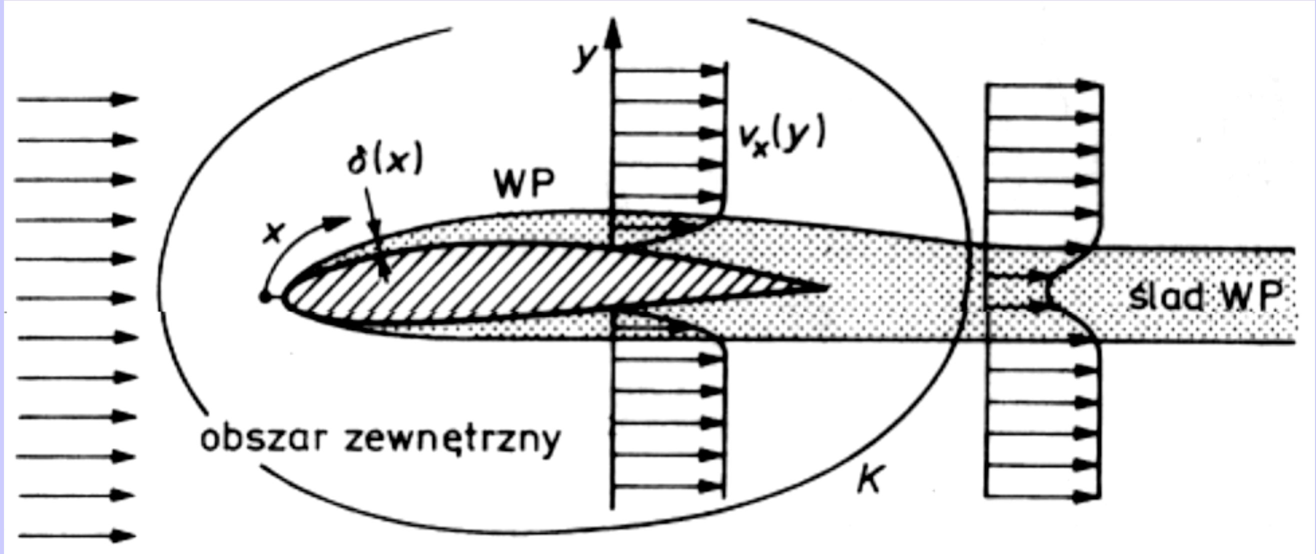
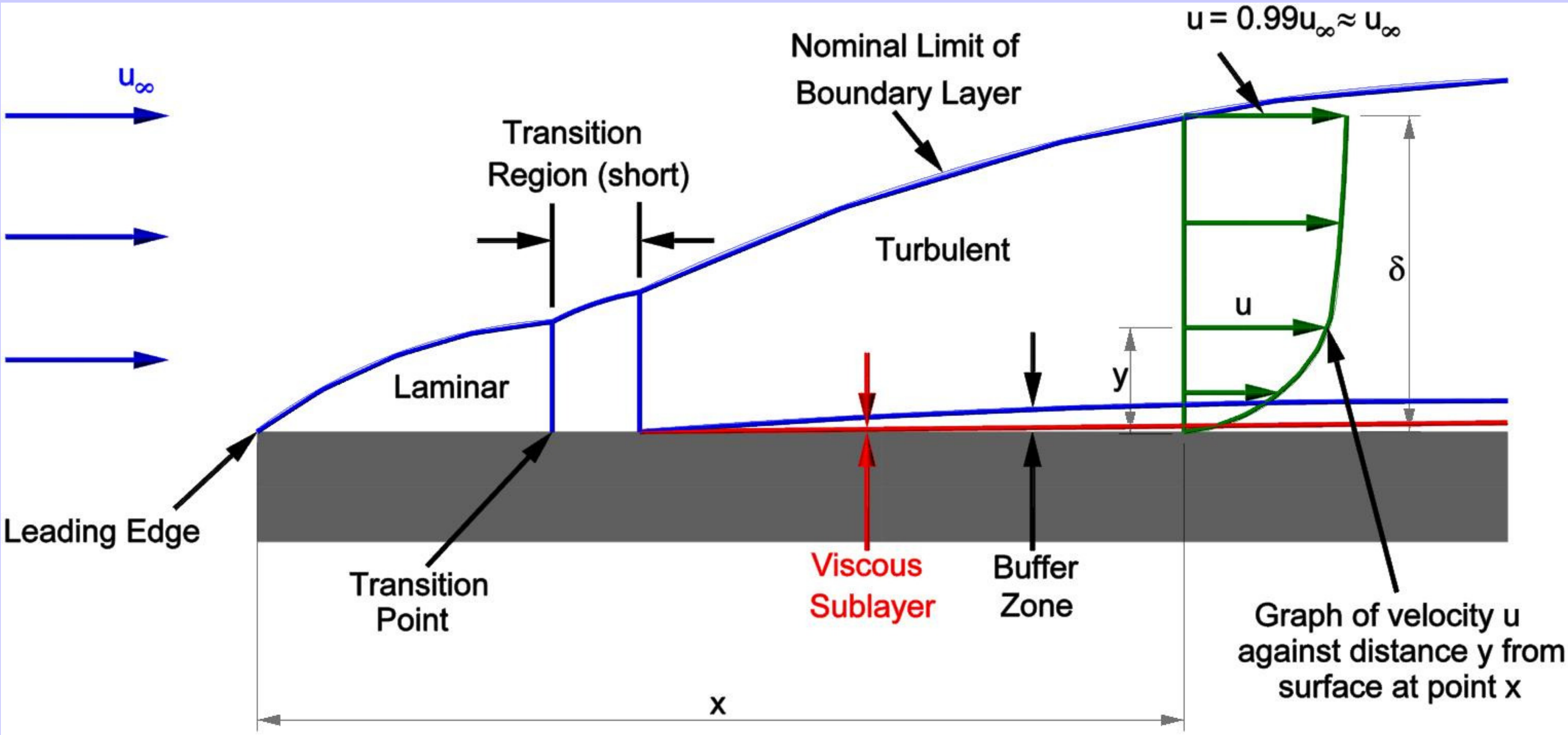
J. Szantyr – Lecture No. 19 – Boundary layers and wakes 1

