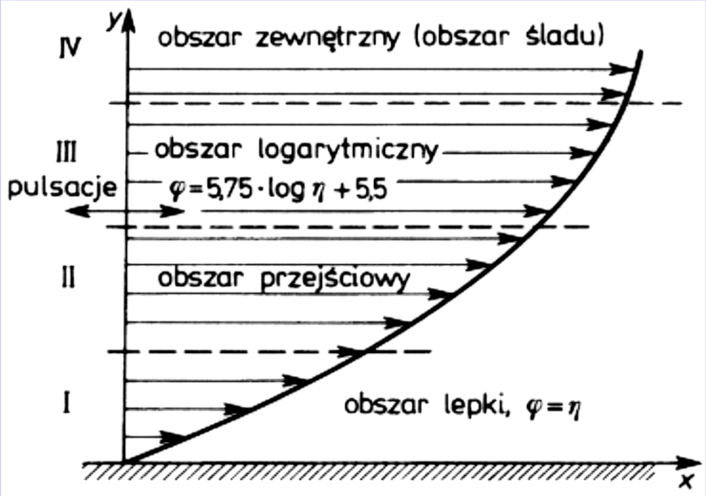
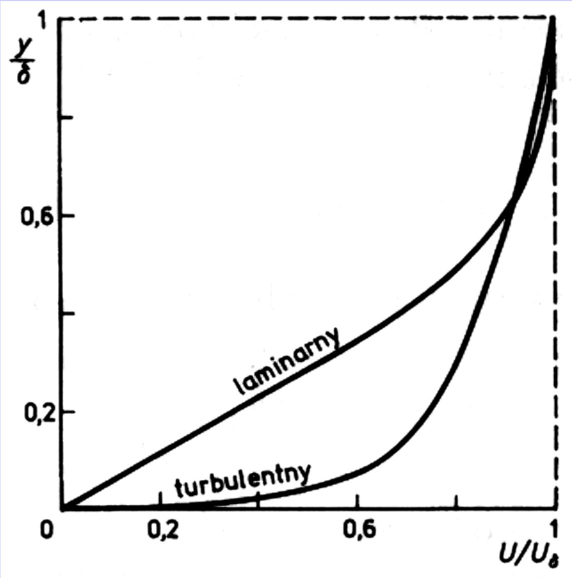
J. Szantyr – Lecture No. 20 – Boundary layers and wakes 2

