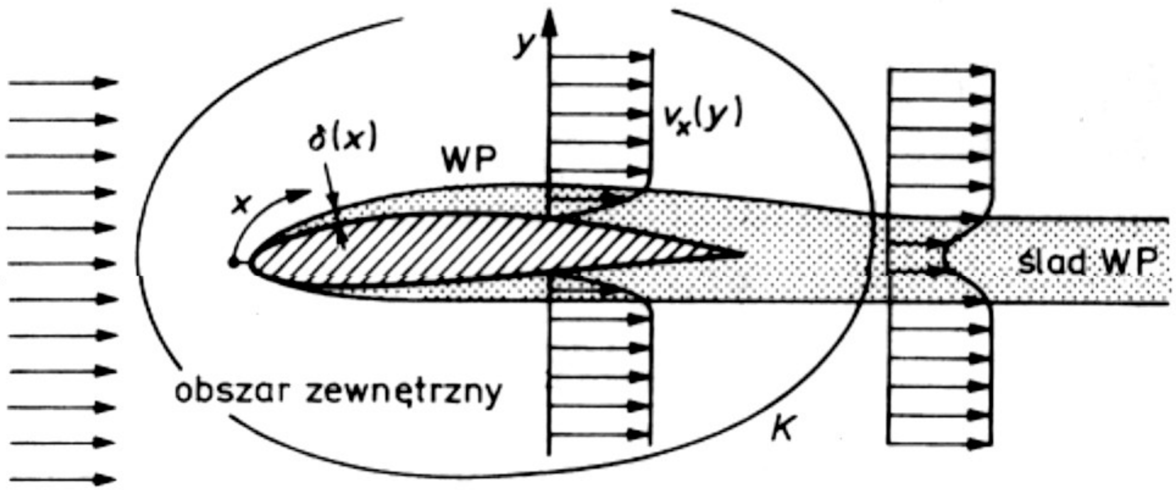
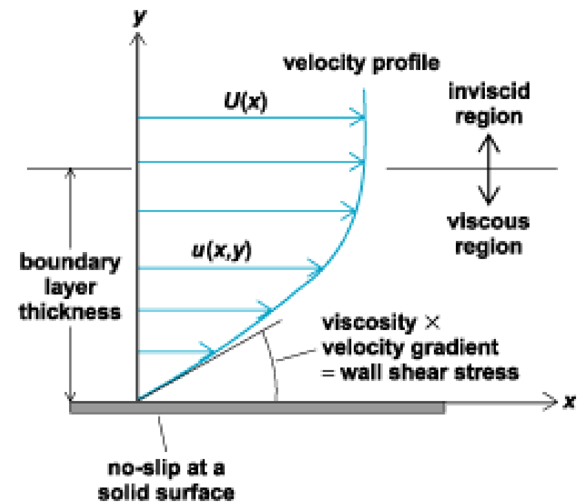
J. Szantyr – Lecture No. 5 – Turbulent Boundary Layer

