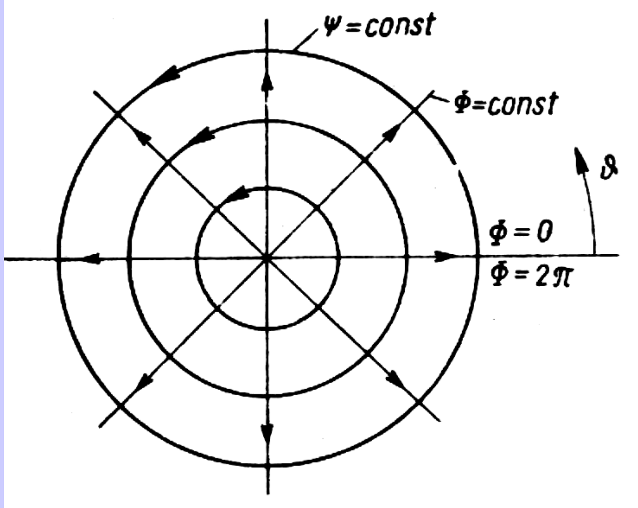
J. Szantyr – Lecture No. 24 – Potential Flows 2

In order to generate hydrodynamic forces on the bodies in potential flows it is necessary to achieve an asymmetrical flow. This is possible by means of another elementary potential flow called a vortex.

<u>The elementary potential flows (continued)</u>

4. The vortex

The vortex is a singular point generating in its neighbourhood a fluid motion along the circular paths.



The vortex is a flow linked to a source flow, because the vortex stream lines are identical to the source equipotential lines and the vortex equipotential lines coincide with the source stream lines.

The vortex potential: $\varphi = A \cdot \theta$ The vortex stream function:

$$u_r = \frac{\partial \varphi}{\partial r} = 0 \qquad u_\theta = \frac{1}{r}\frac{\partial \varphi}{\partial \theta} = \frac{A}{r} \qquad \psi = A \cdot \ln r$$

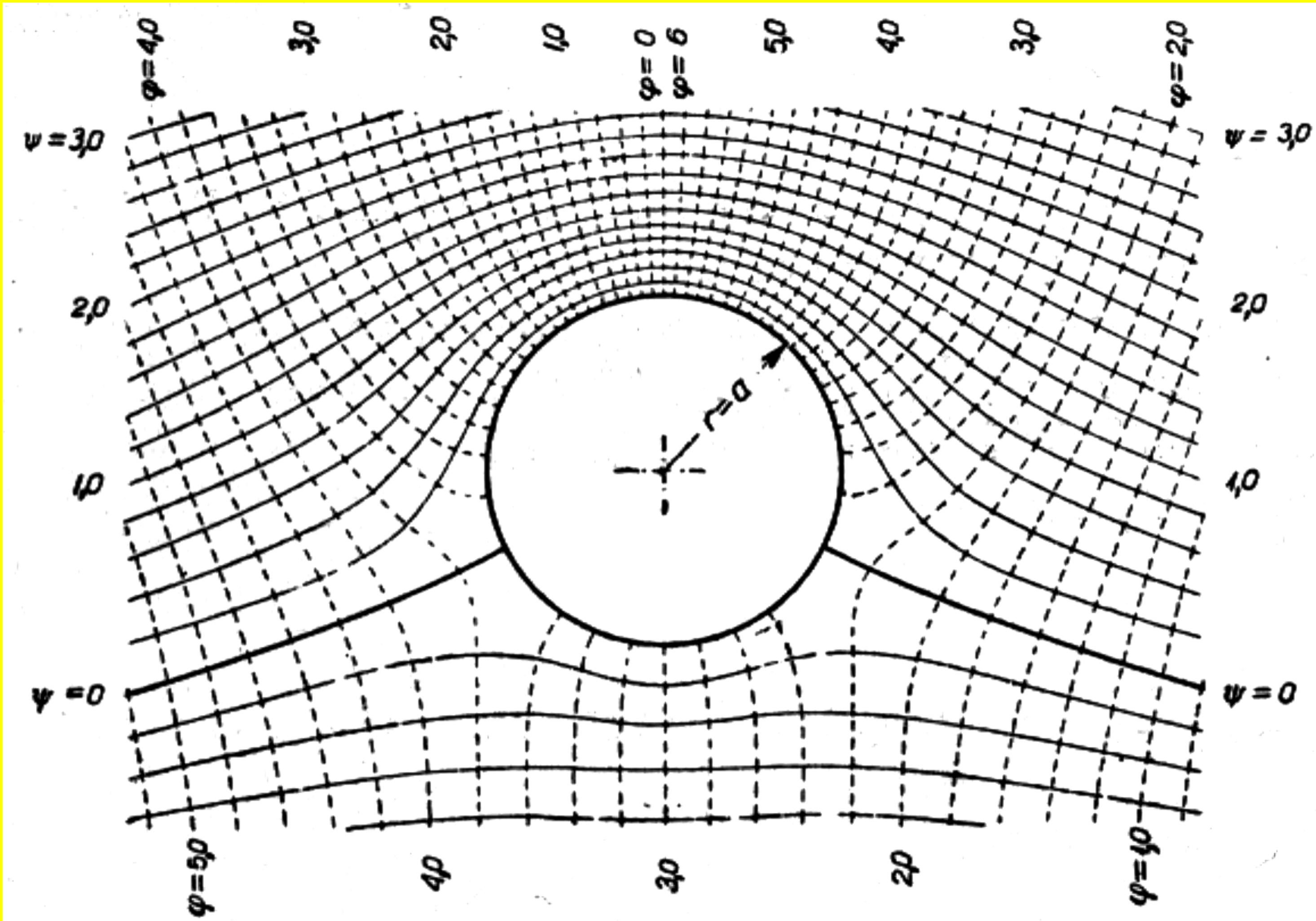
The constant A is connected with the velocity circulation along the contour C encompassing the vortex:

$$\Gamma_C = \oint_C \overline{u} \bullet d\overline{s} = \int_0^{2\pi} \overline{u} \bullet d\overline{s} = \int_0^{2\pi} \frac{A}{r} \bullet r d\theta = \int_0^{2\pi} A d\theta = 2\pi A \rightarrow A = \frac{\Gamma_C}{2\pi}$$

Thus we get: $$u_\theta = \frac{\Gamma}{2\pi r}$$

It should be noticed that the flow generated by the vortex is <u>irrotational</u> in the entire space except the vortex itself. The computation of circulation along any contour not containing the vortex gives zero. Hence we have <u>an isolated vortex</u> at *x=0, y=0* and an <u>irrotational flow in its neighbourhood</u>. This allows us to treat the entire flow as <u>a potential flow.</u>

Example: Rotational flow around a circular cylinder



Superposition of an uniform flow, a dipole and a vortex.

The potential: $\varphi = u_\infty \left(r + \frac{a^2}{r^2} \right) \cos\theta - \frac{\Gamma}{2\pi}\theta$ where: $\theta = arctg\,\frac{y}{x}$

The stream line: $\psi = u_\infty \left(r - \frac{a^2}{r^2} \right) \sin\theta + \frac{\Gamma}{2\pi} \ln\frac{r}{a}$

The velocity components:
$$u_r = u_\infty \left(1 - \frac{a^2}{r^2} \right) \cos\theta$$
$$u_\theta = -u_\infty \left(1 + \frac{a^2}{r^2} \right) \sin\theta - \frac{\Gamma}{2\pi r}$$

On the cylinder surface we have: $u_r = 0$ $\qquad u_\theta = -u_\infty \left(2\sin\theta + \frac{\Gamma}{2\pi a u_\infty} \right)$

The pressure distribution on the cylinder according to the Bernoulli equation:
$$p_\theta = p_\infty + \frac{\rho u_\infty^2}{2} \left(1 - \frac{u_\theta^2}{u_\infty^2} \right)$$

The pressure distribution on the cylinder surface, defined in the form of a non-dimensional coefficient:

$$C_p = \frac{p_\theta - p_\infty}{\frac{1}{2}\rho u_\infty^2} = 1 - \left(2\sin\theta + \frac{\Gamma}{2\pi a u_\infty} \right)^2$$

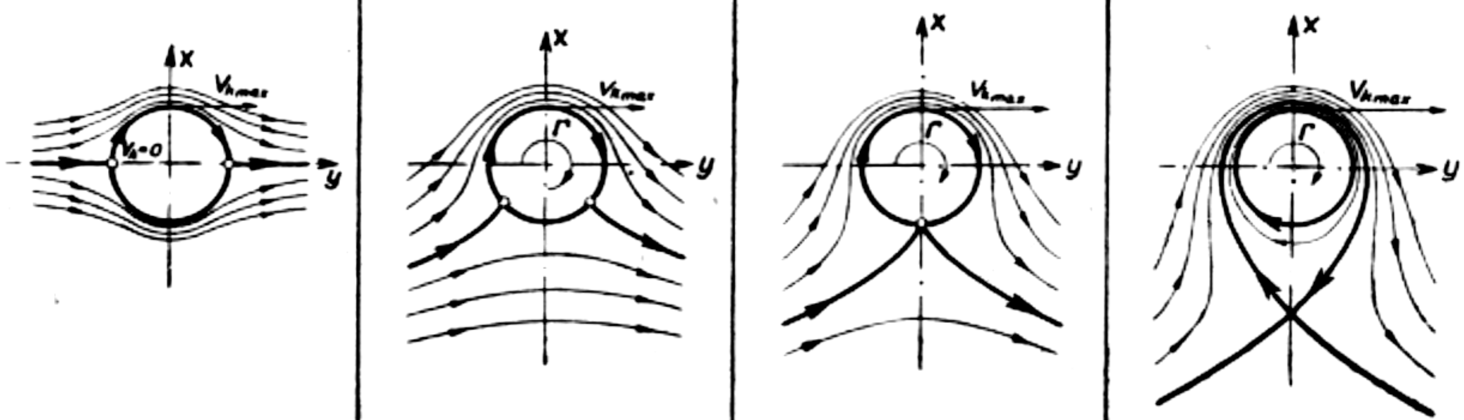
On the basis of the pressure distribution the components of the resultant hydrodynamic force on the cylinder may be calculated:

$$P_x = -a\int_0^{2\pi} p_\theta \cos\theta d\theta = 0$$ - the drag force

$$P_y = -a\int_0^{2\pi} p_\theta \sin\theta d\theta = \rho u_\infty \Gamma$$ - the lift force