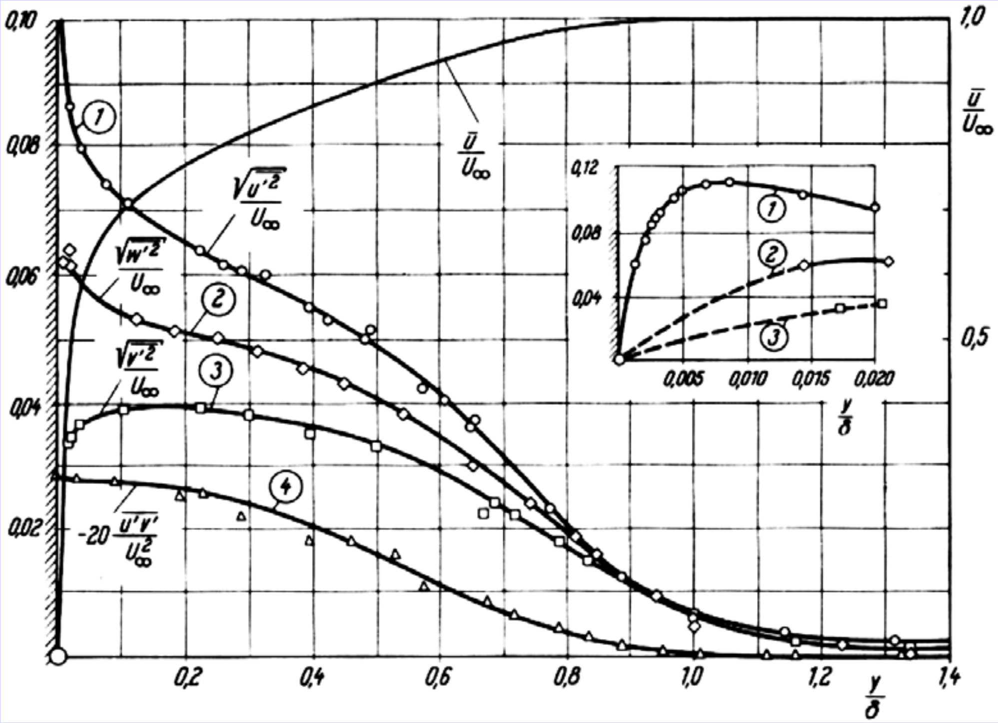


In the turbulent boundary layer several different regions may be distinguished, each with the different physical mechanisms governing the flow.

Generally, the boundary layer may be divided into the inner region of thickness about 0.2δ and the outer region (IV). In the outer region the inertia forces dominate the flow. The internal flow may be divided into the viscous sub-layer (I) of thickness about 0.02δ, where the viscous forces and inertia forces are of the same order and where the viscous mechanism of momentum and energy transport is dominant, and the transitional (II) and „logarithmic" (III) region, where the turbulent stresses and turbulent mechanism of mass, momentum and energy transport dominate the flow.

Due to the combined effect of the viscous and turbulent mechanisms of momentum transport the velocity profile in the turbulent layer is „fuller” than in the laminar layer.





In the turbulent boundary layer strong, three-dimensional fluctuations of velocity occur. They attain maximum amplitudes near the wall i.e. in the region of high gradient of the mean velocity profile.

Some practically useful formulae have been developed using empirical-theoretical approach:

$$\delta_{turb} = \frac{0.37 \cdot L}{\sqrt[5]{\mathrm{Re}}}$$

$$C_{fturb} = \frac{0.074}{\sqrt[5]{\mathrm{Re}}}$$ for Reynolds numbers $5 \cdot 10^5 < \mathrm{Re} < 10^6$

$$C_{fturb} = \frac{0.455}{(\log \mathrm{Re})^{2.58}} - \frac{A}{\mathrm{Re}}$$ for $3 \cdot 10^5 < \mathrm{Re} < 10^9$

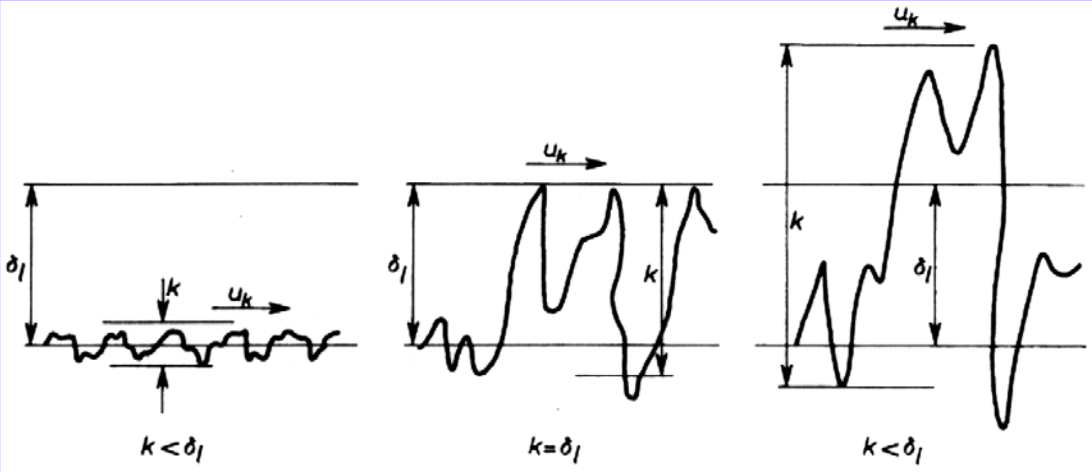
Where the constant A is determined on the basis of the (upper) critical value of the Reynolds number according to the table:

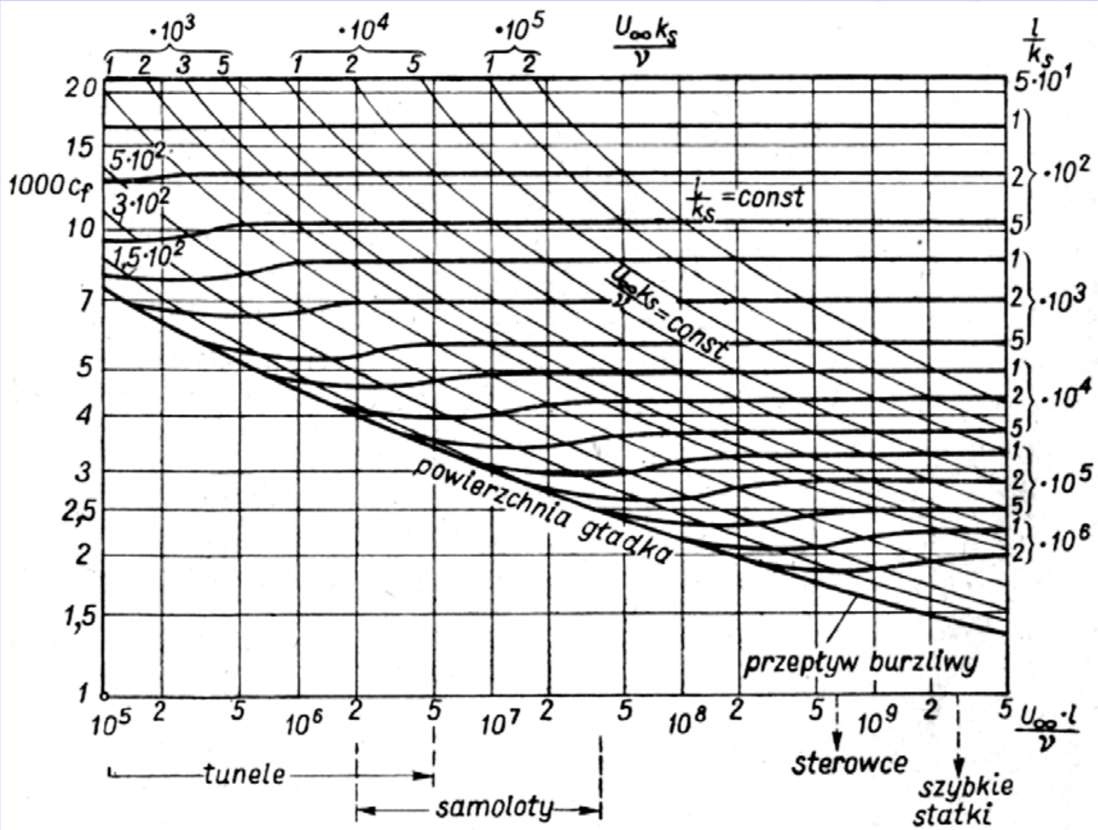
Re_{kryt}	A
$3 \cdot 10^5$	1050
$5 \cdot 10^5$	1700
10^6	3300
$5 \cdot 10^5$	5700

The above formulae for the friction coefficient are valid for a smooth wall. <u>In the turbulent flow this coefficient depends also on the wall surface roughness</u>

The measure of the surface roughness is the mean roughness height k_s

From the point of view of the frictional drag, the ratio of the mean roughness height to the thickness of the viscous sub-layer in the turbulent boundary layer is important. If the roughness is totally contained in this sub-layer, then the roughness does not change the velocity profile in the boundary layer and does not affect the frictional drag – such a surface is called hydrodynamically smooth. When the roughness height exceeds the thickness of the viscous sub-layer, it changes the velocity profile in the boundary layer and increases the frictional drag.