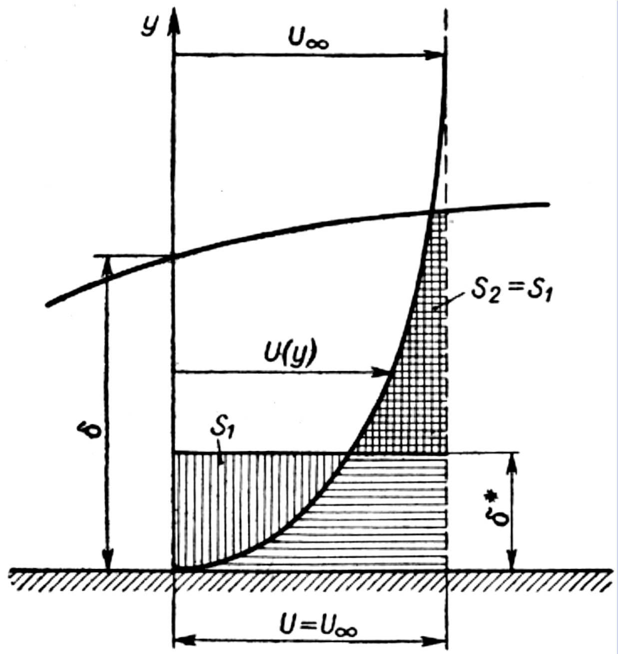


The boundary layer is the region of flow directly neighbouring on the surface of the solid body. In the boundary layer the viscosity forces play a meaningful role and the significant transverse flow velocity gradients occur. Outside the boundary layer the flow may be practically regarded as inviscid. Behind the solid body the boundary layer forms the so called wake.

The flow in the layer may be laminar or turbulent. The layer thickness δ is determined by attaining the velocity $u_\delta = 0.99 u_\infty$

A typical boundary layer on the wall of an object in contact with the flowing fluid includes the zone of laminar flow near the leading edge, the transition region and the turbulent zone. In the turbulent zone a very thin viscous sublayer is located at the wall, followed by a buffer zone further from the wall and by the dominating fully turbulent region.





