# J .Szantyr – Lecture No. 17 – Similarity of flows II

## Dimensional analysis of the energy conservation equation

The initial form of the energy conservation equation:

$$\rho'\left[\frac{\partial}{\partial t'}\left(\frac{u'^2}{2}+c'T'\right)+\left(\overline{u}'\bullet grad\right)\left(\frac{u'^2}{2}+c'T'\right)\right]=\rho'\overline{f}'\bullet\overline{u}'-div\left(p'[E]\overline{u}'\right)+$$

$$-div\left(\frac{2}{3}\mu' div\overline{u}'[E]\overline{u}'-2\mu'[D]'\overline{u}'\right)+div\left(\lambda' gradT'\right)$$

It is necessary to introduce the additional scale coefficients:

$$c'=\alpha_c c \qquad T'=\alpha_T T \qquad \lambda'=\alpha_\lambda \lambda$$

The identity of the energy conservation equation in both scales leads to the condition:

$$\frac{\alpha_\rho \alpha_u^2}{\alpha_l}=\frac{\alpha_\rho \alpha_c \alpha_T}{\alpha_t}=\frac{\alpha_\rho \alpha_u^3}{\alpha_l}=\frac{\alpha_\rho \alpha_u \alpha_c \alpha_T}{\alpha_l}=\alpha_\rho \alpha_f \alpha_u=\frac{\alpha_\rho \alpha_u}{\alpha_l}=\frac{\alpha_\mu \alpha_u^2}{\alpha_l^2}=\frac{\alpha_\lambda \alpha_T}{\alpha_l^2}$$

From the above condition we obtain the already known Strouhal, Froude, Euler and Reynolds numbers, together with two new criteria of similarity:

Eckert number: $$Ec = \frac{u^2}{cT} = \frac{u'^2}{c'T'}$$

Eckert number defines the ratio of the macroscopic kinetic energy of the fluid motion to the energy of molecular motion (internal energy) of the fluid.

Prandtl number: $$\Pr = \frac{c\mu}{\lambda} = \frac{c'\mu'}{\lambda'}$$

Prandtl number defines the ratio of the intensity of the fluid momentum transport to the intensity of the fluid energy transport

Prandtl number is the only criterial number composed solely of the <u>material constants.</u>

Using the criterial numbers, the energy conservation equation may be written in the non-dimensional form:

$$Sh\rho \frac{\partial}{\partial t}\left(\frac{u^2}{2}\right) + \frac{Sh}{Ec}\rho \frac{\partial}{\partial t}(cT) + \rho(\bar{u} \bullet grad)\frac{u^2}{2} + \frac{1}{Ec}\rho(\bar{u} \bullet grad)(cT) =$$

$$= \frac{1}{Fr}\rho\bar{f} \bullet \bar{u} - Eu \cdot div(\rho[E]\bar{u}) - \frac{1}{\mathrm{Re}} div\left(\frac{2}{3}\mu div\bar{u}[E]\bar{u} - 2\mu[D]\bar{u} + \right)$$

$$+ \frac{1}{\mathrm{Pr} \cdot \mathrm{Re} \cdot Ec} div(\lambda gradT)$$

All flow parameters in the above equation are related to their characteristic values, what makes them non-dimensional.

**Dimensional analysis of the balance of entropy equation**

The initial form of the balance of entropy equation:

$$\rho'\left[\frac{\partial e'}{\partial t'}+\overline{u}'\bullet grade'\right]=T'\dot{s}'_m+\frac{p'}{\rho'}\left(\frac{\partial \rho'}{\partial t'}+\overline{u}'\bullet grad\rho'\right)+\lambda'\Delta T$$

The balance of entropy equation does not require introduction of any new scale coefficients. Using the already known scale coefficientwe obtain the following identity condition in two different scales:

$$\frac{\alpha_\rho \alpha_c \alpha_T}{\alpha_t}=\frac{\alpha_\rho \alpha_u \alpha_c \alpha_T}{\alpha_l}=\frac{\alpha_\mu \alpha_u^2}{\alpha_l^2}=\frac{\alpha_p}{\alpha_t}=\frac{\alpha_p \alpha_u}{\alpha_l}=\frac{\alpha_\lambda \alpha_T}{\alpha_l^2}$$

This condition does not lead to any new criterial numbers. The balance of entropy equation may be presented in the non-dimensional form using the already known criterial numbers.

The non-dimensional balance of entropy equation:

$$Sh\cdot\rho\frac{\partial}{\partial t}(cT)+\rho(\overline{u}\bullet grad)(cT)=\frac{Ec}{\mathrm{Re}}T\dot{s}_m+Eu\cdot Sh\cdot Ec\frac{p}{\rho}\frac{\partial\rho}{\partial t}+$$

$$+Eu\cdot Ec\frac{p}{\rho}(\overline{u}\bullet grad)\rho+\frac{1}{\mathrm{Pr}\cdot\mathrm{Re}}\lambda\Delta T$$

**Summary**

The non-dimensional form of the fluid mechanics equations enables an easy assessment of the relative importance of the respective terms of equations in description of a given flow. The small value of the coefficient composed of the criterial numbers may be the ground for introducing simplification through removing the respective term from the equation. However, such a simplification should not change the order of the equation. For example, dropping the viscosity terms from the energy conservation equation reduces the order of the equation, which may lead to compromising of the boundary conditions.

Solution of the non-dimensional fluid mechanics equations has a general form:

$$F\left(Sh, Fr, Eu, \mathrm{Re}, Ec, \mathrm{Pr}\right) = 0$$

If all criterial numbers included in the above formula have identical values for flows in different scales, then these flows are similar to each other.