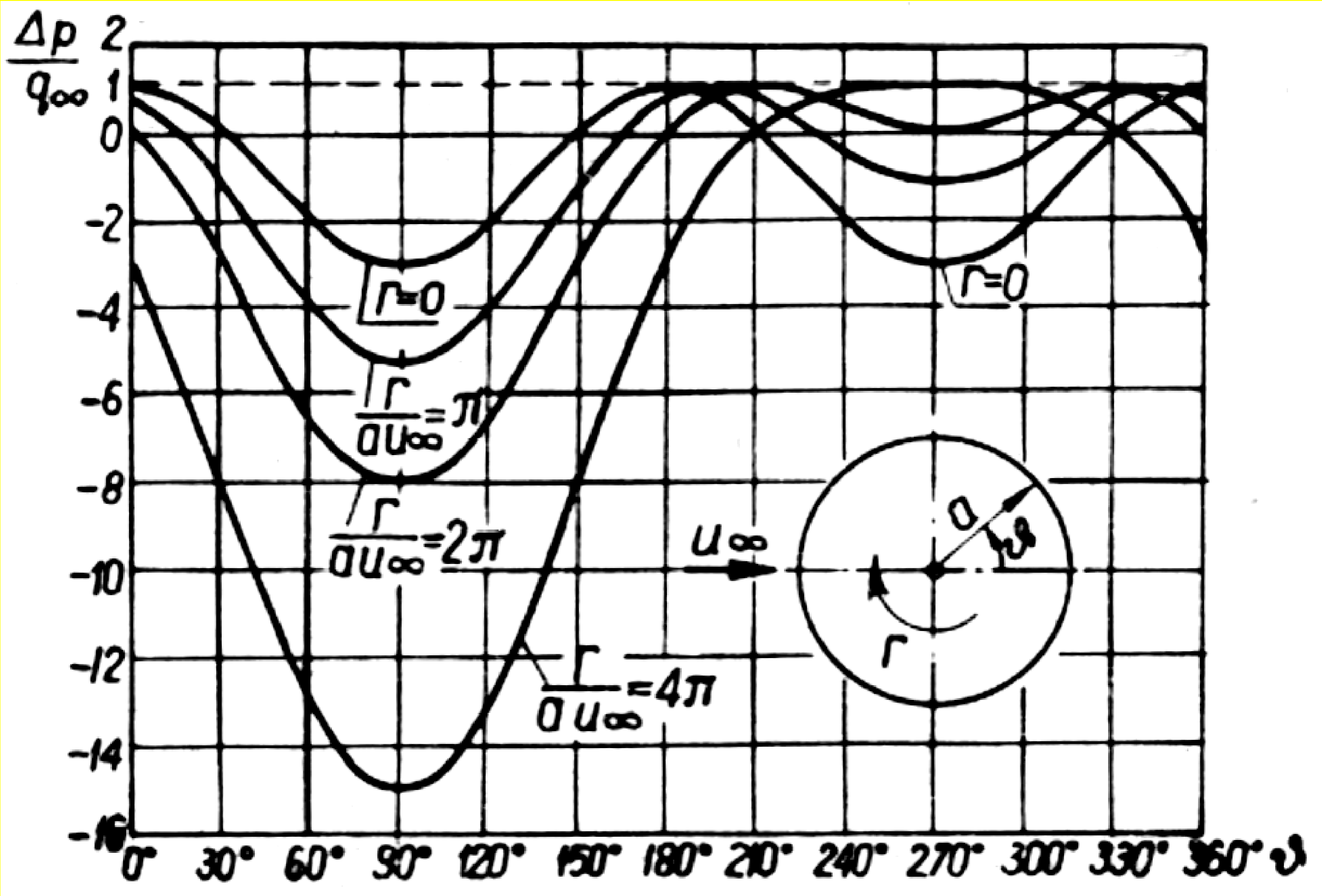
The Joukovsky theorem: the lift force acting on the unit span of the cylinder is equal to the multiple of the fluid density, velocity of the undisturbed flow and the circulation of velocity around the cylinder.