As the layer thickness δ is difficult to determine accurately, the so called displacement thickness has been introduced:

$$\delta^* = \int_0^\delta \left(1 - \frac{\rho u}{\rho_\infty u_\infty}\right) dy$$

The laminar boundary layer

The flow in a two-dimensional laminar boundary layer is described by the **Prandtl equations.** Prandtl has simplified the Navier-Stokes equation on the basis of the following assumptions:

- the layer thickness is much smaller than the wall length,

- the velocity normal to the wall is much smaller than the velocity along the outer limit of the layer.

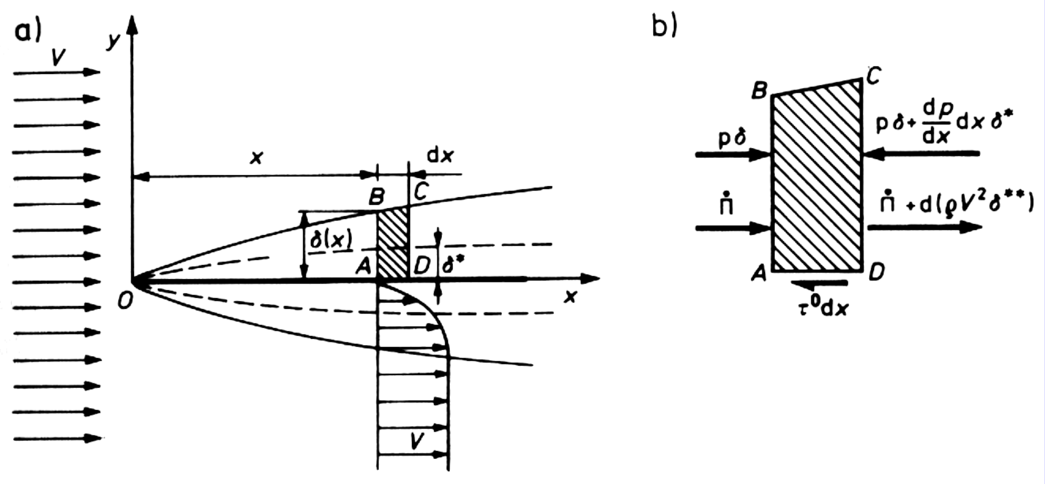
These simplifications lead to the following equations:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\frac{\partial^2 u}{\partial x^2}$$ direction x

$$\frac{\partial p}{\partial y}=0$$ direction y

Conclusion 1: the pressure on the wall of the solid body is equal to the pressure in the corresponding point on the outer limit of the boundary layer.

Conclusion 2: the pressure distribution on the outer limit of the boundary layer may be determined from the Bernoulli equation (in case of a steady flow).



The Prandtl equations may be solved analytically for a steady flow along a flat plate (without the pressure gradient along the plate) – conf. Fig. a)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$ mass conservation equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}$$ momentum conservation equation

Boundary conditions: $u \rightarrow u_\infty$ at $y \rightarrow \infty$

$u = v = 0$ at $y = 0$

The solution leads to the practically useful relations:

Thickness of the laminar boundary layer on a plate:

$$\delta(x) = \frac{5x}{\sqrt{\mathrm{Re}_x}} \rightarrow \frac{\delta(x)}{x} = \frac{5}{\sqrt{\mathrm{Re}_x}}$$

where: $\mathrm{Re}_x = \dfrac{u_\infty x}{\upsilon}$

Frictional drag coefficient on the plate surface:

$$C_f = \frac{1.328}{\sqrt{\mathrm{Re}_L}} \rightarrow R_f = C_f \cdot \frac{1}{2}\rho u_\infty^2 S$$