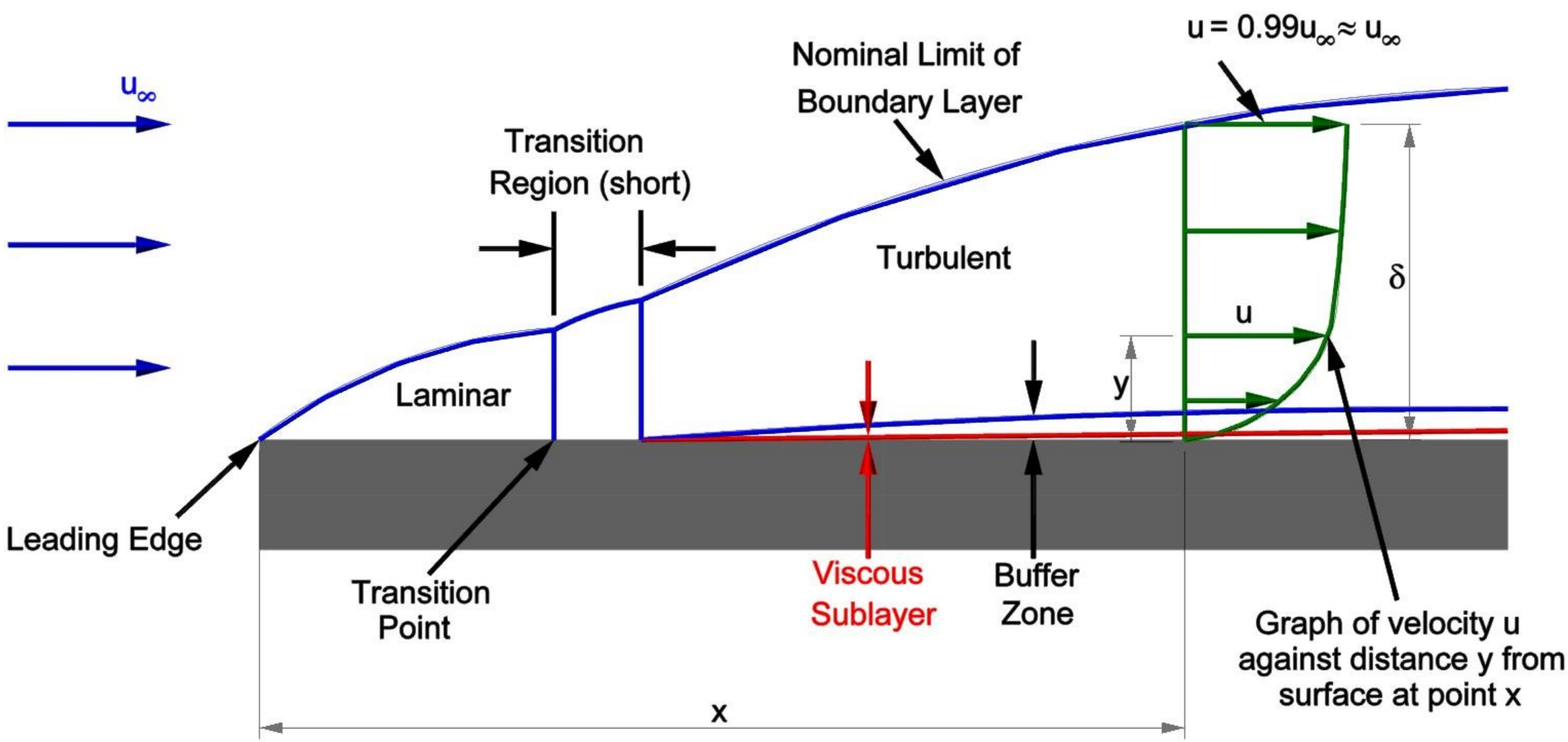




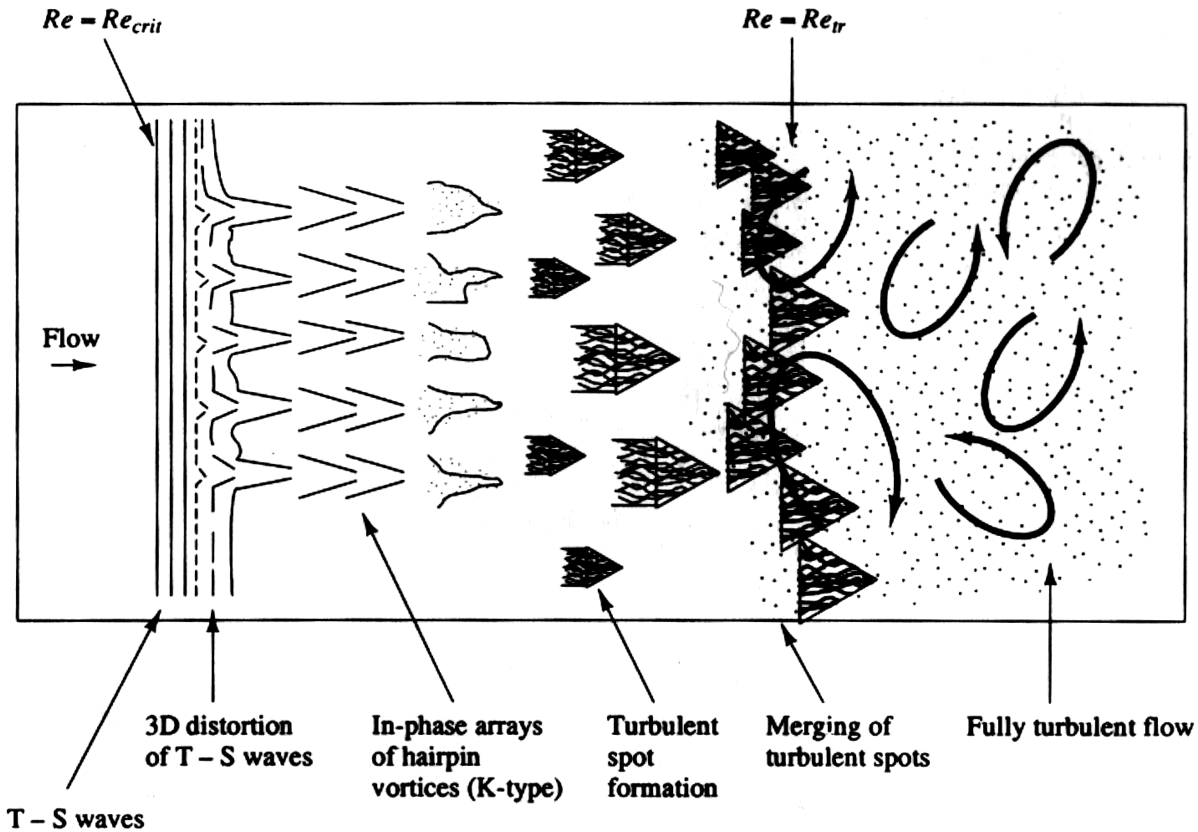
Boundary layer is the part of the flow domain immediately adjacent to the surface of the immersed body. In the boundary layer viscous forces play an important role and significant transverse gradients of the flow velocity are present. Outside the boundary layer the flow may be practically regarded as inviscid. Behind the immersed body the boundary layer forms a viscous wake.

The flow in the boundary layer may be either laminar or turbulent. The boundary layer thickness δ is determined by reaching $u_\delta = 0{,}99 u_\infty$

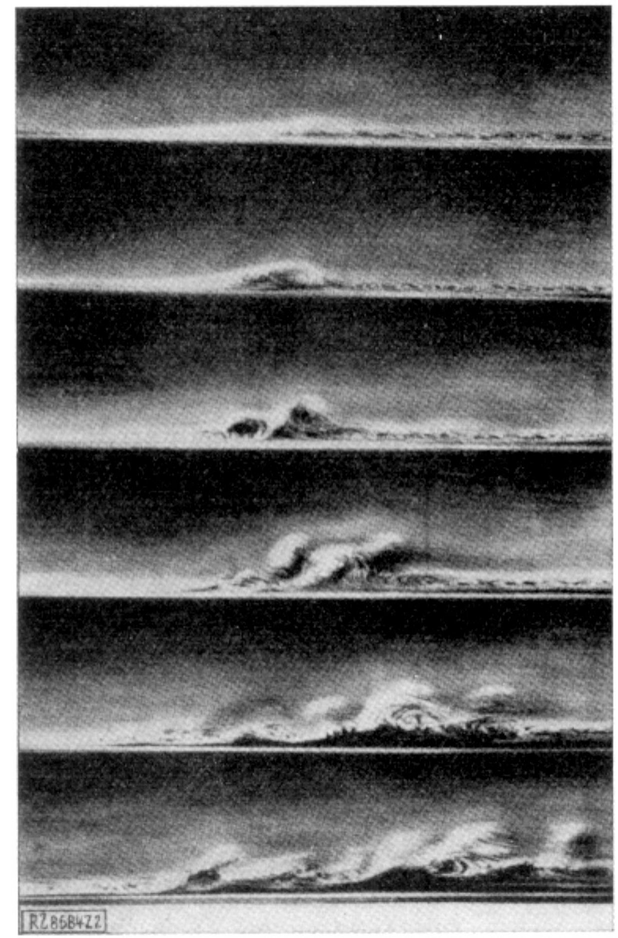
A typical boundary layer on the surface of an immersed body consists of the laminar zone near the leading edge, transition zone and turbulent zone. In the turbulent zone a very thin viscous (laminar) sub-layer exists near the surface, a transition region (buffer zone) located further from the surface and the dominating fully turbulent region.