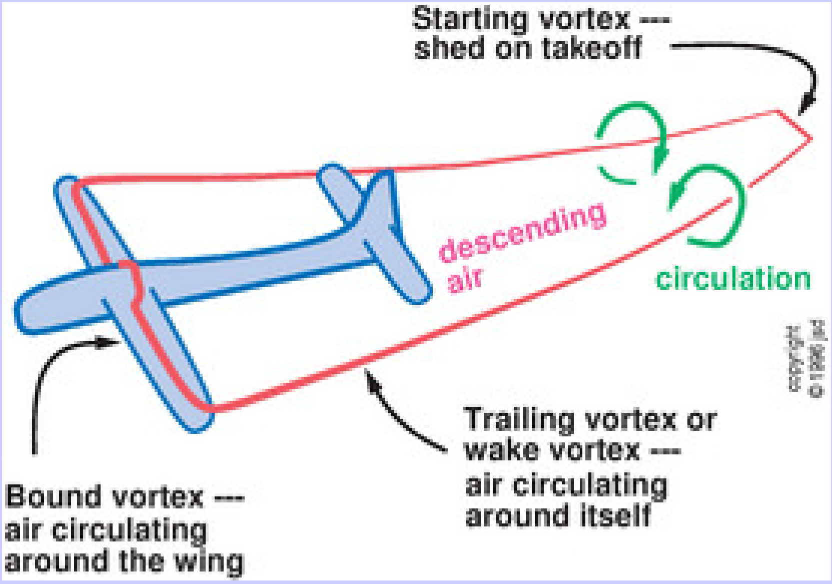
Γ	0	$2\pi a u_\infty$	$4\pi a u_\infty$	$> 4\pi a u_\infty$
$V_{k_{max}}$	$2u_\infty$	$3u_\infty$	$4u_\infty$	
$\sin\vartheta = -\frac{\Gamma}{4\pi a u_\infty}$	0	$-\frac{1}{2}$	-1	
ϑ dla $V_k = 0$	0 π	$\frac{7}{6}\pi$ $\frac{4}{6}\pi$	$\frac{3}{2}\pi$	



The rotational flow around a cylinder depending on the value of the circulation of velocity

The asymmetrical pressure distribution around the cylinder depends on the value of circulation. In order to determine the value of circulation an additional condition is necessary, which defines the location of the stagnation point on the cylinder.





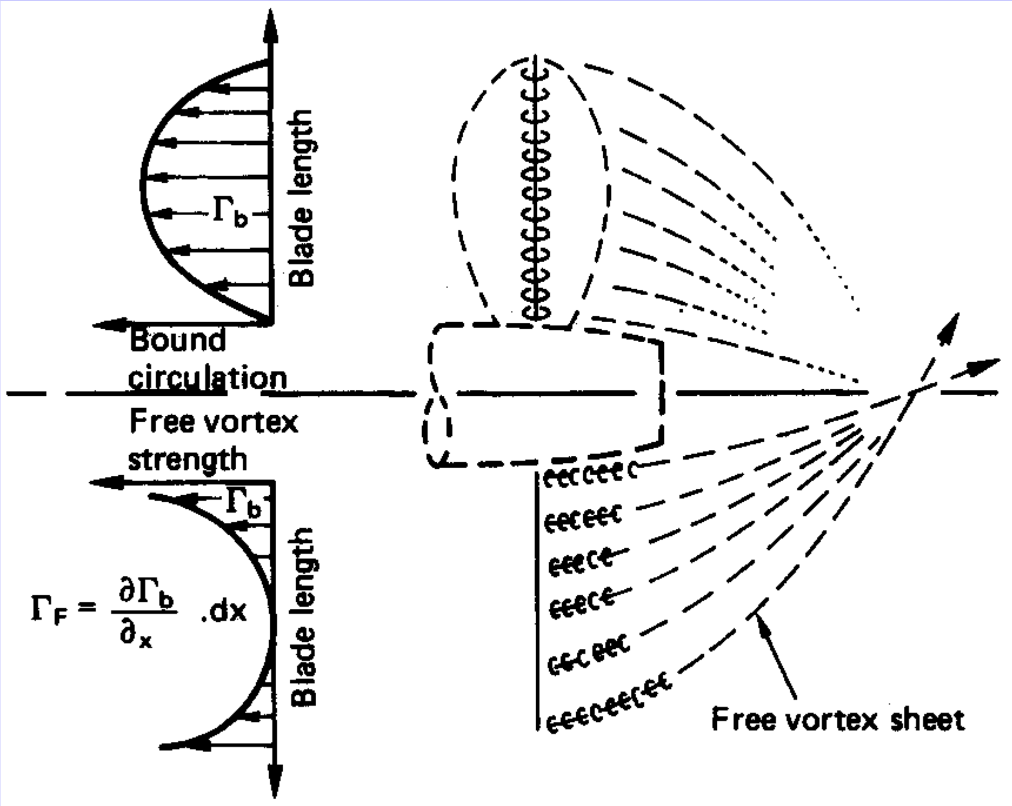
The Joukovsky theorem may be employed e.g. for determination of the lift on the wing of an aircraft, according to:

$$\overline{L} = \rho \overline{U}_{\infty} \times \overline{\Gamma}$$

The above relation determines not only the value of the lift force, but also its direction.