where: $\mathrm{Re}_L = \dfrac{u_\infty L}{\upsilon}$

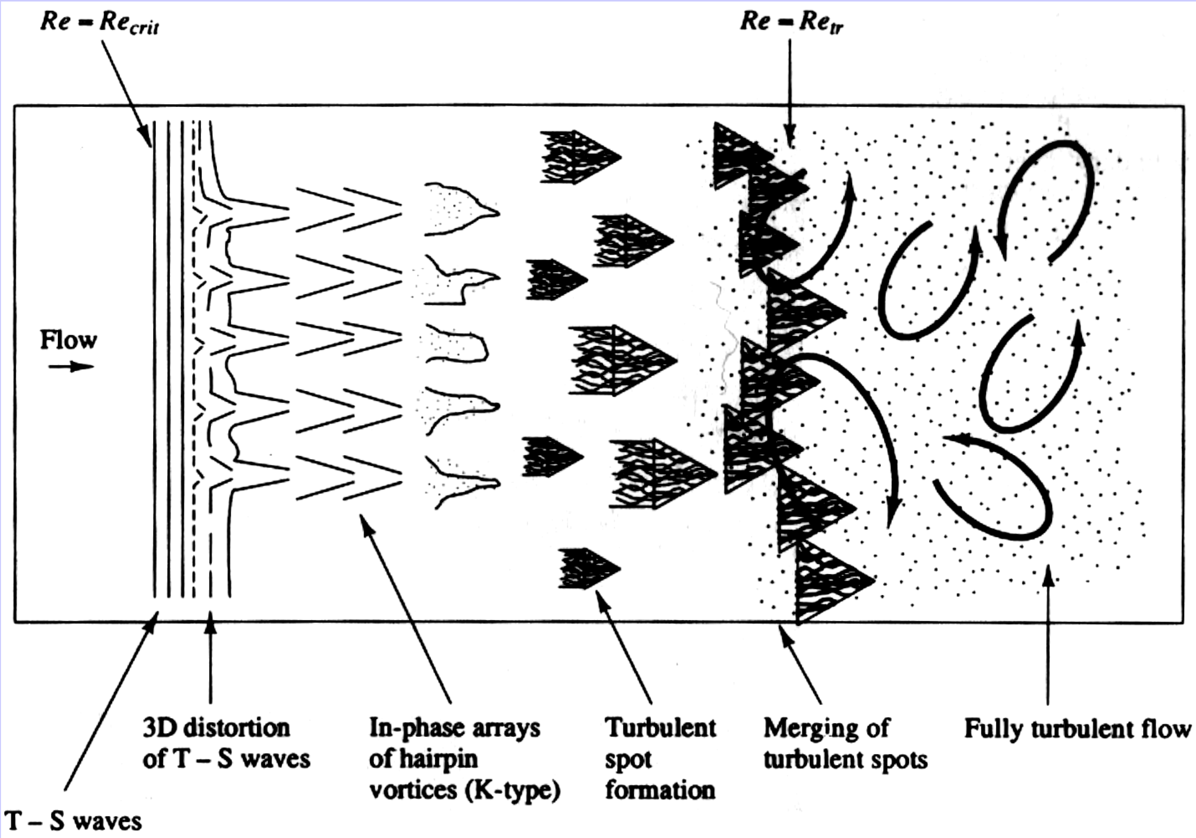
where: R_f - frictional drag of a plate of area S (both sides!)

Velocity profile inside the boundary layer:

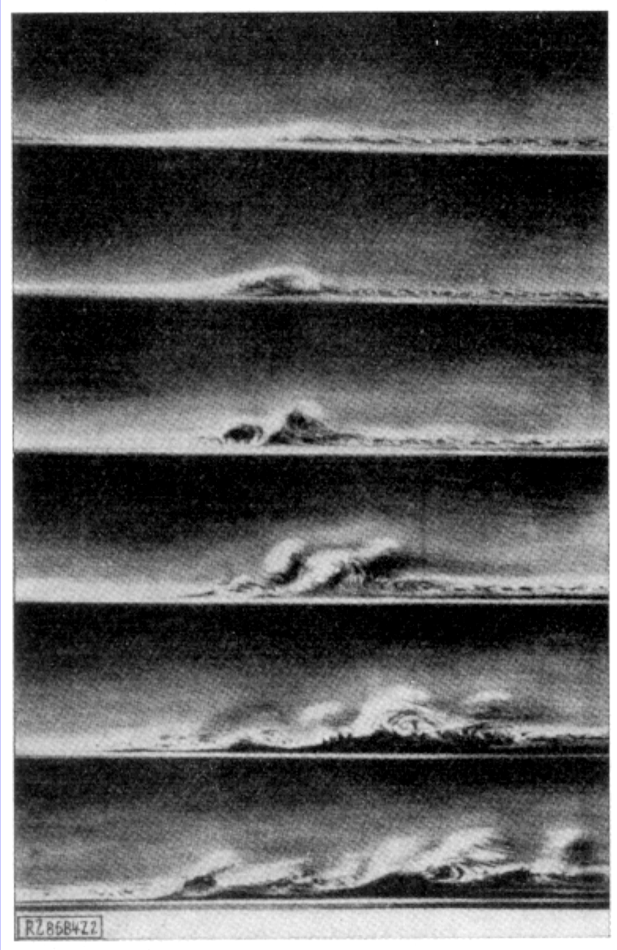
$$u(y) = u_\infty \left[\frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \right]$$

