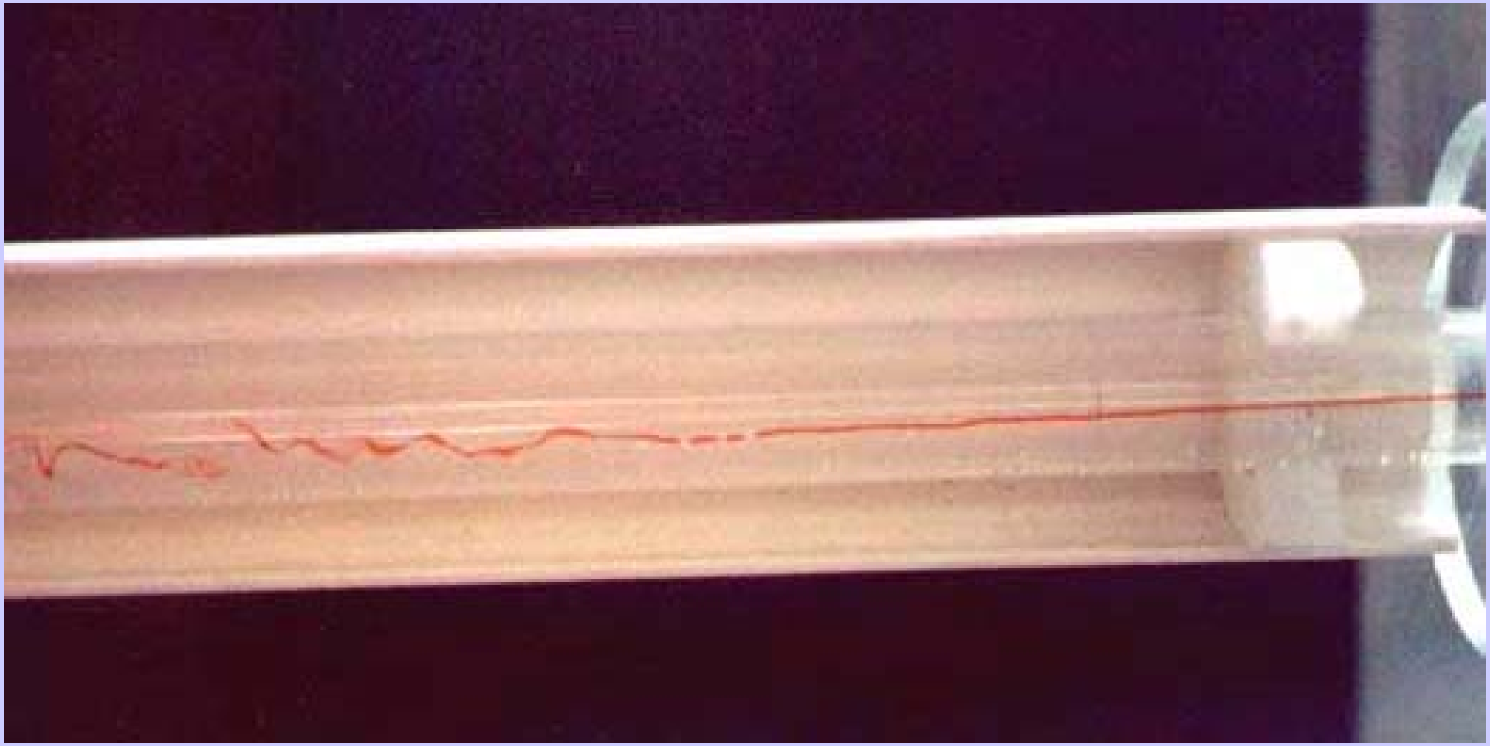
Occurence of the two disctinct types of flow, namely laminar and turbulent, was discovered by Osborne Reynolds in his well-known experiment concerning the flow in a pipe in 1883. He came to the conclusion that the laminar flow occurs up to the value of Re=2300. Above that value the fluid motion becomes unstable and intensive mixing of fluid occurs due to vortex structure of the turbulent flow.