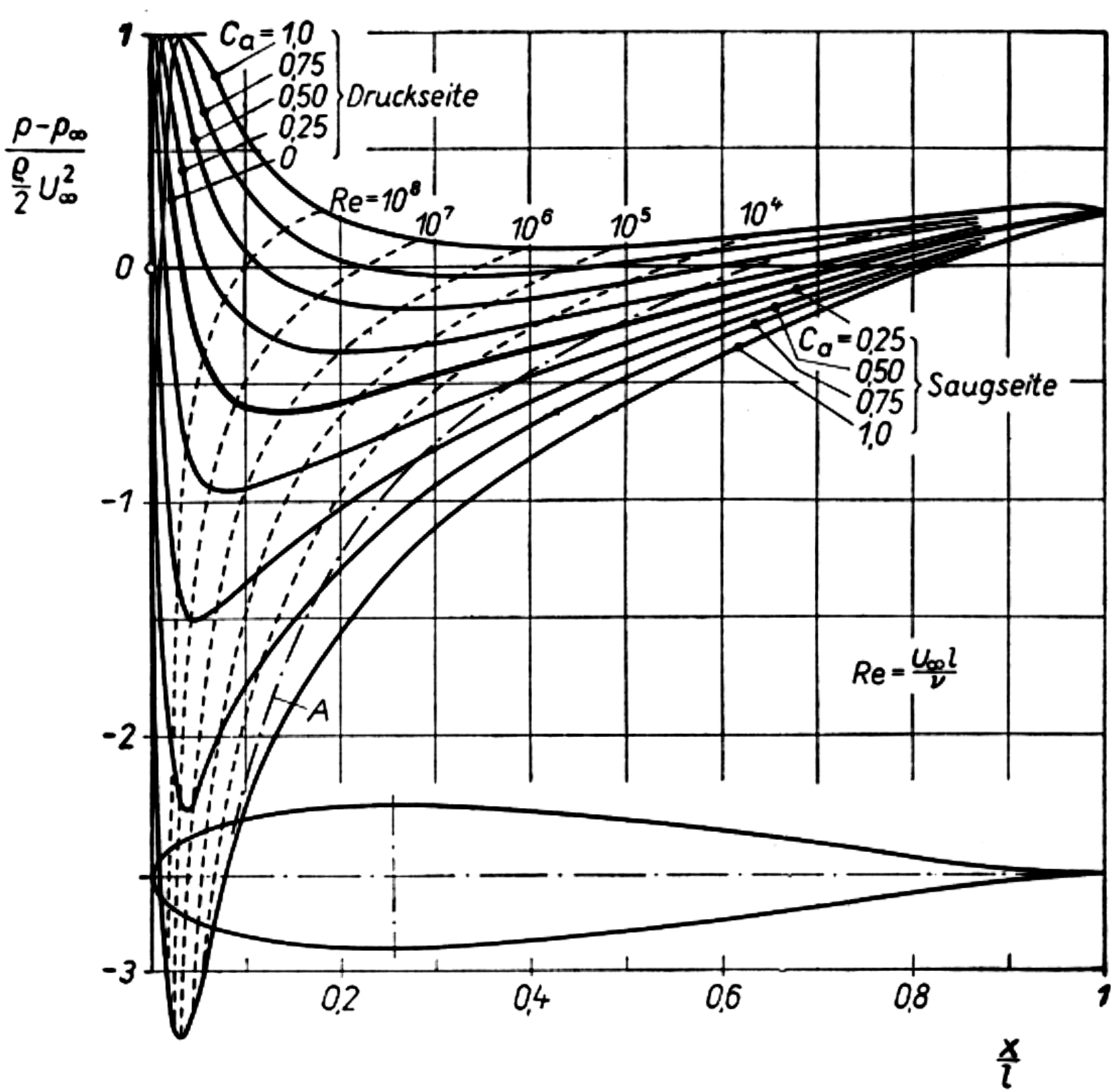
Increase of the Reynolds number leads to destabilization of the laminar boundary layer and to the gradual development of turbulence up to the fully turbulent boundary layer.



Scheme of the process of transition from the laminar to turbulent boundary layer.



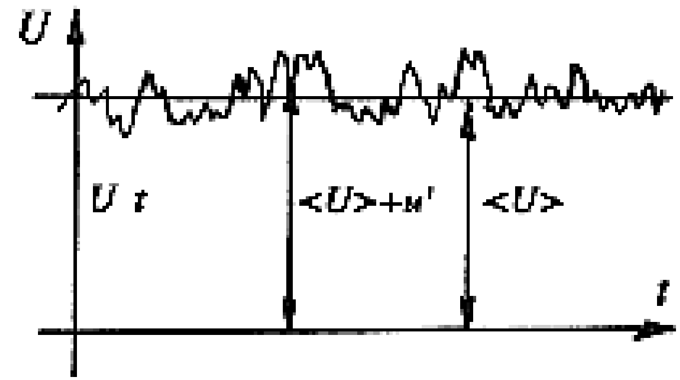
Visualization of the process of turbulence generation



Location of the transition point depends not only on the Reynolds number, but also on the pressure gradient along the boundary layer. The sketch shows this phenomenon on a symmetrical profile set at different angles of attack. This changes the pressure gradient along the profile. The dotted lines show the varying location of the transition points at different Reynolds numbers.

Turbulent boundary layer

Equations analogical to Prandtl equations for a two-dimensional turbulent boundary layer may be developed from the Reynolds equations, treating all parameters as the effect of superposition of their mean (strictly: slowly varying) values and turbulent fluctuations.



$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0$$ <- mass conservation equation

momentum conservation equations:

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right)-\frac{\partial}{\partial y}\left(u'\upsilon'\right)-\frac{\partial}{\partial x}\left(u'^2\right)$$ direction x

$$U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\nu\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right)-\frac{\partial}{\partial x}\left(u'\upsilon'\right)-\frac{\partial}{\partial y}\left(\upsilon'^2\right)$$ direction y

The appropriate assessment of the relative magnitude of terms in these equations enables introduction of simplifications (discarding relatively small terms), what finally leads to the equation describing flow inside a turbulent boundary layer:

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0$$

<- mass conservation equation

$$U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\frac{\partial^2 U}{\partial y^2}-\frac{\partial}{\partial y}\left(u'\upsilon'\right)$$

direction x

$$\frac{\partial P}{\partial y}=0$$

direction y

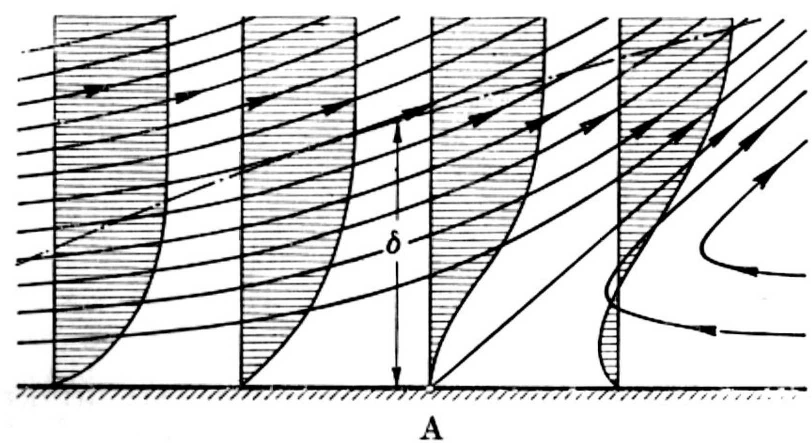
<- momentum conservation equations

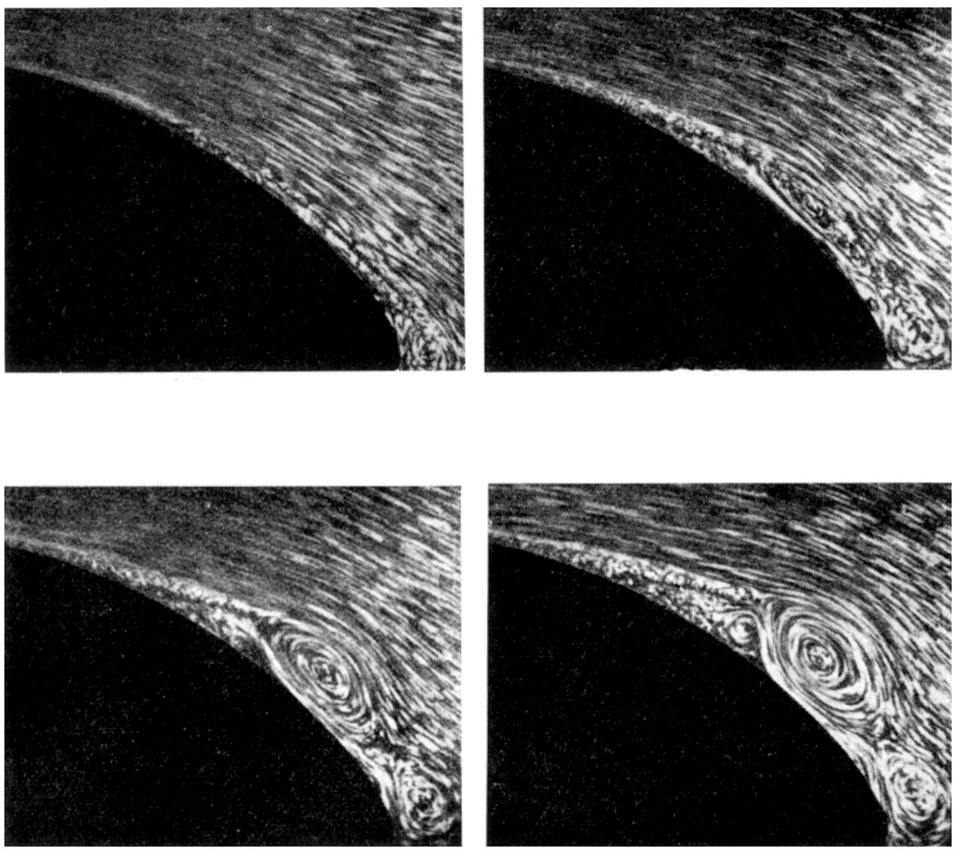
Similarly as in the case of a laminar boundary layer **the mean pressure** remains constant across the layer. The term of turbulent Reynolds stressess requires application of the appropriate **turbulence model** to close the above system of equations.

Generation of a **positive pressure gradient along the boundary layer** (i.e. increase of pressure in the direction of flow), may lead to the so called **separation of the boundary layer**. The motion of the fluid element at the surface is retarded both by the viscous forces and the pressure forces, what leads to its stopping and starting to move in the opposite direction.

At separation point there is: $$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0$$

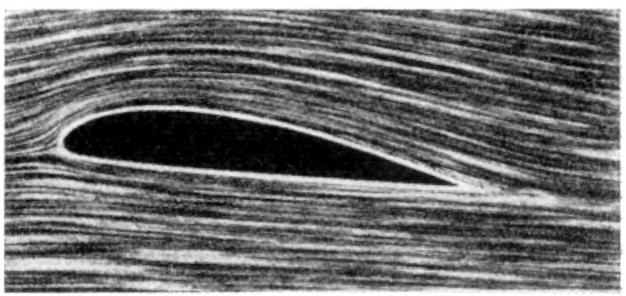
Moreover, the viscous stress at the surface is zero $$\tau_w = 0$$



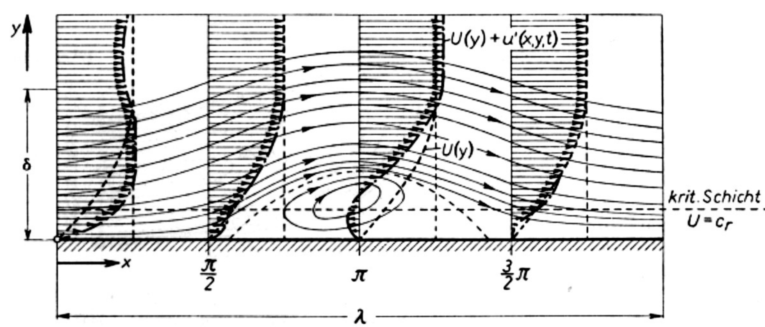


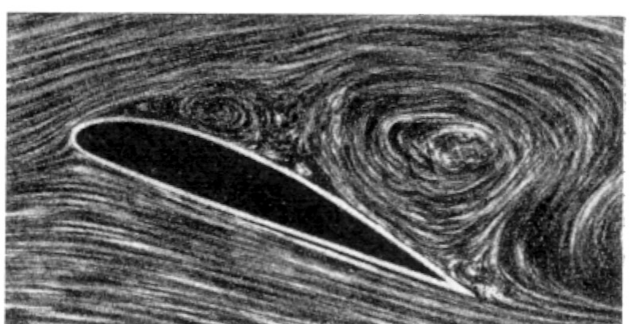
Development of separation with time

Separation may occur both in the laminar and turbulent boundary layer (in the turbulent layer it occurs later, i.e. at the higher positive pressure gradient). Separation of the boundary layer is a detrimental phenomen, it interferes with the operation of the fluid flow machinery and reduces its efficiency, simultaneously generating noise and vibration. The fluid flow machines should be designed in such a way that separation of flow is avoided, at least at their design operating condition.