Moreover it may be calculated that: $\delta^* \approx 0.33\delta$

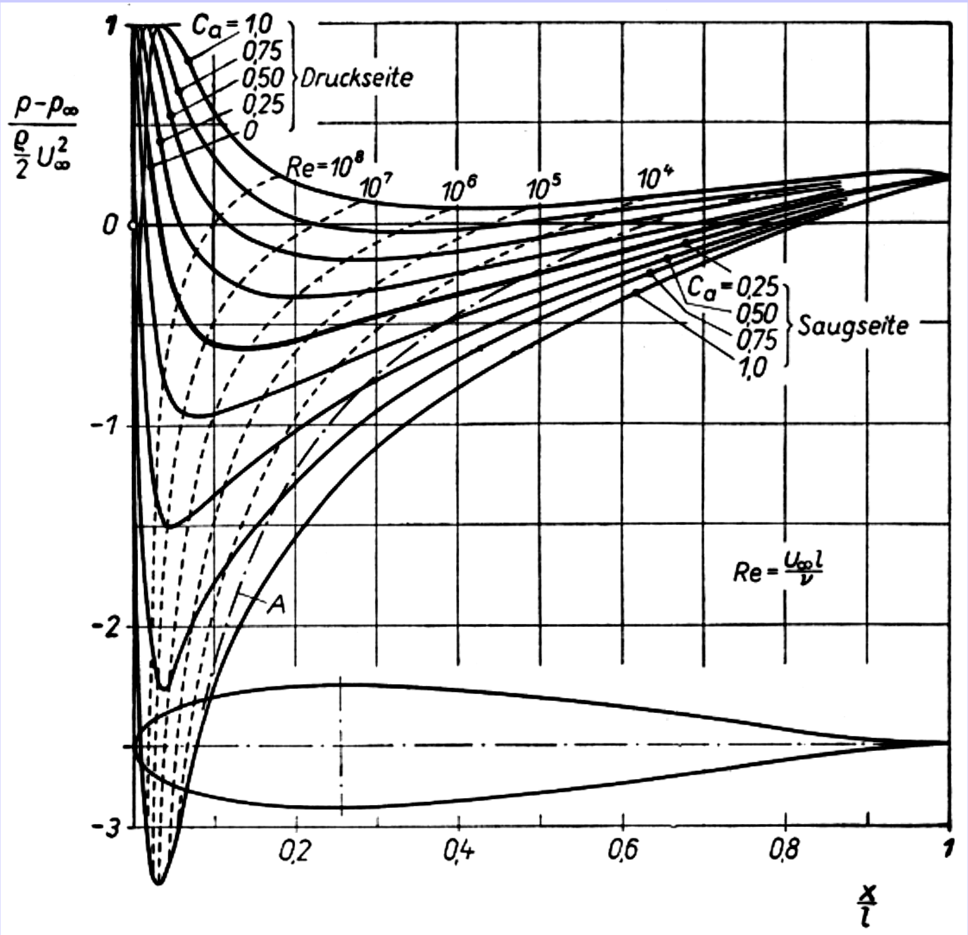
Increasing of the Reynolds number leads to the losing of stability of the laminar boundary layer and to the gradual development of turbulence up to the fully developed turbulent boundary layer.



Scheme of the process of tubulization of the boundary layer.