$$\mathrm{Re} = \frac{uD}{\upsilon}$$

The ratio of inertia forces and viscosity forces in the fluid flow, expressed by the Reynolds number, influences strongly the character of the flow. At low Reynolds numbers, i.e. with relatively high viscosity forces, the flow has an orderly character – the fluid elements move along parallel tracks and no mutual mixing occurs. Such a flow is called the **laminar flow** or the layered flow. Above a certain value of the Reynolds number (called the **lower critical number**), due to the increasing role of the inertia forces, such a flow loses stability and disturbances exhibiting stochastic fluctuations of the flow velocity appear. With further increase of the Reynolds number (above so called **upper critical number**) the disturbances fill the entire flow, which is then called the **turbulent flow** The critical Reynolds numbers are different for different flows, for example they are different for a pipe flow and different for a flow along the plane wall.

The laminar flow – an orderly fluid motion along parallel paths, the fluid elements do not mix with each other, a purely viscous mechanism of transport of momentum and energy dominates the flow.

The turbulent flow – a chaotic fluid motion of a stochastic character, unsteady even with the steady boundary conditions, the fluid elements mix with each other, what leads to an intensive process of transport of mass, momentum and energy.

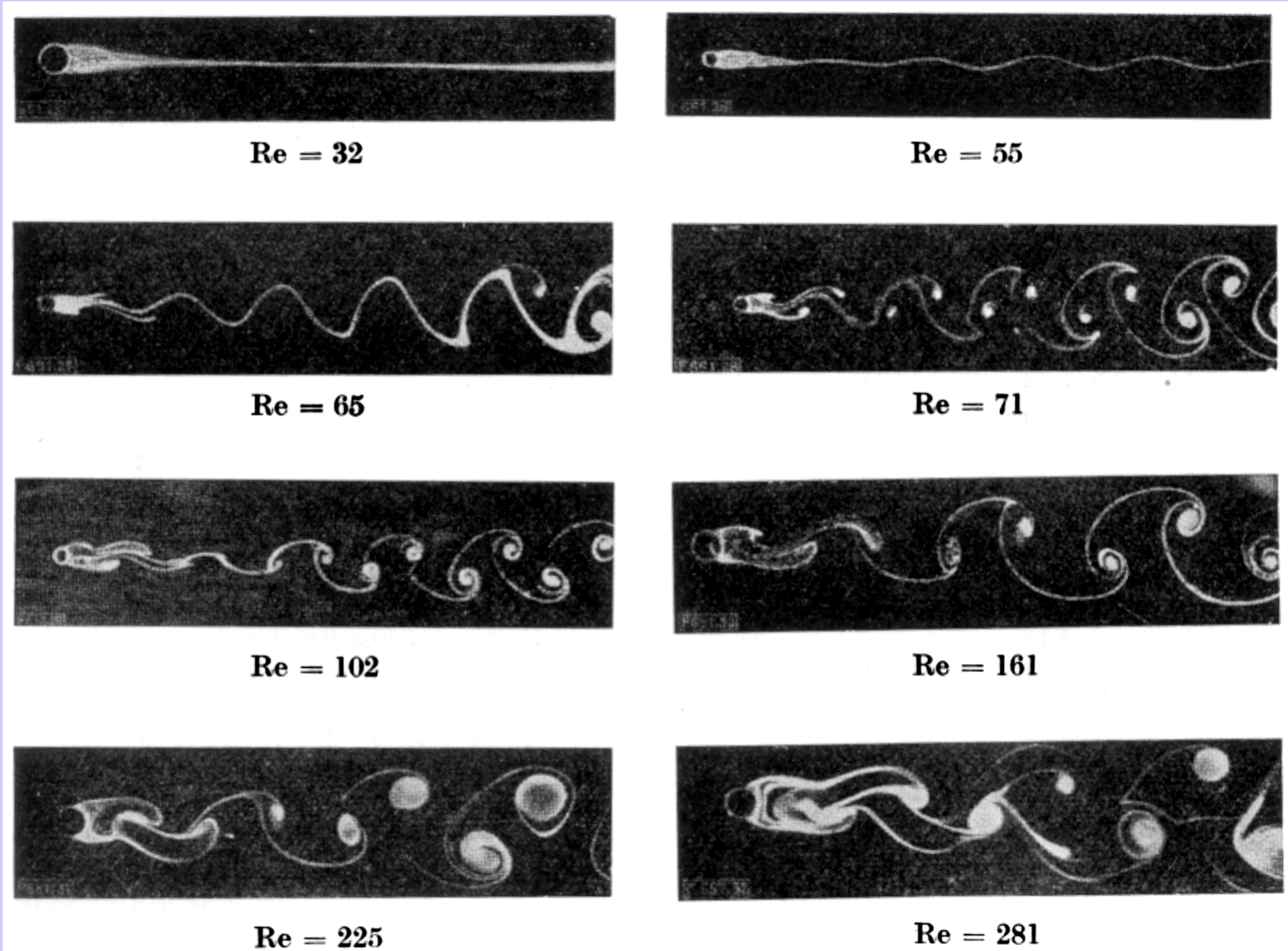
$$\mathrm{Re} = \frac{u \cdot l}{\nu}$$

Re=inertia forces/viscosity forces

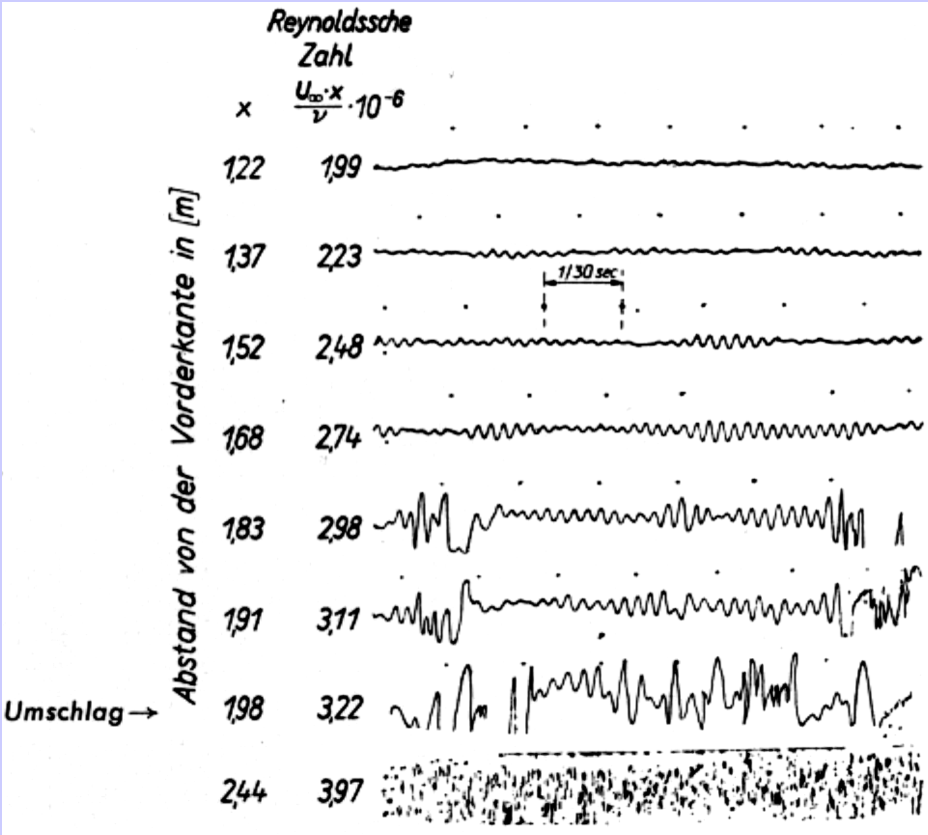
u – the characteristic velocity

l – the characteristic linear dimension

ν – the kinematic viscosity coefficient



The above picture shows an experiment concerning the flow around a thin rod, placed perpendicularly to the direction of velocity. The consecutive photographs show the gradual de-stabilisation of the flow as the Reynolds number increases.