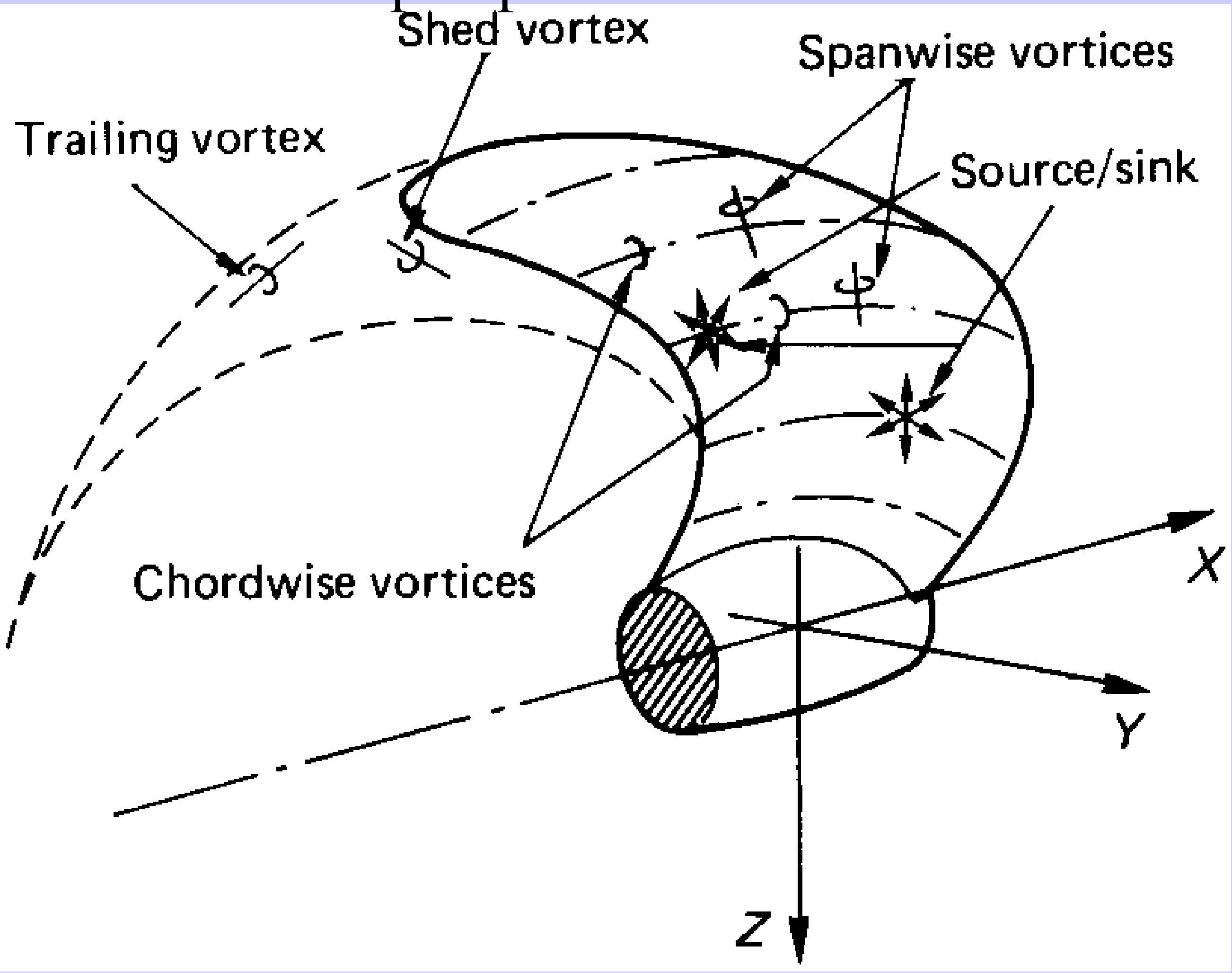
Contemporary methods for determination of potential flows

- the lifting line method
- the lifting surface method
- the boundary element method

