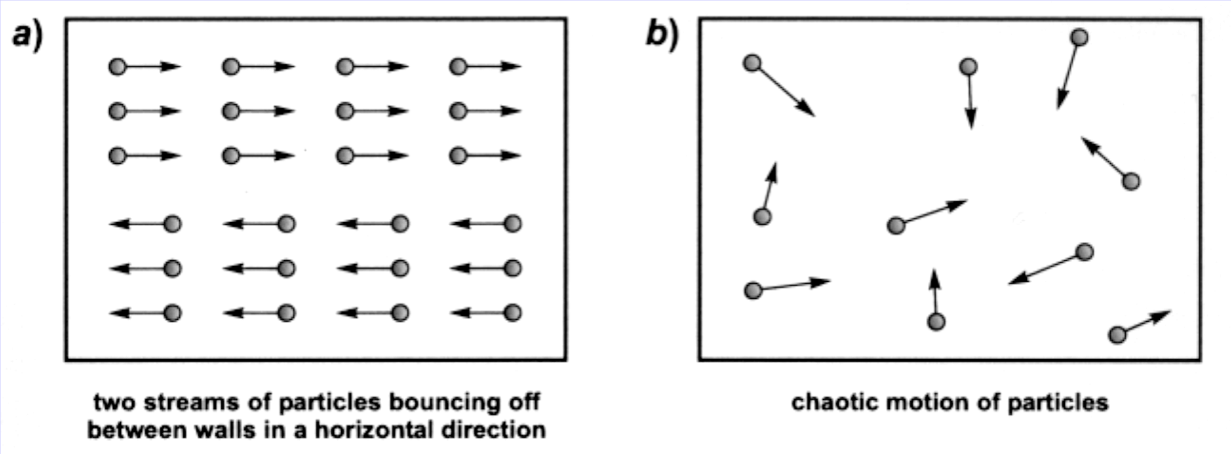
J. Szantyr – Lecture No. 13 – Balance of entropy equation

Entropy is the function of the fluid state parameters and it is the measure of chaos in the molecular motion and the measure of „useless” energy of the given system.



a) low entropy system   b) high entropy system

Unit of entropy S $\left[\frac{J}{K}\right]$

Unit of the specific entropy s $\left[\frac{J}{kg \cdot K}\right]$

Features of entropy

Entropy is transported with heat according to the Clausius formula:

$$j_s = \frac{1}{T} j$$

where:
j_s stream of entropy

j stream of heat

T temperature at which transport takes place

Entropy changes with the fluid state parameters (Gibbs formula):

$$T \frac{Ds}{Dt} = \frac{De}{Dt} + p \frac{D}{Dt}\left(\frac{1}{\rho}\right)$$

where: p - pressure
e – fluid internal energy
ρ - fluid density

The second law of thermodynamics: in any real process the sum of changes of entropy of all the bodies taking part in the process is always positive.

The rate of change (i.e. the material derivative) of entropy in the fluid volume *V(S)* is equal to the production of entropy inside this volume and the stream of entropy through the fluid surface *S*.

$$\frac{D}{Dt}\int_V \rho s dV = \int_V \dot{s} dV - \int_{S(V)} \bar{j}_s \bullet \bar{n} dS$$

Where: $\dot{s}$ volumetric intensity of the entropy sources

The above equation may be converted into the form of a single volumetric integral:

$$\int_V \left(\rho \frac{Ds}{Dt} - \dot{s} + div \frac{\bar{j}}{T} \right) dV = 0$$

As the fluid volume V was arbitrarily selected, the function under the intergal must be also equal zero, what leads to the balance of entropy equation in the differential form (i.e. for a fluid element):

$$\rho \frac{Ds}{Dt} = \dot{s} - div \frac{\bar{j}}{T}$$

If we use the Gibbs formula, we obtain:

$$\dot{s} = \frac{\rho}{T}\frac{De}{Dt} - \frac{p}{\rho T}\frac{D\rho}{Dt} + div\frac{\bar{j}}{T}$$

The above equation may be transformed using the equations of mass conservation, momentum conservation, energy conservation and the thermal conductivity theorem of Fourier:

$$\dot{s} = \dot{s}_M + \dot{s}_T = \frac{\mu}{T}\left[\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)^2 + \left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right)^2 + \left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right)^2 + \right.$$

$$\left. + \frac{2}{3}\left(\frac{\partial u_x}{\partial x} - \frac{\partial u_y}{\partial y}\right)^2 + \frac{2}{3}\left(\frac{\partial u_x}{\partial x} - \frac{\partial u_z}{\partial z}\right)^2 + \frac{2}{3}\left(\frac{\partial u_y}{\partial y} - \frac{\partial u_z}{\partial z}\right)^2\right] + \frac{\lambda}{T^2}(gradT)^2$$

Fourier theorem: $\bar{j} = -\lambda gradT$

The balance of entropy equation in the above form illustrates the continuous process of **dissipation of the mechanical energy** of the flowing fluid and its conversion into heat.

Remarks:

-both terms of the intensity of the volumetric sources of entropy are always non-negative,

-mechanical sources of entropy $\dot{s}_M$ are equal zero when μ=0, and thermal sources of entropy $\dot{s}_T$ are equal zero when λ=0 , what leads to the model of inviscid and non-conducting fluid,

-the above equation shows that the internal fluid energy depends on:

a) entropy processes (combustion, chemical reactions, internal friction of the fluid),

b) change of fluid density (compression or expansion),

c) addition or subtraction of heat.