J. Szantyr –Lecture No. 2 - Introduction

There are three forms of matter: solids – fluids, including liquids and gases.

-Solids support external loads in such a way, that they undergo deformation as long as the force is acting on them. When the force is removed, they return to the previous shape (assuming that the elasticity limit has not been exceeded).

-Fluids under external loads undergo continuous deformation and they do not return to the previous shape when the load is removed.

-Fluids do not have their „own" shape as the solids do – they take the shape of the container in which they are placed. The gases always fill the entire container, while the liquids usually form free surfaces, which divide them from gases (or vacuum).

Characteristic features at the molecular level

In the **liquids** the molecules perform an oscillatory motion around certain semi-permanent location, and from time to time they jump between these locations. The following molecular parameters are important because they influence the macroscopic features of liquids:

-The size of molecules and mean distance between them

-The mean distance of the „jump" (0.0000003 mm) for water

-The mean time of the „stationary life" in the given location (0.0000000001s for water, and 1s for tar)

In the **gases** the molecules perform a continuous chaotic motion. The following molecular parameters are important because they influence the macroscopic features of gases:

-The size of molecules and mean distance between them

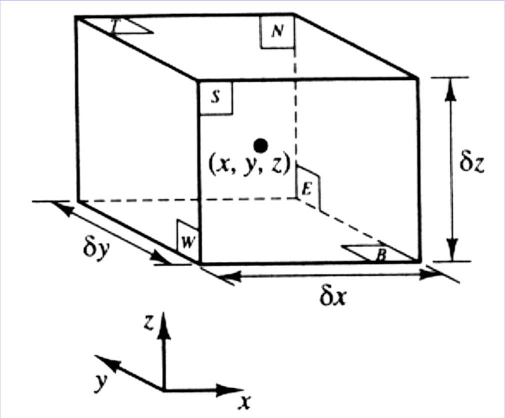
-The mean free path (0.00005 mm for air)

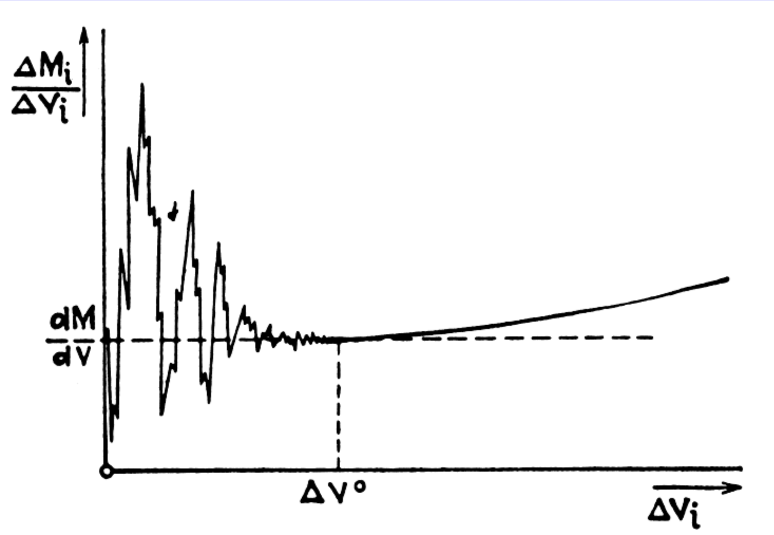
Fluid as the **model of the liquid or gas state of the matter** possesses the features of **fluidity and continuity.**

Fluidity postulates proportionality between the velocity of fluid deformation and the deforming force. In case of liquids fluidity becomes visible provided the time of force action is markedly longer than the time of „stationary life” of the molecules of a particular liquid.

Continuity postulates that the gases or liquids fill the space in a continuous way. The necessary condition for continuity is that the characteristic dimension of the flow in question should be markedly larger (at least 100 times) than the mean free path of molecules in case of gases or the mean distance of the „jump” of molecules in case of liquids. Continuity enables the application of the differential calculus for description of the fluid motion.

The **fluid element** is the part of the fluid mass having infinitely small dimensions with respect to the fluid mass in motion or with respect to the dimensions of the object moving in the fluid, and simultaneously sufficiently large in comparison with the mean free path or distance of the „jump" of the molecules.





The basis for determination of the correct size of the fluid element may be the calculation of the fluid density through division of the fluid mass by the volume of the element.