The picture shows an increase of the turbulent fluctuations of velocity of the flow along a flat plate, i.e. with the increasing value of the Reynolds number calculated on the basis of the distance from the leading edge of the plate.

For the flow in a pipe of a circular cross-section (D – the diameter) we have:

the lower critical value: $$\mathrm{Re}_{kr1} = \frac{u \cdot D}{\nu} = 2000$$

the upper critical value: $$\mathrm{Re}_{kr2} = \frac{u \cdot D}{\nu} = 50000$$

In the flow along a flat plate (x – the distance from the leading edge) we have:

the lower critical value: $\mathrm{Re}_{kr1} = \frac{u \cdot x_1}{\nu} = 90000$

the upper critical value: $\mathrm{Re}_{kr2} = \frac{u \cdot x_2}{\nu} = 1000000$

In a turbulent flow we have: $\overline{u} = \overline{U} + \overline{u}'$ or:

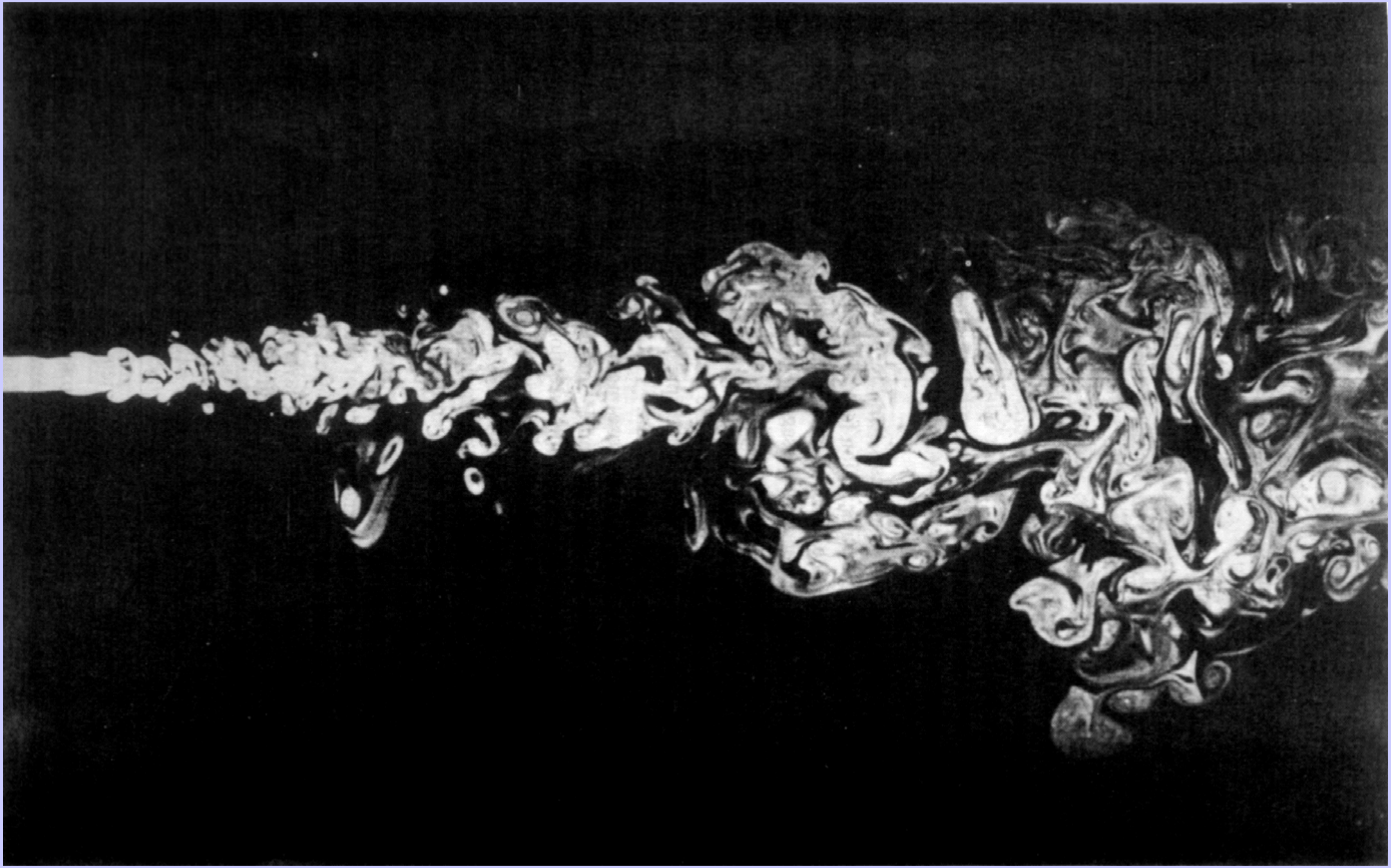
time-dependent velocity=mean velocity+turbulent fluctuation