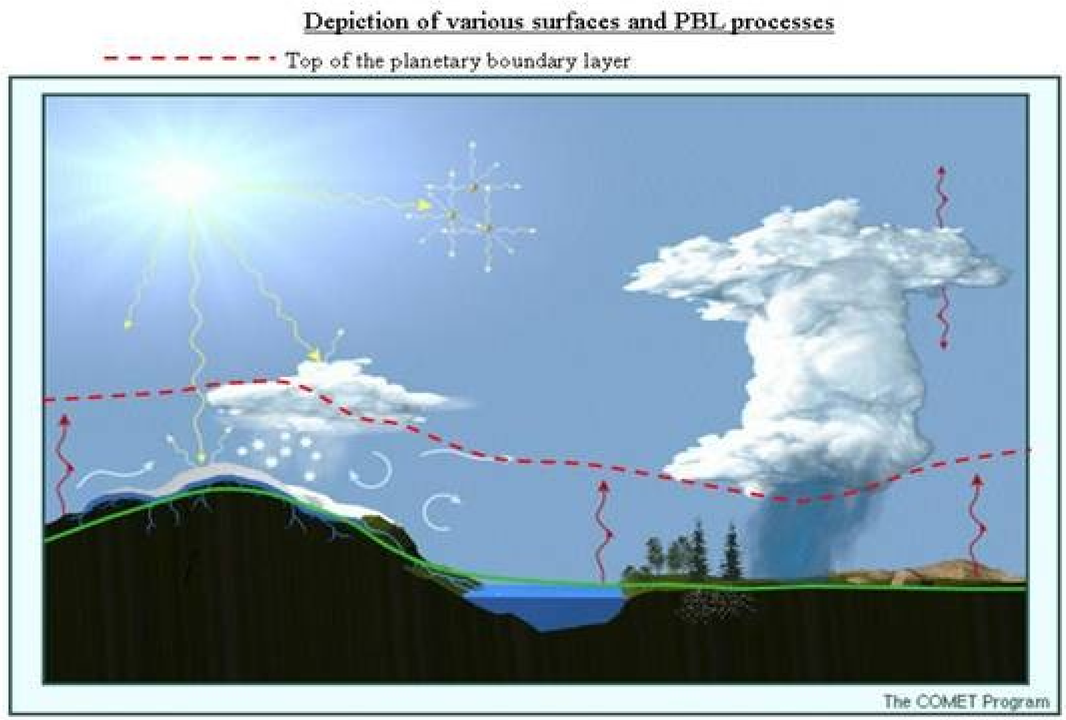
The separation bubble

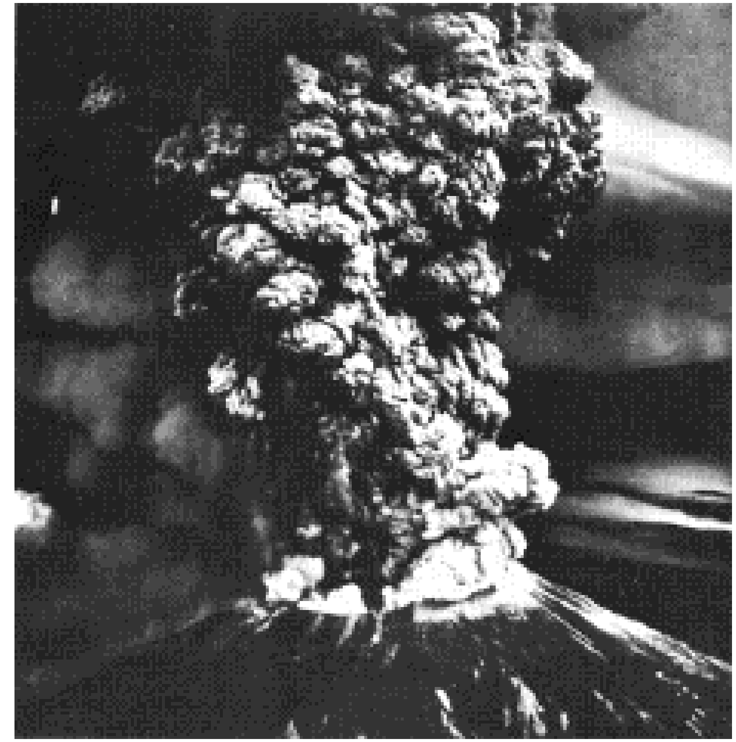




<Separation of the boundary layer on an airfoil at high angle of attack (bottom picture)

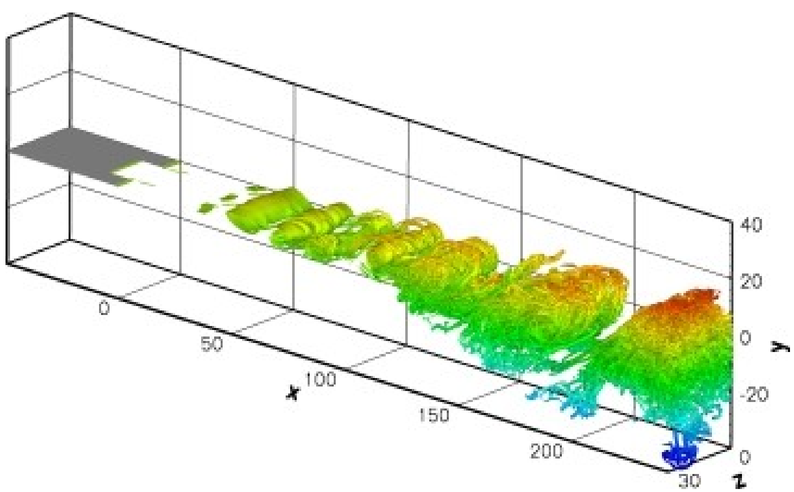
Examples of laminar and turbulent flows in the boundary layers and wakes