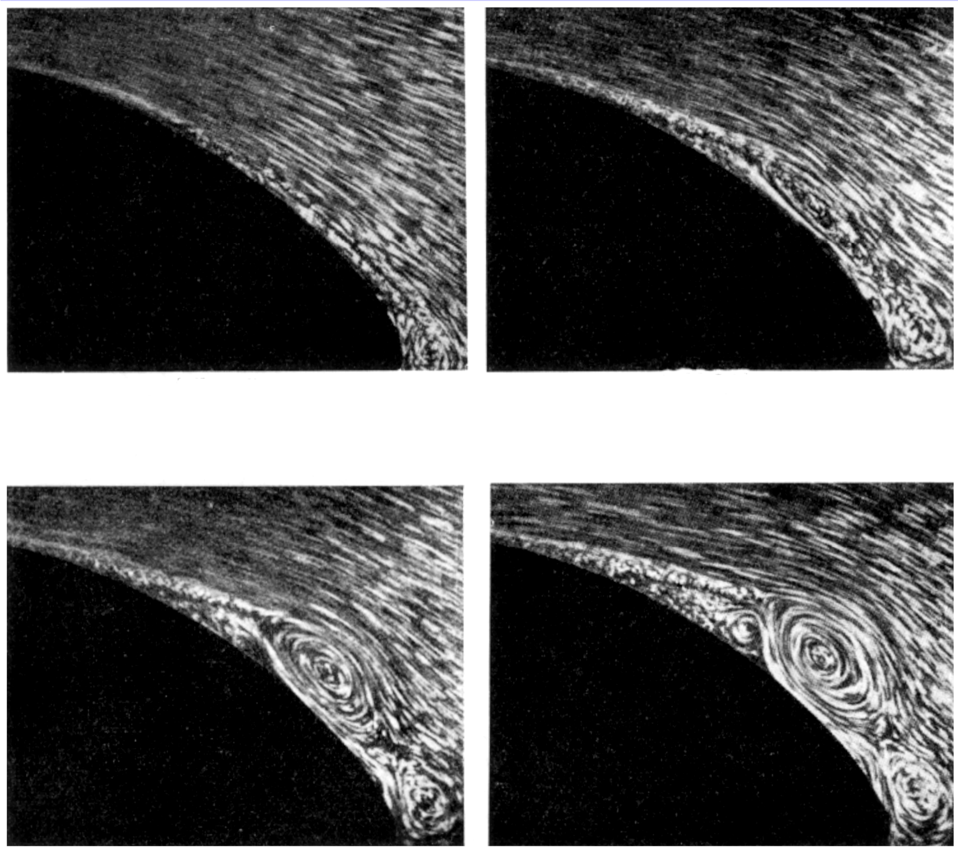


Visualisation of the generation of turbulence



The location of the laminar-turbulent transition depends both on the Reynolds number and on the pressure gradient along the boundary layer. The picture shows this phenomenon on an asymmetric profile set at the different angles of attack, what changes the pressure gradient. The broken lines show the location of the laminar-turbulent transition at different values of the Reynolds number.

The **positive pressure gradient along the boundary layer** (i.e. increase of pressure in the flow direction), may lead to the so called **separation** of the boundary layer. The mechanism is explained in Fig b) on slide 4. The fluid element near the wall is retarded by the viscosity forces and pressure forces, what leads to its stopping and then inducing its motion against the direction o flow.