The measure of the turbulence intensity is the degree of turbulence ε:

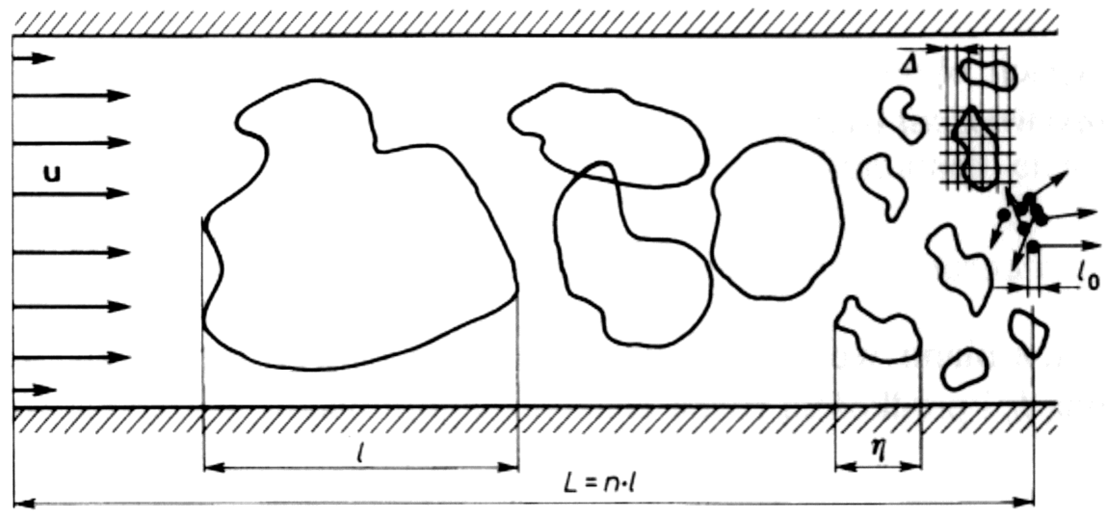
$$\varepsilon = \frac{\sqrt{\frac{1}{3}\left[\left(u_x'\right)^2 + \left(u_y'\right)^2 + \left(u_z'\right)^2\right]}}{\left|\overline{U}\right|}$$

The kinetic energy of turbulence *k* is given by the expression:

$$k = \frac{1}{2}\left[\left(u_x'\right)^2 + \left(u_y'\right)^2 + \left(u_z'\right)^2\right]$$



Visualisation of a turbulent flow shows the specific vortex structures of different scales, known as the turbulent eddies.