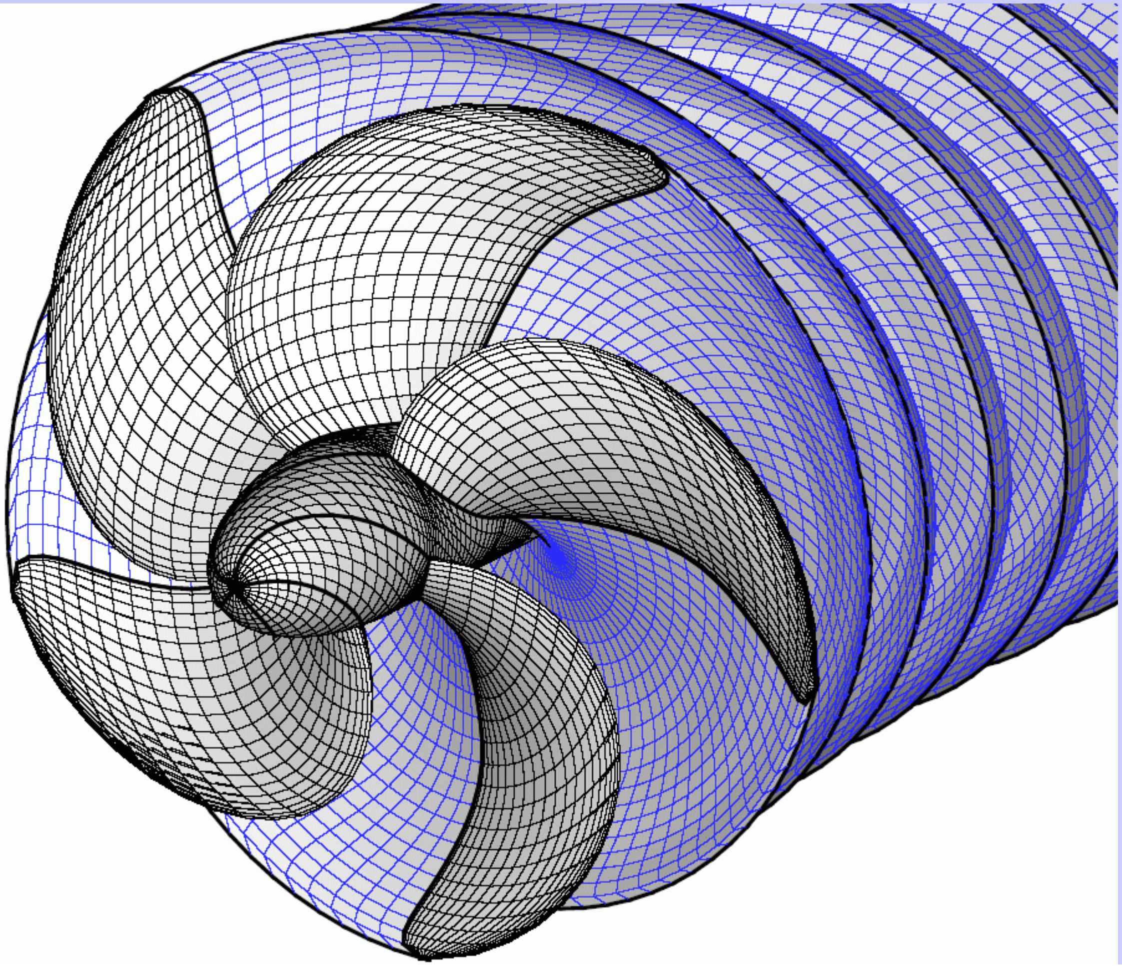


The lifting line method is based on substituting the lifting foil with a single vortex line, so called bound vortex, which generates lift according to the Joukovsky theorem. This vortex must be supplemented with the system of free vortices. The lifting line method is well suited for modelling flows around foils of high aspect ratio, e.g. aircraft wings or airscrew blades.

The lifting surface method is based on the distribution of vortices, sources and dipoles on an infinitely thin surface, bounded by the true foil outline. This method is well suited for modelling of flow around the foils of low aspect ratio, e.g. marine propeller blades or turbine and pump blades etc.



The boundary element method is based on distribution of vortices, sources and dipoles on the true surface of objects, that is on both sides of an aircraft wing or turbine blade etc. This method is well suited for deremination of flows around complicated objects, e.g. complete aircraft, vehicles or ships. The modelling of such flows requires using many thousands of elements.