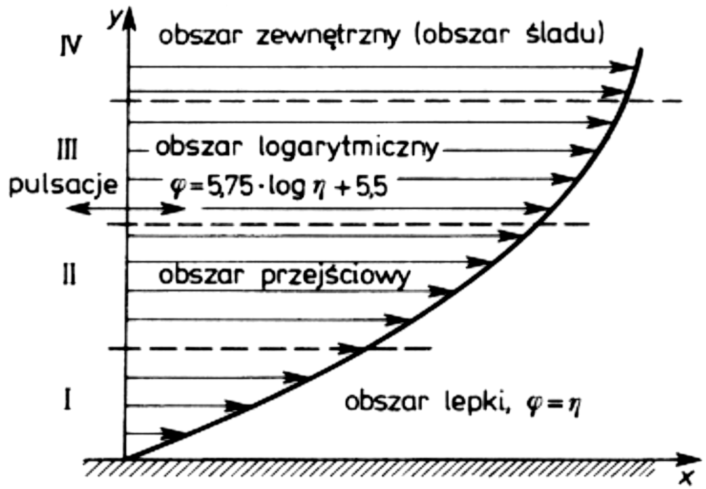
Boundary layer in the atmosphere

Turbulent outflow form a volcano



<- Turbulent wake behind a flat plate calculated using LES method (Large Eddy Simulation)

Structure of the turbulent boundary layer



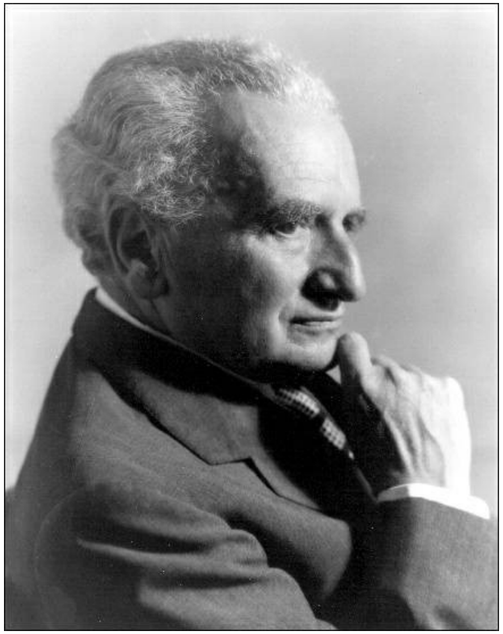
In a turbulent boundary layer several zones may be distinguished across its thickness, with different physical mechanisms governing the flow.

Generally, the boundary layer may be divided into an internal zone covering about 0,2δ from the wall and the external zone outside it. In the external zone the inertia forces dominate the flow. The internal zone may be divided further into a viscous sublayer of thickness about 0,02δ, where the viscous forces and inertia forces are approximately of the same order and a purely viscous mechanism of transport of momentum and energy dominates the flow, and a transitional and „logarithmic" zone, where the turbulent stresses and turbulent mechanism of transport of mass, momentum and energy starts to dominate.

Theodore von Karman has introduced non-dimensional flow velocity and non-dimensional distance from the wall into the mathematical description of the boundary layer:

$$y^+ = \frac{y \cdot u_\tau}{\nu}$$ non-dimensional distance from the wall

where: $$u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$



Theodore von Karman
1881 - 1963

$$u^+ = \frac{u}{u_\tau}$$ non-dimensional velocity of flow

where: ρ – fluid density

ν – kinematic viscosity coefficient of the fluid

τ_w- viscous stress at the wall

Then in the viscous sub-layer there is: $u^{+} = \varphi = y^{+} = \eta$

In the „logarithmic” zone there is: $$u^{+} = \frac{1}{\kappa} \ln y^{+} + C$$

Initially von Karman has established experimentally the constants:

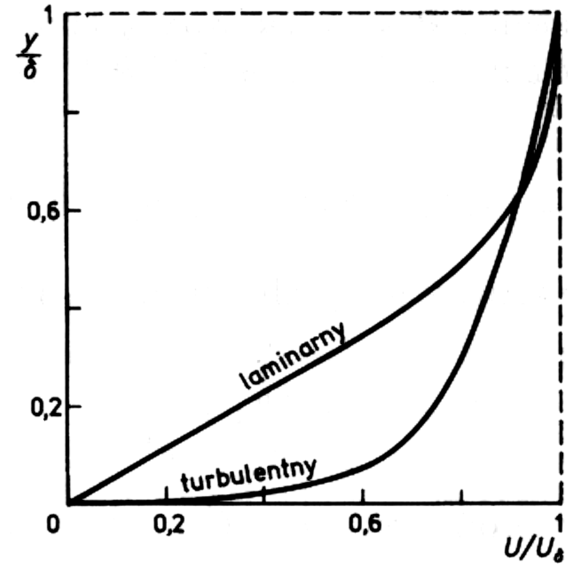
$\kappa = 0{,}41$ $\quad C \approx 5{,}0$ $\quad$ (for smooth walls)

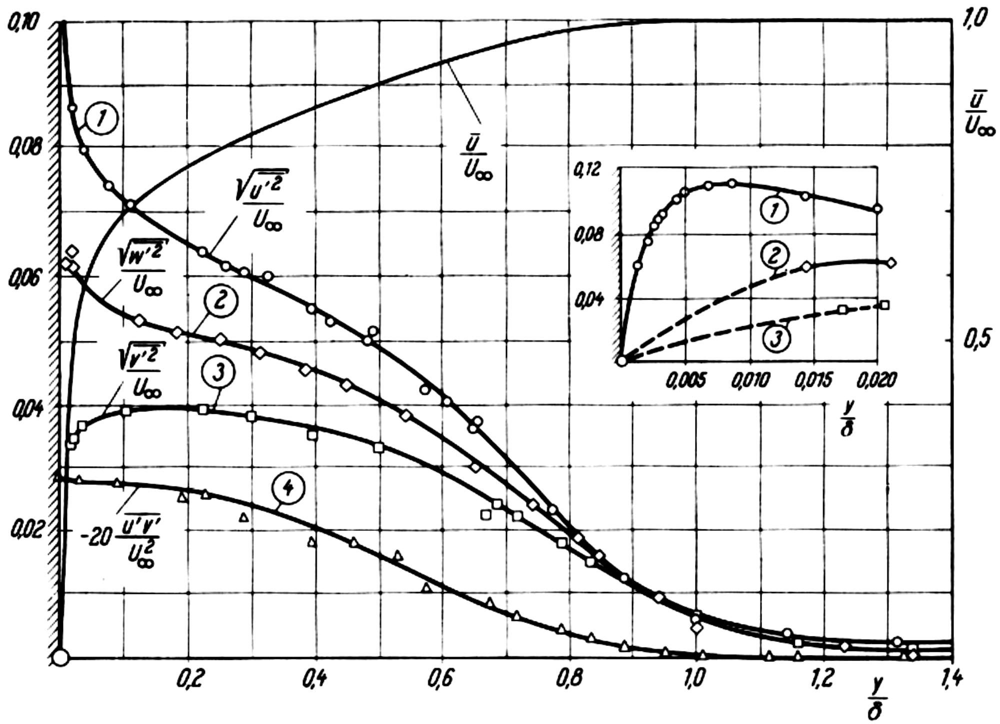
In the transitional zone none of the above relations agrees exactly with reality. A limiting value of the non-dimensional distance from the wall is:

$$y^{+} = 11{,}0$$

Below this value the formula for viscous sub-layer is closer to reality, above this value – the logarithmic formula is more realistic. The above relations form the basis for the so called **law of the wall,** used for correction of the turbulence models employed in the Computational Fluid Dynamics calculations in the regions directly adjacent to the wall.

