J. Szantyr – Lecture No. 10 – State of stress in the fluid

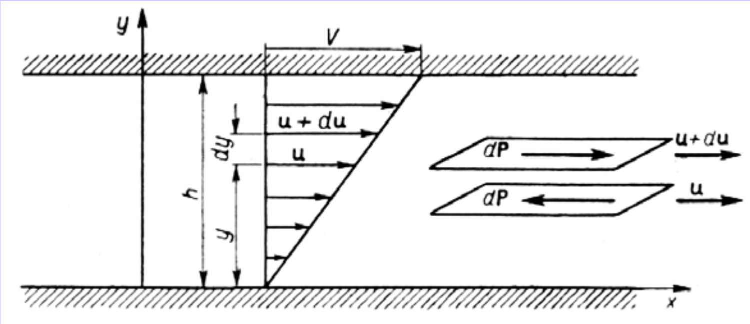
It may be proved that the tensor of stress in the fluid is symmetrical, i.e.: $\tau_{xy} = \tau_{yx}$ etc.

This reduces the number of unknown viscous stresses to 6, which must be determined on the basis of the selected model of the fluid. In most cases the Newtonian model of fluid is employed.

The Newtonian model of fluid is based on the following assumptions:

-the fluid is isotropic, i.e. it has the same properties in all directions,

-the stresses in the fluid are linear functions of the rate of strain.



$$\tau_{yx} = \mu \frac{\partial u}{\partial y}$$

where:

μ -the dynamic viscosity coefficient

In the three-dimensional flow of a compressible fluid the Newtonian fluid model is described by the following relations:

$$\tau_{xx} = 2\mu \frac{\partial u}{\partial x} + \lambda div\bar{u}$$

$$\tau_{yy} = 2\mu \frac{\partial v}{\partial y} + \lambda div\bar{u}$$

$$\tau_{zz} = 2\mu \frac{\partial w}{\partial z} + \lambda div\bar{u}$$

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

$$\tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\right)$$

$$\tau_{xz} = \tau_{zx} = \mu\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

where:

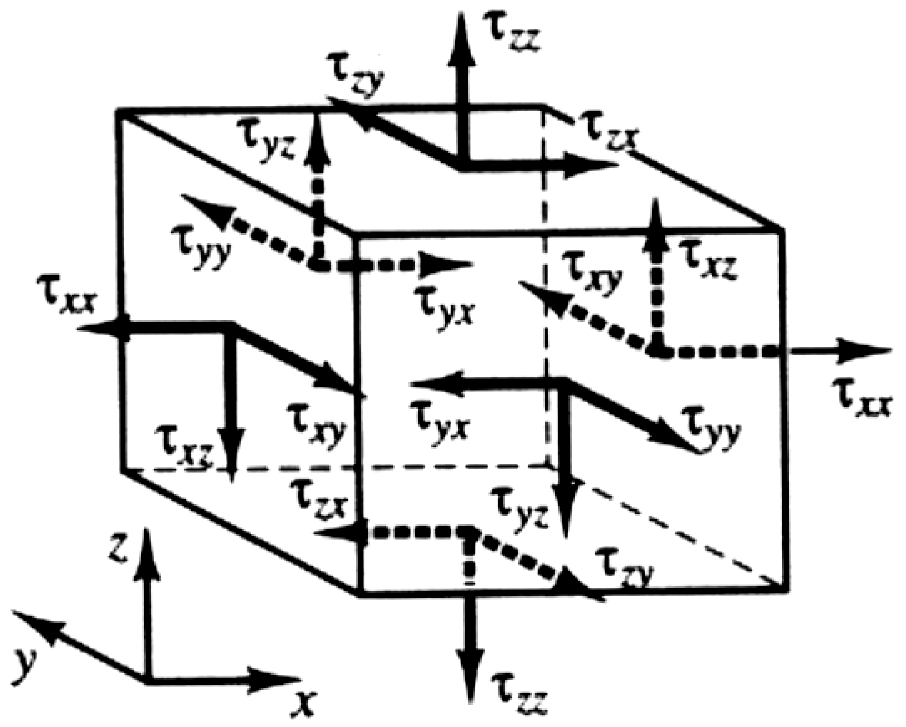
$$div\bar{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

λ – volumetric viscosity coefficient

According to the Stokes hypothesis we have:

$$\lambda = -\frac{2}{3}\mu$$

In the incompressible fluid $div\bar{u} = 0$ so the second terms of normal stresses are reduced to zero.



The above relations describe the normal and shearing viscous stresses as shown in the picture. As these relations link the stress field to the velocity field, their substitution to the momentum conservation equation reduces the number of unknowns. This is demonstrated in the next lecture.

In tensor notation we have: $$[P] = -\left(p + \frac{2}{3}\mu div\bar{u}\right)[E] + 2\mu[D]$$

where:

$[E]$ - unit tensor

$[D]$ - rate of strain tensor

In the incompressible fluid we have: $[P] = -p[E] + 2\mu[D]$