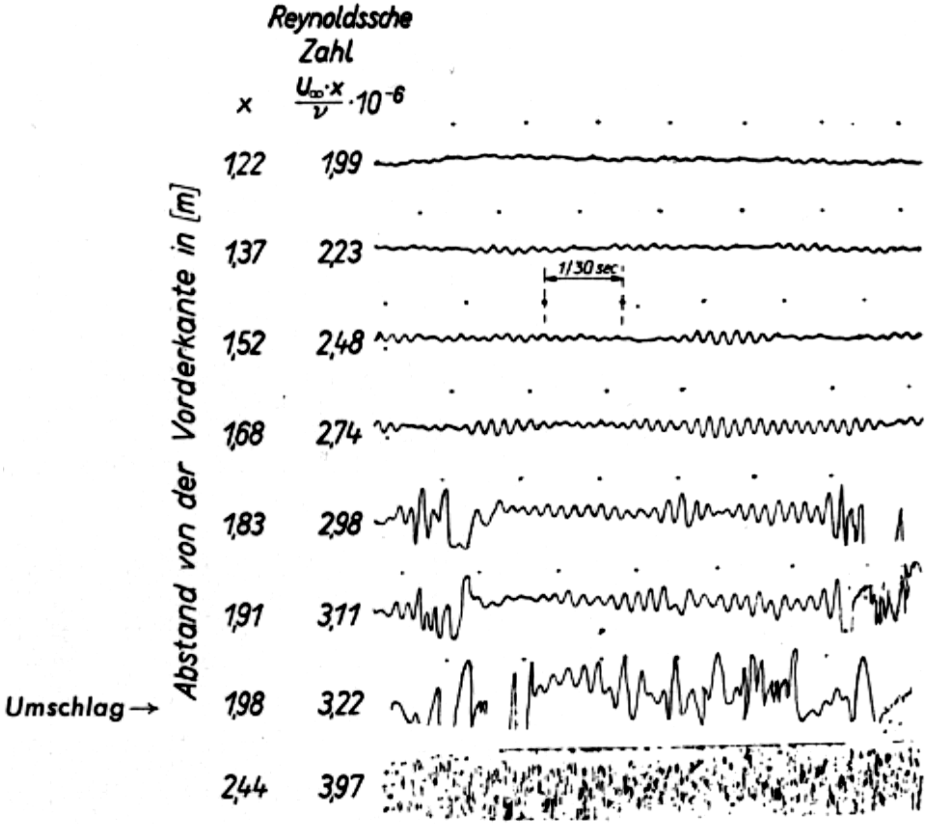
And so on to viscosity

(in the molecular sense)

L.F. Richardson (1922)



The experiment presented above shows the flow around a thin rod placed perpedicularly to the velocity. The consecutive photographs show the gradual loss of flow stability and development of a turbulent flow due to the increase of a Reynolds number.



The drawing shows the process of increasing of the turbulent fluctuations in a flow along a flat plate, i.e. with the growing Reynolds number calculated on the basis of a distance from the plate leading edge (flow direction vertically down).

For the flow in a pipe having circular cross-section (D – diameter) there is:

critical value:

$$\mathrm{Re}_{kr1} = \frac{u \cdot D}{\nu} = 2000$$

transition value:

$$\mathrm{Re}_{kr2} = \frac{u \cdot D}{\nu} = 50000$$

In the flow along a flat plate (x – distance from the edge) there is:

critical value: $$\mathrm{Re}_{kr1} = \frac{u \cdot x_1}{\nu} = 90000$$

transition value: $$\mathrm{Re}_{kr2} = \frac{u \cdot x_2}{\nu} = 1000000$$

In a turbulent flow there is: $$\overline{u} = \overline{U} + \overline{u}'$$ or:

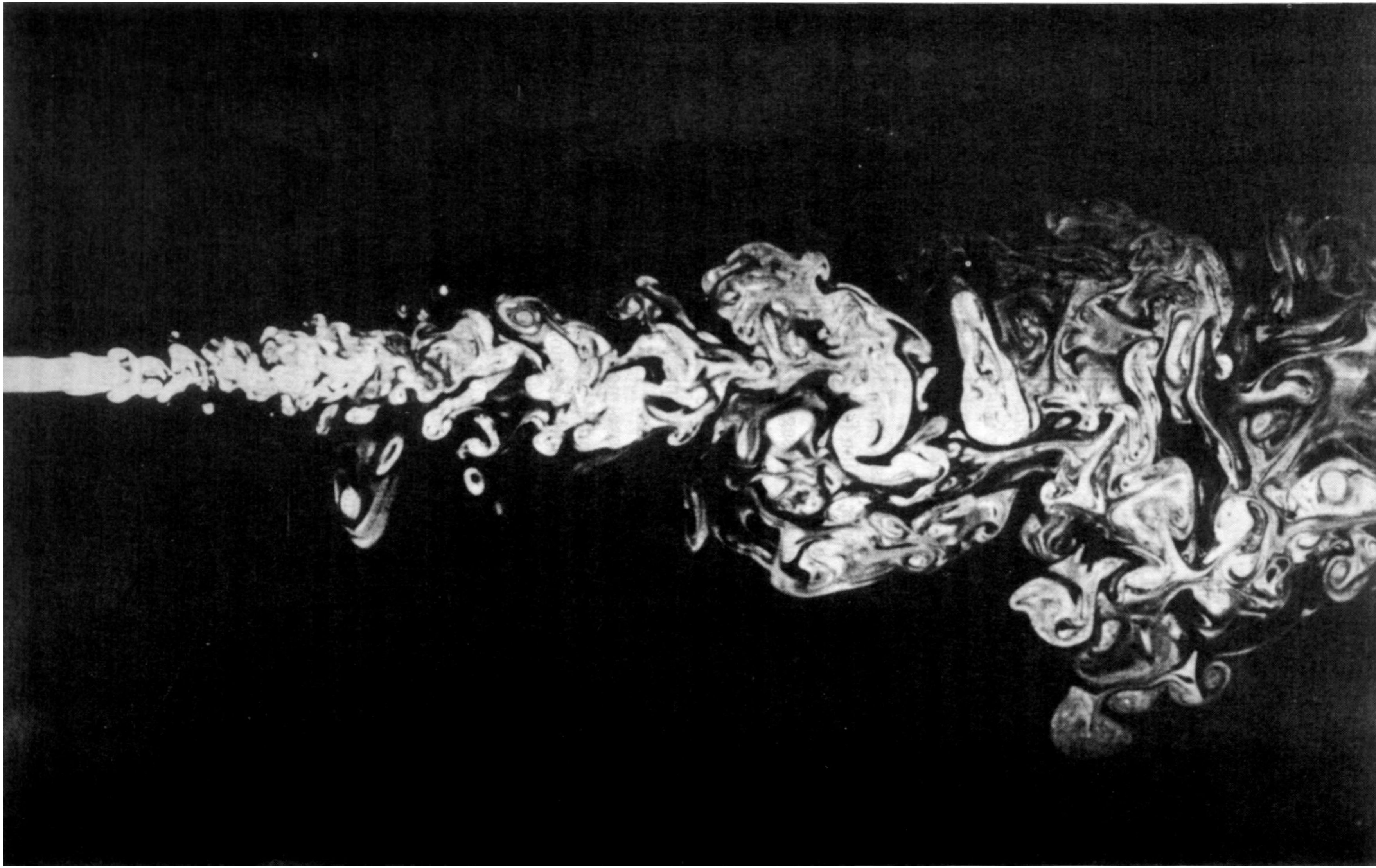
actual velocity=mean velocity+turbulent fluctuation

The measure of turbulence intensity is the degree of turbulence ε:

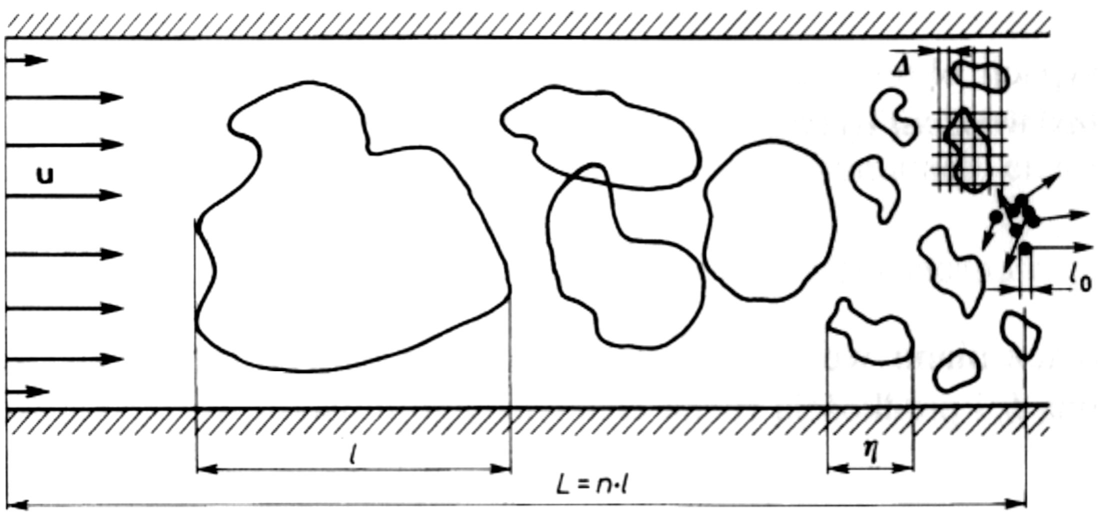
$$\varepsilon = \frac{\sqrt{\frac{1}{3}\left[(u'_x)^2 + (u'_y)^2 + (u'_z)^2\right]}}{\left|\overline{U}\right|}$$

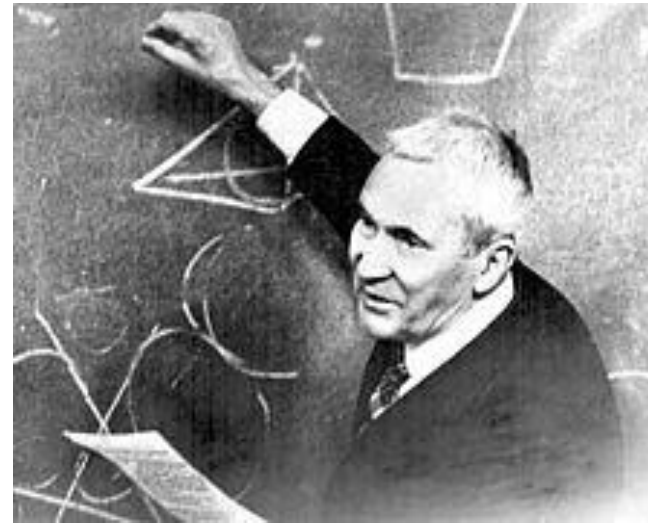
The kinetic energy of turbulence may be defined as:

$$k = \frac{1}{2}\left[(u'_x)^2 + (u'_y)^2 + (u'_z)^2\right]$$



Visualization of a turbulent flow shows vortex structures of different sizes, called the turbulent vortices.





Andrej Kolmogorow
1903 - 1987

The model of Kolmogorov (1941) treats turbulence as a cascade of vortices, transmitting the energy of flowing fluid from the main flow to the molecular motion level.

The largest vortices interact with the main flow and extract their energy from this flow. Their characteristic velocity and characteristic dimension are of the same order as in the main flow (high Re number). This means that they are dominated by inertia, with negligible viscous forces. This leads to disintegration of larger vortices into the smaller and faster rotating ones. The smallest vortices have Re=1 with diameter η=0,1-0,01 mm and frequency10 kHz. The motion of these vortices is retarded by viscous forces (equal to inertia forces) and their energy is dissipated and converted into heat (i.e. internal energy of the molecular motion).