Due to the joint action of the viscous and turbulent mechanisms of transport of momentum the velocity profile in the turbulent boundary layer is „fuller” than in the laminar boundary layer.





In the turbulent boundary layer very intensive three-dimensional fluctuations of velocity take place. Their amplitudes reach maximum close to the wall. i.e. in the region of the highest gradient of the mean flow velocity.

Several practically useful formulae have been developed in an empirical-theoretical way:

$$\delta_{turb} = \frac{0{,}37 \cdot L}{\sqrt[5]{\mathrm{Re}}}$$

$$C_{fturb} = \frac{0{,}074}{\sqrt[5]{\mathrm{Re}}}$$ for Reynolds numbers $5 \cdot 10^5 < \mathrm{Re} < 10^6$

$$C_{fturb} = \frac{0{,}455}{(\log \mathrm{Re})^{2{,}58}} - \frac{A}{\mathrm{Re}}$$ for $3 \cdot 10^5 < \mathrm{Re} < 10^9$

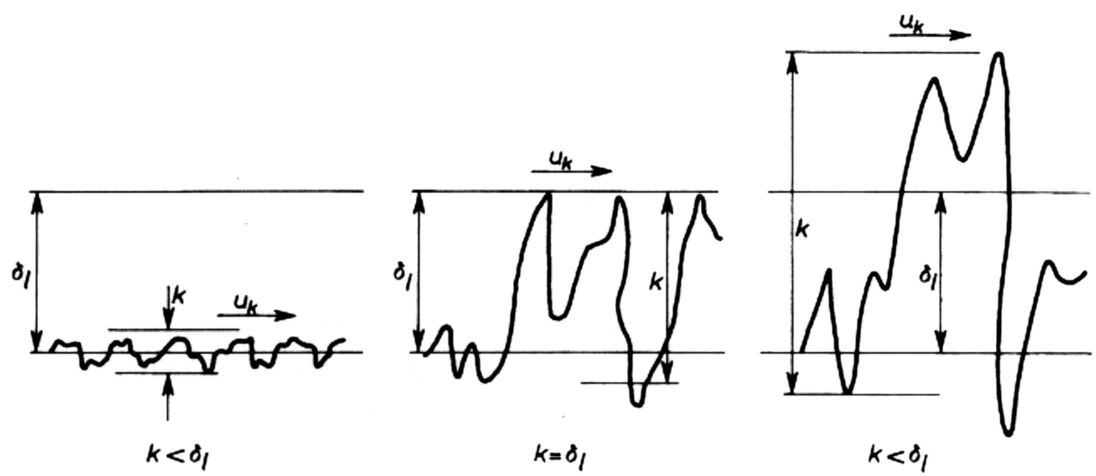
where the constant A is determined in relation to the transition value of the Reynolds number according to the Table:

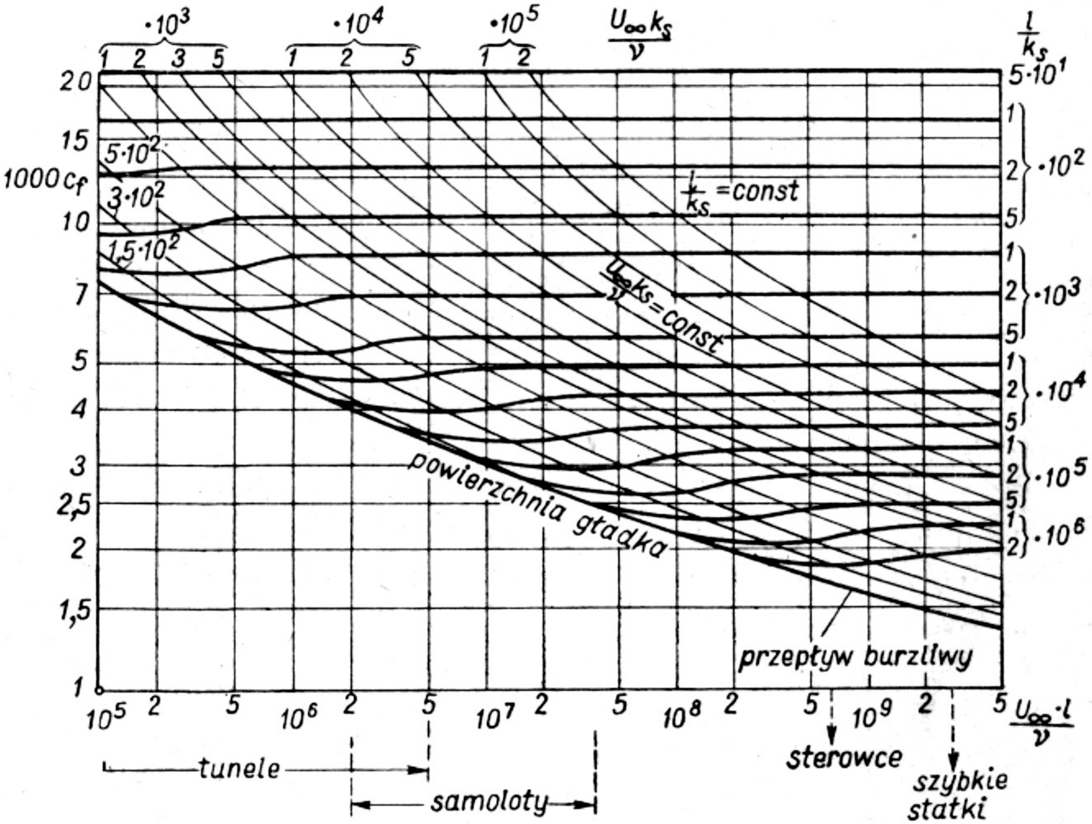
Re_{trans}	A
$3 \cdot 10^5$	1050
$5 \cdot 10^5$	1700
10^6	3300
$5 \cdot 10^6$	5700

The above formulae for the friction coefficient are valid for the smooth surface. In the turbulent flow <u>this coefficient depends also on the surface roughness.</u>

The measure of surface roughness is the mean roughness height k_s

From the point of view of friction resistance the relation between the mean roughness height and the viscous sub-layer thickness in the turbulent boundary layer is very important. If the roughness is completely immersed in the viscous sub-layer, then it does not influence both the velocity profile and friction resistance – such a surface is called **hydrodynamically smooth**. When the surface roughness reaches above the viscous sub-layer, it changes the velocity profile and causes an increase in the friction resistance.