The diagram shows the dependence of the frictional drag coefficient on the inverted relative roughness (i.e. related to the characteristic linear dimension L). Reynolds numbers based on the roughness height are also marked in the diagram.

There are empirical relations, which enable determination of the frictional drag coefficient on a rough surface in the turbulent boundary layer, for example:

$$C_{fchrop} = C_{fturb} + \Delta C_f$$

where:

$$\Delta C_f = \left(1.89 + 1.62 \log \frac{l}{k_s}\right)^{-2.5} \quad \text{with} \quad 10^2 < \frac{l}{k_s} < 10^6$$

Example 1

A thin flat plate of dimensions 1.0*1.0 [m] is placed at a zero angle of attack in the flow of water having velocity 10 [m/s]. Determine the frictional drag of the plate in two cases: a) for a smooth plate, b) for a plate with relative roughness 0.0001.

Given: the kinematic viscosity coefficient ν=0.000001 $\left[\frac{m^2}{s}\right]$

the density of water ρ=1000.0 $\left[\frac{kg}{m^3}\right]$

Case a

$$Re = \frac{uL}{\upsilon} = \frac{10.0 \cdot 1.0}{0.000001} = 10000000$$

high value of the Reynolds number requires using the more complicated formula

$$C_{fturb} = \frac{0.455}{(\log Re)^{2.58}} - \frac{A}{Re} = \frac{0.455}{\left(\log 10^7\right)^{2.58}} - \frac{1050}{10^7} = 0.00263$$

$$R_{fturb} = C_{fturb}\frac{1}{2}\rho u^2 S = 0.00264\cdot 0.5\cdot 1000.0\cdot 10.0^2\cdot 2.0 = 264[N]$$

Case b

$$\Delta C_f = \left(1.89 + 1.62\log 10000\right)^{-2.5} = 0.00494$$

$$C_{fchrop} = C_{fturb} + \Delta C_f = 0.00264 + 0.00494 = 0.00758$$

$$R_{fchrop} = C_{fchrop}\frac{1}{2}\rho u^2 S = 0.00758\cdot 0.5\cdot 1000.0\cdot 10.0^2\cdot 2.0 = 758[N]$$

Conclusion: surface roughness has a meaningful effect on the frictional drag in the turbulent boundary layer and it may lead to the significant increase of this drag with respect to a smooth surface.

Example 2

Laminar or turbulent boundary occurs alternatively on a plate having length L=1 [m] in the flow at Re=100000. What are the thicknesses of both layers at the end of the plate?

The laminar layer:

$$\delta_{lam} = \frac{5L}{\sqrt{Re}} = \frac{5 \cdot 1}{\sqrt{100000}} = 0.0158[m]$$

The turbulent layer:

$$\delta_{turb} = \frac{0.37L}{\sqrt[5]{Re}} = \frac{0.37 \cdot 1}{\sqrt[5]{10^5}} = 0.037[m]$$

Conclusion: At comparable flow conditions the turbulent boundary layer is more than twice thicker than the laminar boundary layer. This is the consequence of the more intensive transport of momentum and energy in the turbulent layer.

The temperature boundary layer

In certain problems (eg. in heat exchangers) it is important to determine the temperature distribution in the boundary layer. If we assume that the flow is steady and that the Reynolds number is larger than 1000, then the following relation may be developed:

$$\theta = \frac{T_w - T(y)}{T_w - T_\infty} = \frac{u(y)}{u_\infty} \qquad \text{when} \qquad \Pr = \frac{c\mu}{\lambda} = 1{,}0 \quad \text{(Prandtl number)}$$

where:

θ – non-dimensional temperature

T_w - temperature on the wall

T_∞ - temperature far from the wall

If in a steady flow the Prandtl number is equal 1, then the profile of non-dimensional temperature θ in the boundary layer is identical with the profile of non-dimensional velocity. If there is Pr>1, the temperature gradient in the inner region of the layer is greater than the velocity gradient. If there is Pr<1, the temperature gradient is smaller.