The analysis of physical mechanisms acting in the turbulent motion of fluid leads to the following expressions defining the characteristic magnitude of vortices in the Kolmogorov cascade:

$$\frac{\eta}{l} = \mathrm{Re}_L^{-\frac{3}{4}}$$

where: η – scale of the smallest vortices – Kolmogorov scale

$$\frac{\eta}{l_0} = \frac{\mathrm{Re}_L}{Ma}$$

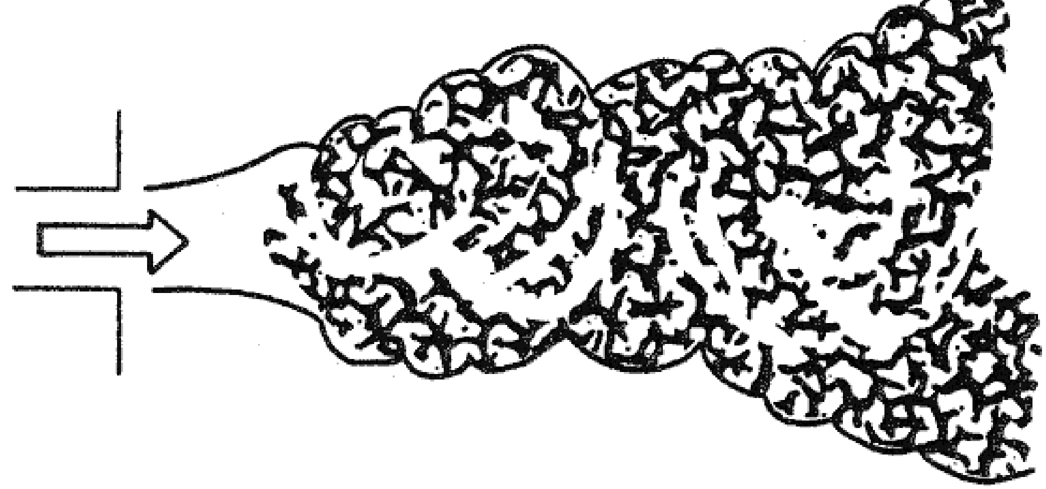
where: *Ma* –Mach number based on the molecular motion velocity

Numerical assessment of the above formulae leads to the following approximate values:

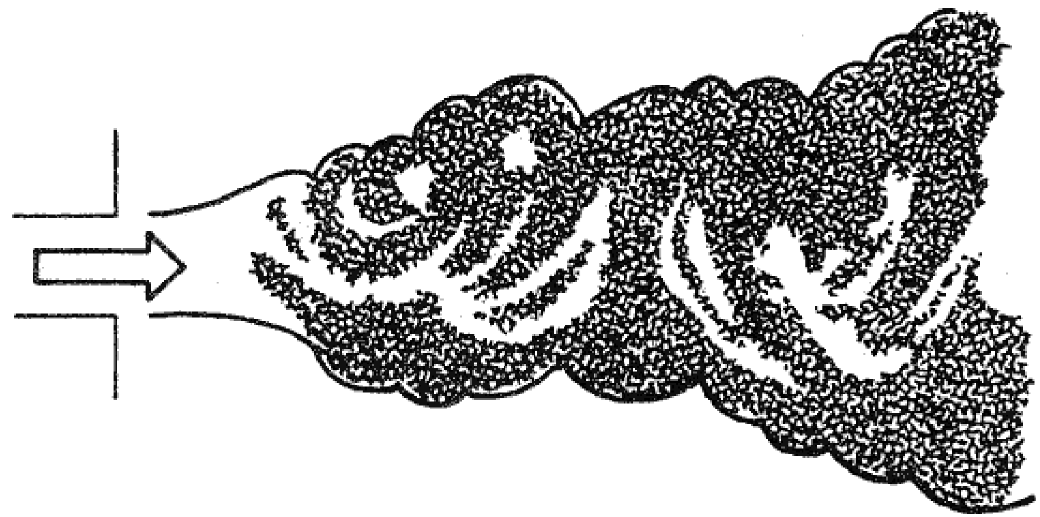
$$\frac{\eta}{l} \approx 10^{-6} \qquad \frac{\eta}{l_0} \approx 10^{2} \qquad \text{or:} \qquad l >> \eta > l_0$$

It should be stressed that the distance between l and η is covered in a continuous way by different vortex sizes, but between η and l_0 there are no intermediate vortex scales.