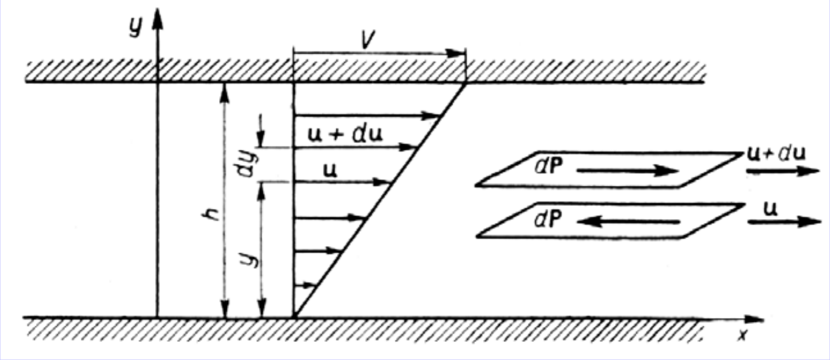
Parameters describing the macroscopic features of the fluids

A) State of the fluid in the given point of space

velocity	$\bar{u}\left[\frac{m}{s}\right]$	ratio of distance to time
density	$\rho\left[\frac{kg}{m^3}\right]$	ratio of mass to volume
pressure	$p\left[\frac{N}{m^2}\right]\left[Pa\right]$	ratio of force to area
temperature	$T\left[K\right]$	measure of internal energy

B) Transport coefficient

Diffusion coefficient $D\left[\frac{m^2}{s}\right]$



Measure of mass transport through diffusion

Dynamic viscosity coefficient $\mu\left[\frac{kg}{m*s}\right]$ $\tau = \mu\frac{\partial u}{\partial l}$

Coefficient of proportionality between viscous shear stresses and velocity of fluid deformation

Heat conductivity coefficient $\lambda\left[\frac{kg*m}{s^3*K}\right]$ $q = -\lambda\frac{\partial T}{\partial l}$

Coefficient of proportionality between the stream of heat and the gradient of temperature

C) Parameters dependent on the molecular structure of fluids

Specific heat at constant pressure: $c_p \left[\frac{J}{kg \cdot K} \right]$

Specific heat at constant volume: $c_v \left[\frac{J}{kg \cdot K} \right]$

For liquids there is only one value: $c_p = c_v = c$

The specific heat determines the amount of energy necessary for increasing the temperature of 1 kg of fluid by 1 kelvin (i.e. For the corresponding increase of the internal fluid energy)

Categories of flows

One-dimensional flows – flows with one dominating direction of velocity, for example flows inside pipelines with circular cross-section of constant area.

Two-dimensional flows – flows with two equally important directions of velocity, for example flow around aerofoils or flows in pipelines of rapidly varying cross-sections.

Three-dimensional flows – flows with three equally important directions of velocity, for example flows around three-dimensional bodies such as cars, aircraft or ships.

NB!: in physical reality **all flows are three-dimensional,** the other above mentioned categories are simplified models, acceptable under specific conditions.

Categories of flows (continued)

Each of the above mentioned categories of flows may be additionally treated alternatively as:

Stationary (or steady) flow – the flow in which the describing parameters do not change with time.

Instationary (or unsteady) flow – in which the describing parameters are functions of time

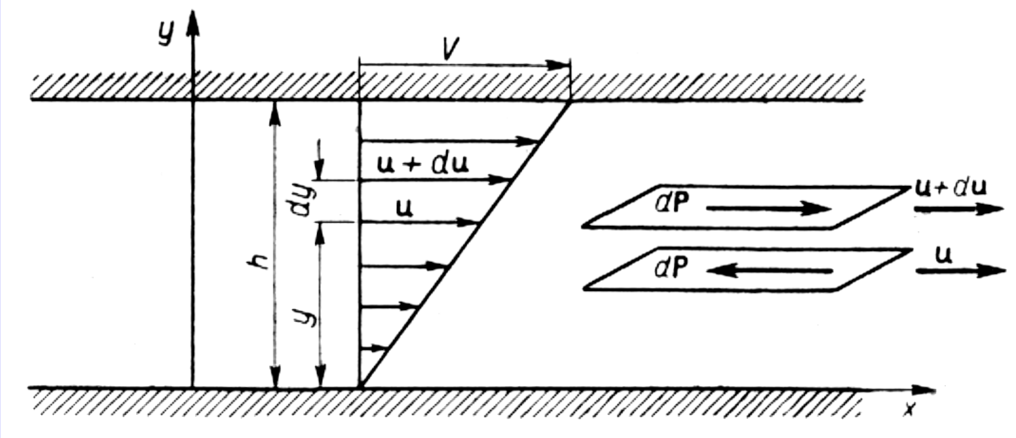
NB!: the above distinction is subjective, that is **the same physical flow** may be **steady** in one system of co-ordinates and **unsteady** in another system of co-ordinates

Models of fluids

Ideal liquid (Pascal liquid) – incompressible and inviscid fluid

Viscous fluid – (Newton fluid), fluid in which the viscous shear stresses are proportional to the velocity of deformation, eg. water, air, mineral oils.