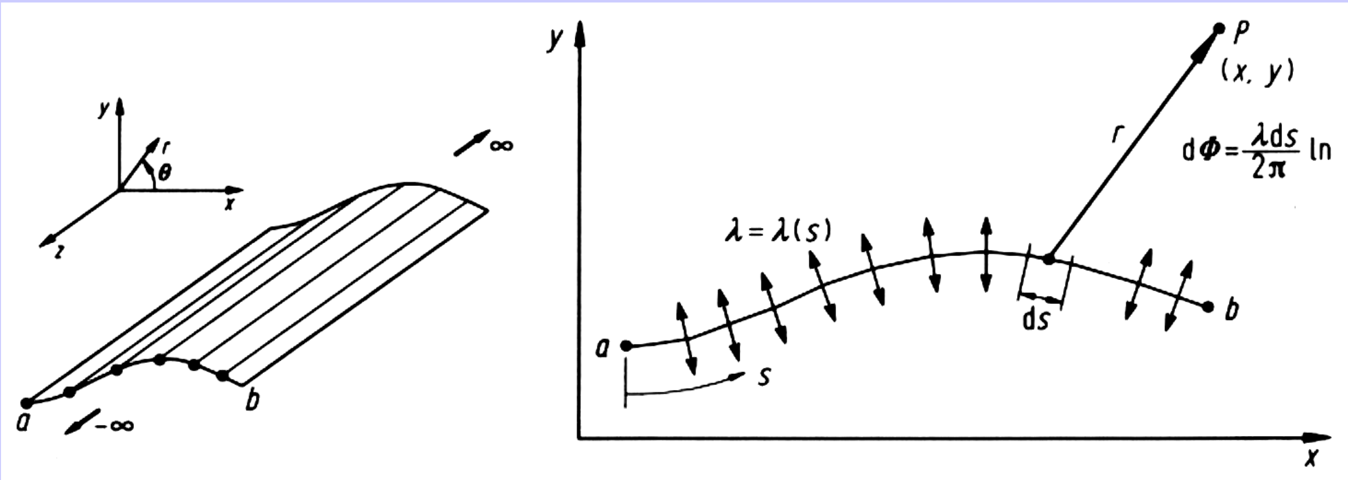


Pressure distribution on the marine propeller blades calculated by means of the boundary element method

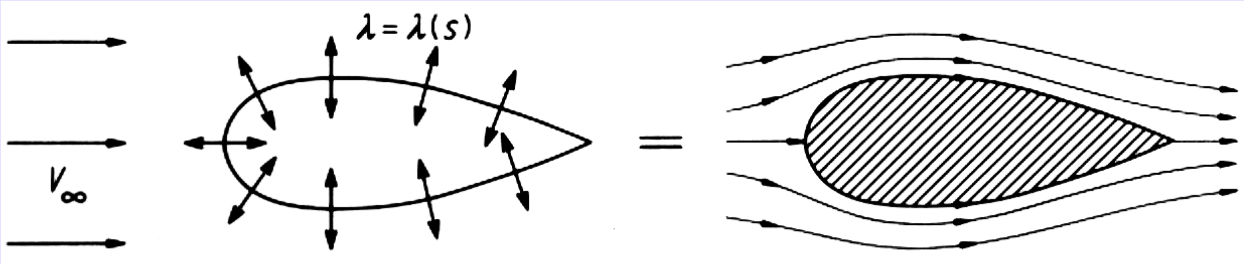
Flows around geometrically the complicated objects may be modelled by continuous distributions of sources, vortices or dipoles. For example, a continuous distribution of sources along the curve a-b may be described by a potential:

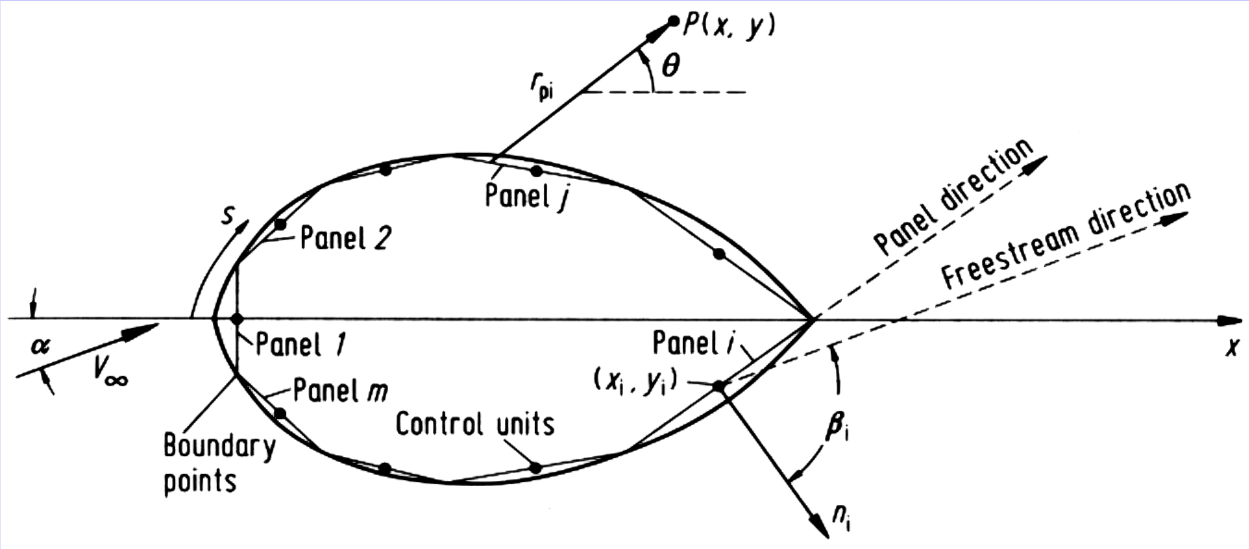
$$\varphi(x, y) = \int_a^b \frac{\lambda ds}{2\pi} \ln r$$

where: $\lambda[m/s]$ - a continuous distribution of sources



In practice, the geometrically complicated surface of the object is divided into a number of elements, so called panels. In the two-dimensional flows the panels are most frequently sections of a straight line, and in three-dimensional flows – sections of flat surfaces.