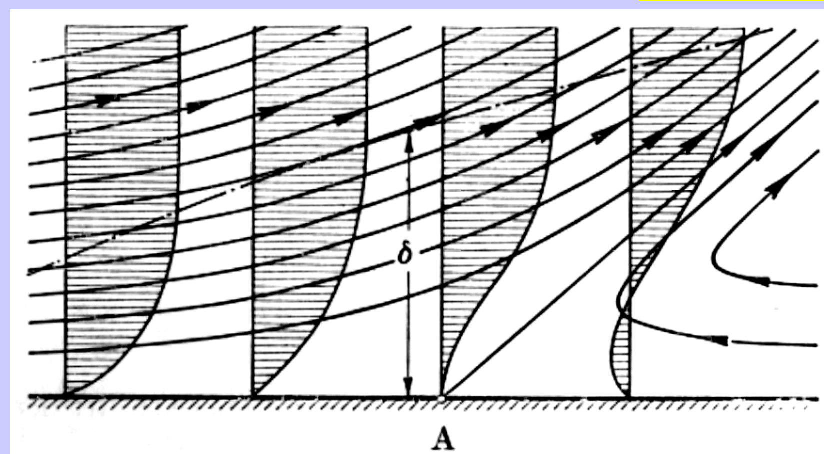


The development od separation

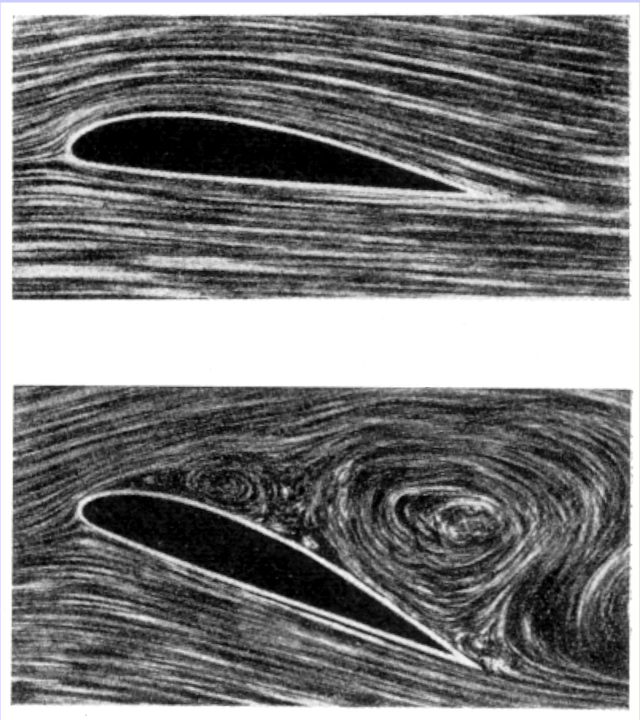
In the separation point A we have:

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0$$

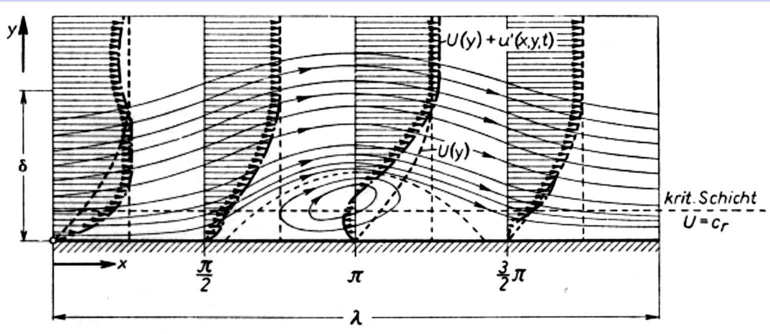
Moreover, the viscous stress on the wall is zero: $\tau_w = 0$



Separation may occur both in the laminar and turbulent boundary layer (in the turbulent one it occurs later, i.e. at the higher pressure gradients). Separation of the boundary layer has many negative consequences, it disturbs the operation of the fluid flow machinery and it reduces their efficiency. The fluid flow machinery should be designed in such a way that the separation of flow is avoided, at least at the design operation parameters of these machines.



The separation bubble



<Separation of the boundary layer on an airfoil at high angle of attack (lower picture)

Examples of the laminar and turbulent flows in the boundary layers and wakes

Example

A thin flat plate of dimensions 0.1*0.5 [m] is placed at zero angle of attack in the flow of water having velocity equal to 0.1 [m/s]. Determine the frictional drag of the plate in two cases: a) when the longer side of the plate is perpendicular to the direction of velocity, b) when the shorter side is perpendicular to the velocity.

Given: the kinematic viscosity coefficient ν=0.000001 $\left[m^2/s\right]$

the density of water ρ=1000 $\left[kg/m^3\right]$

Case a

$$\mathrm{Re} = \frac{u \cdot L}{\upsilon} = \frac{0.1 \cdot 0.1}{0.000001} = 10000 \qquad C_f = \frac{1.328}{\sqrt{\mathrm{Re}}} = \frac{1.328}{\sqrt{10000}} = 0.01328$$

$$R_f = C_f \frac{1}{2} \rho u^2 S = 0.01328 \cdot 0.5 \cdot 1000.0 \cdot 0.1^2 \cdot 2 \cdot 0.1 \cdot 0.5 = 0.01328[N]$$

Case b

$$Re = \frac{u \cdot L}{\upsilon} = \frac{0.1 \cdot 0.5}{0.000001} = 50000 \qquad C_f = \frac{1.328}{\sqrt{Re}} = \frac{1.328}{\sqrt{50000}} = 0.00593$$

$$R_f = C_f \frac{1}{2} \rho u^2 S = 0.00593 \cdot 0.5 \cdot 1000.0 \cdot 0.1^2 \cdot 0.1 = 0.00593[N]$$

Conclusion: the change of position of the plate with respect to the flow, with other parameters unchanged, may result in more than **doubling the frictional drag of the plate**