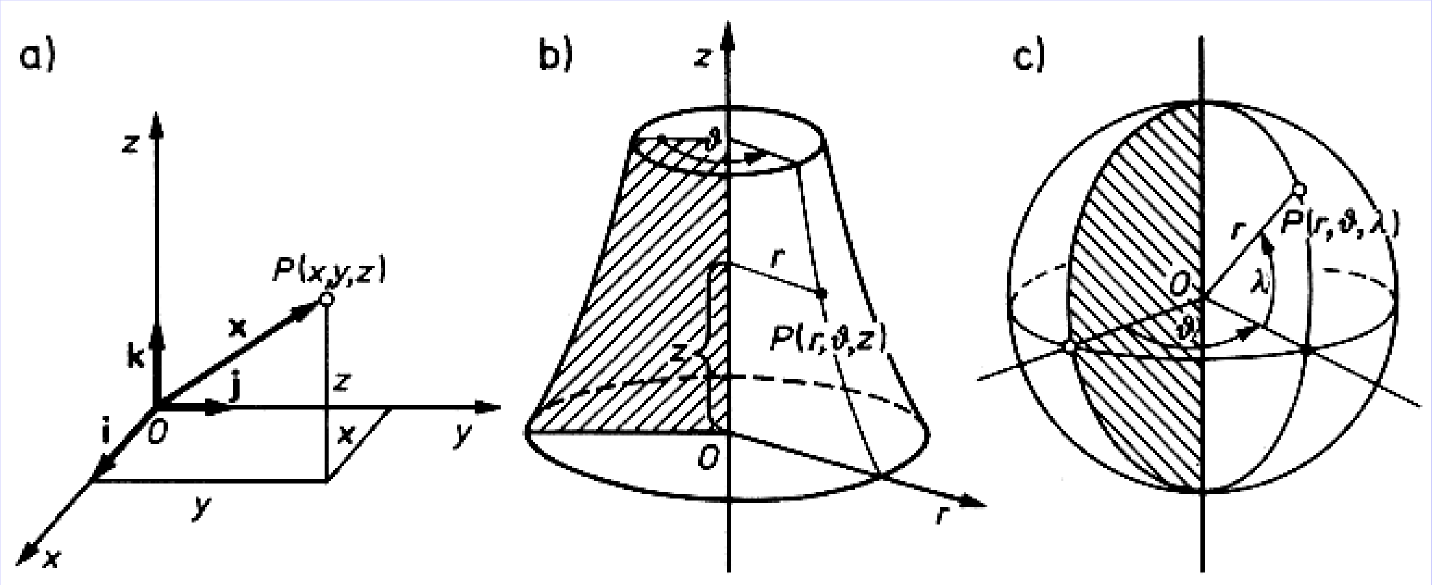
$$\tau = \mu \frac{\partial u}{\partial y}$$



Visco-plastic fluid (Bingham fluid) – fluid which behaves like a solid below certain stress level and like a newtonian fluid above that stress level, eg. paints, pastes or water-cement mixture

$$\tau = \tau_0 + \mu \frac{\partial u}{\partial y}$$

Systems of co-ordinates



a) Cartesian system Oxyz

b) Cylindrical system Orθz

c) Spherical system Orθλ

Physical fields

Scalar field, eg. pressure or temperature

$$S = S(x, y, z)$$

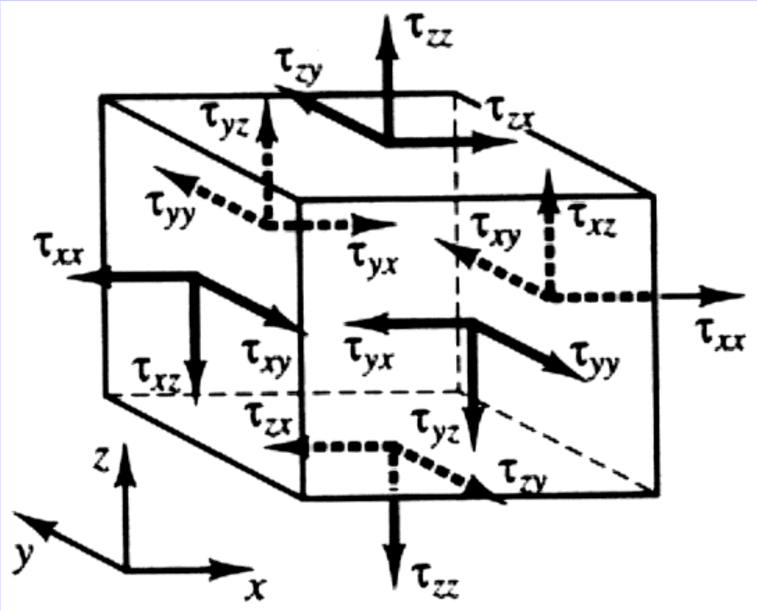
Vector field, eg. velocity

$$\overline{W} = \overline{W}(x, y, z) = \bar{i}W_x(x, y, z) + \bar{j}W_y(x, y, z) + \bar{k}W_z(x, y, z)$$