The outflowing stream a the relatively low Renolds number just above the transition value



The outflowing stream at a high Reynolds number above the transition value



Remarks about the agreement of Kolmogorov theory with the physical reality of turbulent flows:

1. Kolmogorov theory describes quite well the real turbulent flow at high Reynolds numbers above transition.
2. Kolmogorov theory assumes purely stochastic character of turbulence, while in the real turbulent flows there are often large, coherent vortex structures, which behave and may be described in a deterministic way.
3. Kolmogorov theory assumes only unidirectional transport of energy – from large vortices to the small ones – while the experiments confirm the existence of so called backscatter of energy, i.e. transport of energy in the opposite direction.

The mathematical description of the turbulent fluid motion may be done by means of **Reynolds equations**. Reynolds assumed that in a turbulent flow all characteristic parameters (including pressure and velocity) may be presented in the form of sums of their mean values (strictly: slowly varying values) and their turbulent fluctuations:

$$\bar{u} = \overline{U} + \overline{u'} \qquad\qquad p = P + p'$$

where U is the mean flow velocity

$$\overline{U} = \bar{i}U + \bar{j}V + \bar{k}W$$

and u' is the turbulent fluctuation of velocity

$$\overline{u'} = \bar{i}u' + \bar{j}v' + \bar{k}w'$$

Substitution of velocity and pressure defined in the above way into the Navier-Stokes equations leads to the appearance of new surface forces, named the **turbulent stresses:**

$$\rho \frac{DU}{Dt} = \rho f_x - \frac{\partial P}{\partial x} + \mu div grad U + \rho \left[-\frac{\partial \widetilde{u'^2}}{\partial x} - \frac{\partial \widetilde{u'}\widetilde{v'}}{\partial y} - \frac{\partial \widetilde{u'}\widetilde{w'}}{\partial z} \right]$$

$$\rho \frac{DV}{Dt} = \rho f_y - \frac{\partial P}{\partial y} + \mu div grad V + \rho \left[-\frac{\partial \widetilde{u'}\widetilde{v'}}{\partial x} - \frac{\partial \widetilde{v'^2}}{\partial y} - \frac{\partial \widetilde{v'}\widetilde{w'}}{\partial z} \right]$$

$$\rho \frac{DW}{Dt} = \rho f_z - \frac{\partial P}{\partial z} + \mu div grad W + \rho \left[-\frac{\partial \widetilde{u'}\widetilde{w'}}{\partial x} - \frac{\partial \widetilde{v'}\widetilde{w'}}{\partial y} - \frac{\partial \widetilde{w'^2}}{\partial z} \right]$$

The above equations refer to the flow of an incompressible fluid

Normal stresses:

$$\tau_{xx} = -\rho \widetilde{u'^2} \qquad \tau_{yy} = -\rho \widetilde{v'^2} \qquad \tau_{zz} = -\rho \widetilde{w'^2}$$

Tangential (shear) stresses:

$$\tau_{xy} = \tau_{yx} = -\rho \widetilde{u'} \widetilde{v'}$$

$$\tau_{xz} = \tau_{zx} = -\rho \widetilde{u'} \widetilde{w'} \qquad \tau_{yz} = \tau_{zy} = -\rho \widetilde{v'} \widetilde{w'}$$

<u>Turbulent stresses, also known as Reynolds stresses, depend on the values of turbulent fluctuations of velocity and not on the fluid viscosity.</u> It may be proved that they form a symmetrical tensor of turbulent stresses. They constitute 6 additional unknowns in the Reynolds equations describing the turbulent motion of fluid. In order to reduce the number of unknowns and to close the system of equations, an appriopriate model of turbulence must be introduced. <u>Reynolds equations are employed in a majority of commercial CFD codes.</u>