The Kolmogorov model (1941) treats **turbulence as the cascade of vortices**, transferring the energy of the fluid motion from the main flow to the molecular motion.

The largest vortices interact with the main flow and absorb its energy. Their characteristic length and velocity are of the same order as those of the main flow (high Reynolds number). It means that the inertia forces dominate and the viscosity forces are negligible. This leads to the disintegration of the vortices into smaller and faster rotating ones. The smallest vortices have Re=1 with diameter η=0.1-0.01 mm and rotational frequency 10 kHz. The motion of these vortices is retarded by the viscosity forces (equal to the inertia forces), and their energy is dispersed and converted into the internal energy (i.e. heat) .

We have: $l >> \eta > l_0$ $\qquad l/\eta \approx 10^6$ $\qquad \eta/l_0 \approx 10^2$

The turbulent flow is mathematically described by the **Reynolds Equations**. Reynolds has assumed that in the turbulent flow the velocity and pressure may be expressed as sums of their mean values (or rather: slowly changing) and turbulent fluctuations, that is:

$$\bar{u} = \bar{U} + \bar{u}' \qquad p = P + p'$$

where: $\bar{u}' = \bar{i}\,u' + \bar{j}v' + \bar{k}w' \qquad \bar{U} = \bar{i}\,U + \bar{j}V + \bar{k}W$

Substitution of such velocity and pressure into the Navier-Stokes equation leads to the appearance of the new surface forces, called the **turbulent stresses:**

$$\rho\frac{DU}{Dt} = \rho f_x - \frac{\partial P}{\partial x} + \mu divgradU + \rho\left[-\frac{\partial \widetilde{u}'^2}{\partial x} - \frac{\partial \widetilde{u}'\widetilde{v}'}{\partial y} - \frac{\partial \widetilde{u}'\widetilde{w}'}{\partial z}\right]$$

$$\rho\frac{DV}{Dt} = \rho f_y - \frac{\partial P}{\partial y} + \mu divgradV + \rho\left[-\frac{\partial \widetilde{u}'\widetilde{v}'}{\partial x} - \frac{\partial \widetilde{v}'^2}{\partial y} - \frac{\partial \widetilde{v}'\widetilde{w}'}{\partial z}\right]$$

$$\rho\frac{DW}{Dt} = \rho f_z - \frac{\partial P}{\partial z} + \mu divgradW + \rho\left[-\frac{\partial \widetilde{u}'\widetilde{w}'}{\partial x} - \frac{\partial \widetilde{v}'\widetilde{w}'}{\partial y} - \frac{\partial \widetilde{w}'^2}{\partial z}\right]$$

Normal stresses:

$$\tau_{xx} = -\rho\widetilde{u'^2} \qquad \tau_{yy} = -\rho\widetilde{v'^2} \qquad \tau_{zz} = -\rho\widetilde{w'^2}$$

Tangential (shearing) stresses:

$$\tau_{xy} = \tau_{yx} = -\rho\widetilde{u'}\widetilde{v'}$$

$$\tau_{xz} = \tau_{zx} = -\rho\widetilde{u'}\widetilde{w'} \qquad \tau_{yz} = \tau_{zy} = -\rho\widetilde{v'}\widetilde{w'}$$

<u>The turbulent stresses, also known as Reynolds stresses, depend on the values of the velocity fluctuations, not on the fluid viscosity.</u> It may be proved that they form a symmetrical system of stresses. They constitute the additional 6 unknowns in the system of Reynolds equations describing the turbulent motion of the fluid. In order to reduce the number of unknowns and close the system it is necessary to introduce the appropriate **models of turbulence.** The <u>Reynolds equations are the basis of most of the contemporary commercial computer codes in the domain of the Computational Fluid Mechanics.</u>