Tensor field, eg. stress distribution in the fluid

$$\{\mathrm{T}\} = \tau_{ij}$$ where $i,j=x, y, z$

Double-index tensor is describing a matrix of nine coefficients. In the case of stresses in the fluid the first index indicates the direction perpendicular to the wall on which the stress acts and the second – direction of stress.



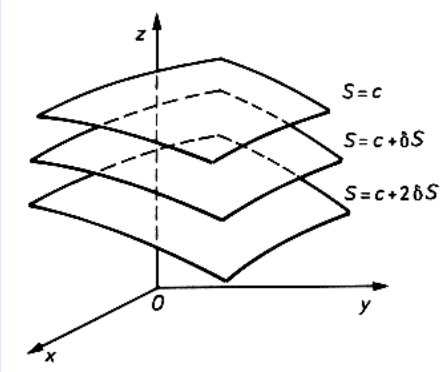
Hamilton operator (nabla)

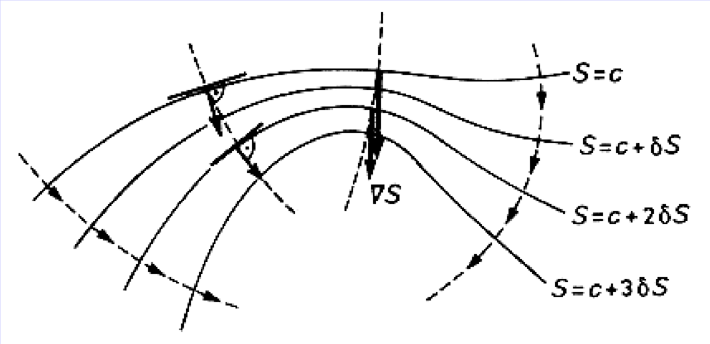
Operator of spatial differentiation, formally may be treated as a vector:

$$\nabla = \bar{i}\frac{\partial}{\partial x} + \bar{j}\frac{\partial}{\partial y} + \bar{k}\frac{\partial}{\partial z}$$

Multiplication of nabla by a scalar gives a vector of gradient of the scalar field:

$$\nabla S = gradS = \bar{i}\frac{\partial S}{\partial x} + \bar{j}\frac{\partial S}{\partial y} + \bar{k}\frac{\partial S}{\partial z}$$





Vector of gradient shows the direction of the maximum increase of the scalar S. The vector field of **gradient** may be assigned to **every scalar field**. **The inverse operation is not always possible**. If there exists a scalar field S such that W=gradS, then the vector field W is a potential field.

Scalar multiplication of nabla by a vector gives the scalar quantity (a number) named the divergence of the vector field.

$$\nabla * \overline{W} = \frac{\partial W_x}{\partial x} + \frac{\partial W_y}{\partial y} + \frac{\partial W_z}{\partial z} = div\overline{W}$$

Divergence describes the rate of change of the volume of a fluid element per unit volume, ie. it may be **different than zero only in a compressible fluid.**

Vector multiplication of nabla by a vector gives a vector quantity named the rotation of the vector field.

$$\nabla \times \overline{W} = \overline{i}\left(\frac{\partial W_z}{\partial y} - \frac{\partial W_y}{\partial z}\right) + \overline{j}\left(\frac{\partial W_x}{\partial z} - \frac{\partial W_z}{\partial x}\right) + \overline{k}\left(\frac{\partial W_y}{\partial x} - \frac{\partial W_x}{\partial y}\right) = rot\overline{W}$$

If the vector field W describes the field of flow velocity, then the non-zero value of rotation of this field indicates the vortex motion of the fluid elements. The field where rotW=0 is named irrotational.

One of the several possible **double multiples** of nabla is the divergence of the gradient of a scalar field S or a Laplace operator:

$$\nabla * \nabla S = \nabla^2 S = \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} + \frac{\partial^2 S}{\partial z^2}$$

Laplace operator is the main part of several fluid mechanics equations, among others the **Laplace equation**, which governs the so called potential flows