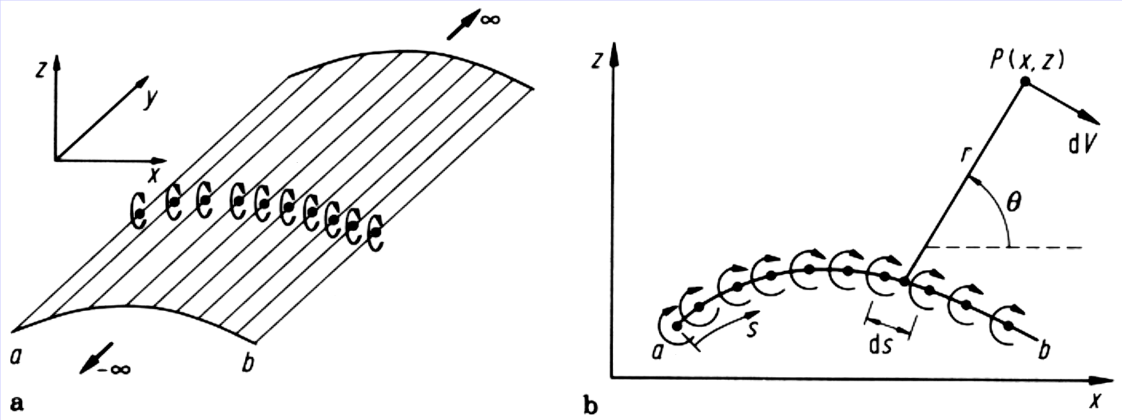


The main role in the solution of such a flow plays the boundary condition, which postulates no flow through the object surface, i.e. normal component of the resultant velocity equal zero, what leads to the equation:

$$\frac{\lambda_i}{2} + \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{\lambda_j}{2\pi} \int_j \frac{\partial}{\partial n_j} \left(\ln r_{ij} \right) ds_j + V_\infty \cos \beta_i = 0$$

Writing such an equation for each panel leads to the system of linear equations for the unknown intensities of the source distribution λ.

Modelling of flows in which lift forces are generated requires using the distributions of vortices or dipoles. In the case of a continuous vortex distribution of intensity γ we have:



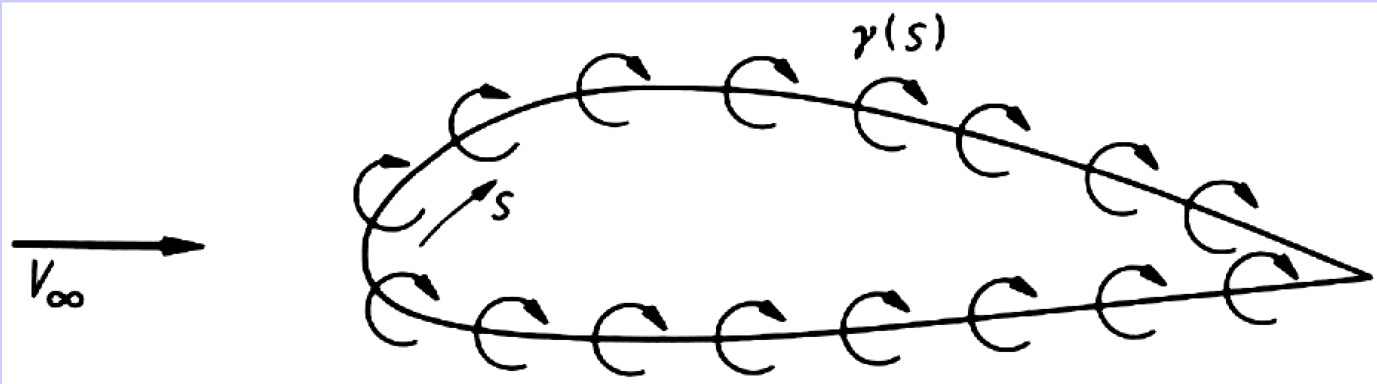
In this case the flow is described by the potential:

$$\varphi(x, y) = -\frac{1}{2\pi}\int_a^b \theta\gamma ds$$

where: $\gamma[m/s]$ - a continuous vortex distribution

After dividing the object into panels we obtain the equation:

$$V_\infty \cos\beta_i - \sum_{j=1}^{n} \frac{\gamma_j}{2\pi}\int_j \frac{\partial\theta_{ij}}{\partial n_i} ds_j = 0$$



In the case of modelling of the flow around an object generating lift force by means of the vortex distribution it is necessary to introduce an additional condition, which enables a unique determination of the vortex intensity. For a profile in most cases it is so called Kutta condition, which postulates that the flow leaves the profile exactly at the trailing edge. After solution of the system of linear equations and calculation of the vortex intensity on all panels, the lift force may be computed using the Joukovsky equation:

$$L = \rho V_\infty \sum_{j=1}^{n} \gamma_j s_j$$