The diagram shows the dependence of the friction coefficient on the inverse relative surface roughness (non-dimensionalised by the wall length l). The values of Reynolds numbers based on the roughness height are also plotted.

There are several empirical formulae for determination of the friction coefficient for rough surface in a turbulent flow, for example:

$$C_{frough} = \left(1{,}89 + 1{,}62 \log \frac{l}{k_s}\right)^{-2{,}5} \quad \text{for} \qquad 10^2 < \frac{l}{k_s} < 10^6$$

Example No. 1

A thin flat plate having dimensions 0.1*0.5 [m] is placed at a zero angle of attack in the flow of water with velocity 10.0 [m/s]. Calculate the friction resistance of the plate for two cases: a) with the longer side perpendicular to the flow direction, b) with the shorter side perpendicular to the flow direction.

Given: kinematic viscosity coefficient ν=0,000001 $\left[m^2/s\right]$

density of water ρ=1000 $\left[kg/m^3\right]$

Case a

$$\mathrm{Re} = \frac{u \cdot L}{\upsilon} = \frac{10.0 \cdot 0.1}{0{,}000001} = 1000000 \qquad C_f = \frac{0.074}{\sqrt[5]{\mathrm{Re}}} = \frac{0.074}{\sqrt[5]{1000000}} = 0{,}00467$$

$$R_f = C_f \frac{1}{2} \rho u^2 S = 0{,}00467 \cdot 0{,}5 \cdot 1000{,}0 \cdot 10.0^2 \cdot 2 \cdot 0{,}1 \cdot 0{,}5 = 23.35[N]$$

Case b

$$\mathrm{Re} = \frac{u \cdot L}{\upsilon} = \frac{10.0 \cdot 0{,}5}{0{,}000001} = 5000000 \quad C_f = \frac{0.074}{\sqrt[5]{\mathrm{Re}}} = \frac{0.074}{\sqrt{5000000}} = 0{,}00338$$

$$R_f = C_f \frac{1}{2} \rho u^2 S = 0{,}00338 \cdot 0{,}5 \cdot 1000{,}0 \cdot 10.0^2 \cdot 0{,}1 = 16.9[N]$$

Conclusion: change of setting of the plate with respect to the flow may cause **a significant increase in the friction resistance** at the same flow parameters.

Example No. 2

A thin flat plate of dimensions 1,0*1,0 [m] is placed at a zero angle of attack in the flow of water with velocity 10 [m/s]. Determine the friction resistance for two cases: a) for a smooth plate, b) for a plate with relative roughness of 0,0001.

Given: kinematic viscosity coefficient ν=0,000001 $\left[\frac{m^2}{s}\right]$

density of water ρ=1000,0 $\left[\frac{kg}{m^3}\right]$

Case a

$$\mathrm{Re} = \frac{uL}{\upsilon} = \frac{10{,}0 \cdot 1{,}0}{0{,}000001} = 10000000$$

High value of the Reynolds number requires a more complex formula

$$C_{fturb} = \frac{0{,}455}{\left(\log \mathrm{Re}\right)^{2,58}} - \frac{A}{\mathrm{Re}} = \frac{0{,}455}{\left(\log 10^7\right)^{2,58}} - \frac{1050}{10^7} = 0{,}00263$$

$$R_{fturb} = C_{fturb}\frac{1}{2}\rho u^2 S = 0{,}00264 \cdot 0{,}5 \cdot 1000{,}0 \cdot 10{,}0^2 \cdot 2{,}0 = 264[N]$$

Case b

$$C_{frough} = \left(1{,}89 + 1{,}62\log 10000\right)^{-2{,}5} = 0{,}00494$$

$$R_{frough} = C_{frough}\frac{1}{2}\rho u^2 S = 0{,}00494 \cdot 0{,}5 \cdot 1000{,}0 \cdot 10{,}0^2 \cdot 2{,}0 = 494[N]$$

Conclusion: surface roughness has a serious effect on the friction resistance in the turbulent boundary layer; it may lead to more than doubling of the friction resistance with respect to the smooth surface.

Example No. 3

On a flat plate of length L=1 [m] in a flow of water at Re=100000 laminar or turbulent boundary layers are both possible. What would be the thickness of each of the boundary layers at the trailing edge of the plate?

Laminar:
$$\delta_{lam} = \frac{5L}{\sqrt{\mathrm{Re}}} = \frac{5 \cdot 1}{\sqrt{100000}} = 0{,}0158[m]$$

Turbulent:
$$\delta_{turb} = \frac{0{,}37L}{\sqrt[5]{\mathrm{Re}}} = \frac{0{,}37 \cdot 1}{\sqrt[5]{10^5}} = 0{,}037[m]$$

Conclusion: in the comparable flows the turbulent boundary layer is more than twice thicker than the corresponding laminar boundary layer. This is the consequence of a more intensive transport of momentum and energy of the fluid in the turbulent layer.

Temperature boundary layer

In certain problems (e.g. in heat exchangers) it is important to determine the temperature distribution in the boundary layer. Assuming that the flow is stationary and the Reynolds number is more than1000, the following relation may be developed:

$$\theta = \frac{T_w - T(y)}{T_w - T_\infty} = \frac{u(y)}{u_\infty} \qquad \text{for} \qquad \Pr = \frac{c\mu}{\lambda} = 1{,}0 \quad \text{(Prandtl number)}$$

where: θ – non-dimensional temperature

T_w - temperature on the wall

T_∞ - temperature far from the wall

If in a stationary flow the Prandtl number is equal 1, then the non-dimensional temperature profile θ in the boundary layer is identical to the profile of non-dimensional velocity. At Pr>1 the gradient of temperature inside the layer is larger than the gradient of velocity, and at Pr<1 – smaller.