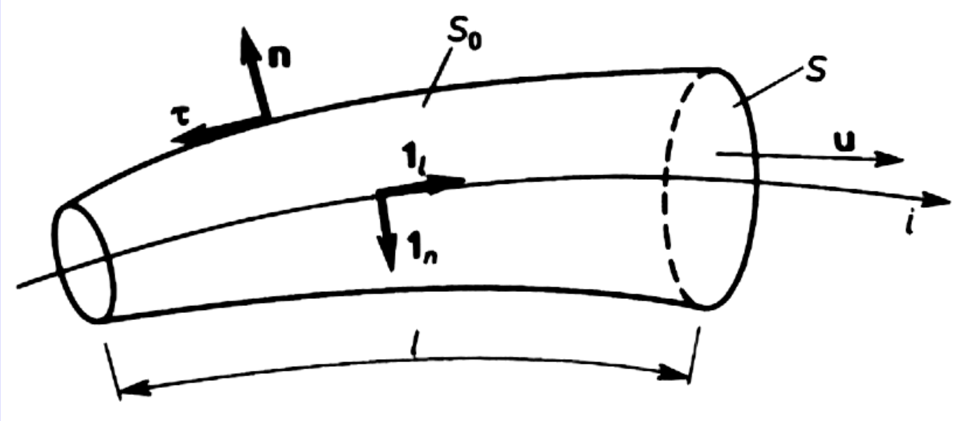
J. Szantyr – Lecture No. 25 – Flows in closed channels 1

A closed channel – channel of an arbitrary cross-section, limited by a closed curve, completely filled with the fluid (without a free surface)



The flow of fluid in a closed channel is described by means of the simplified model of a one-dimensional flow. It is assumed that the axis of the channel is „almost" a straight line and the flow across the section S takes place with a „representative" velocity $\tilde{u}$, that is some mean velocity.

The simplest case: a horizontal channel of constant circular cross-section. Steady flow of an incompressible fluid.

The mass conservation equation (m-mass intensity of flow):

$$\frac{\partial}{\partial l}\left(\rho\tilde{u}S\right)=0 \rightarrow \rho\tilde{u}S = m = const \rightarrow \tilde{u} = \frac{m}{\rho S} = const$$

The momentum conservation equation: $\frac{D}{Dt}\left(\tilde{\rho}\tilde{u}S\right)=\tilde{\rho}\tilde{f}S - \frac{\partial\tilde{p}}{\partial l}S - p_\tau C$

where C – the circumference of the section S

p_τ - the viscous tangential stress

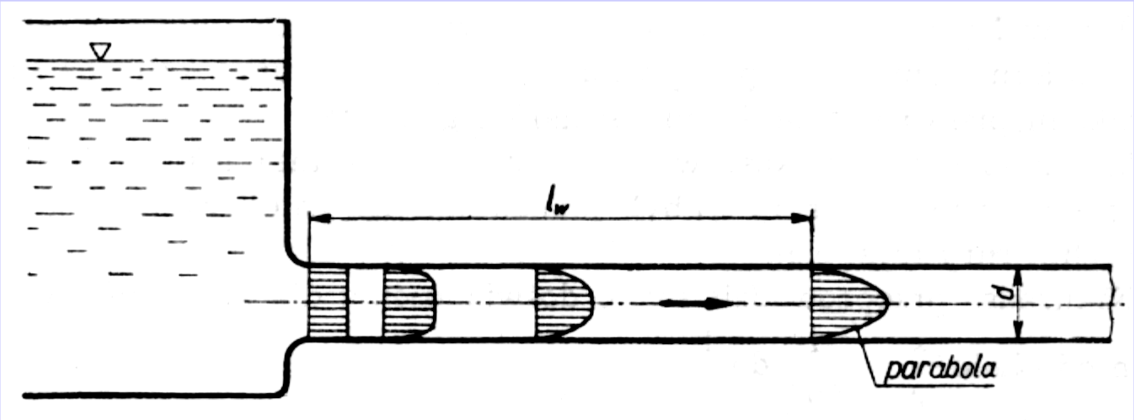
$$0 = -\frac{\partial p}{\partial l}S - p_\tau C \rightarrow \frac{\partial p}{\partial l} = -p_\tau \frac{C}{S} \rightarrow \int_1^2 dp = -\int_1^2 p_\tau \frac{C}{S} dl$$

With constant stresses along the channel of a circular cross-section we have:

$$\boxed{\Delta p = p_1 - p_2 = p_\tau \frac{4l}{d}}$$

Due to the action of viscous forces there is a pressure drop along the channel, directly proportional to l and p_τ and inversely proportional to *d.*

In the case of a fully developed laminar flow, i.e. after the initial section $l_w \approx 0.03 \cdot \mathrm{Re} \cdot d$ an analytical solution of the Navier-Stokes equation is possible, which leads to the formulae for:

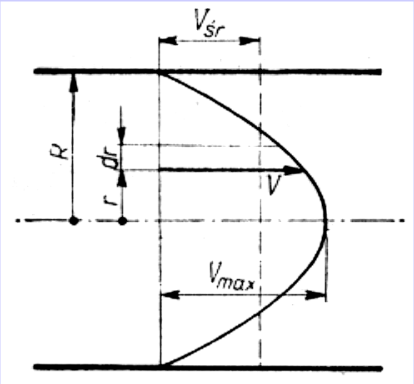


local velocity:

$$u(r) = \frac{\Delta p}{4\mu l}\left(r_0^2 - r^2\right)$$

mean velocity:

$$\tilde{u} = \frac{\Delta p \cdot r_0^2}{8\mu l}$$



Formula for the mean velocity may be transformed into the Darcy-Weisbach formula

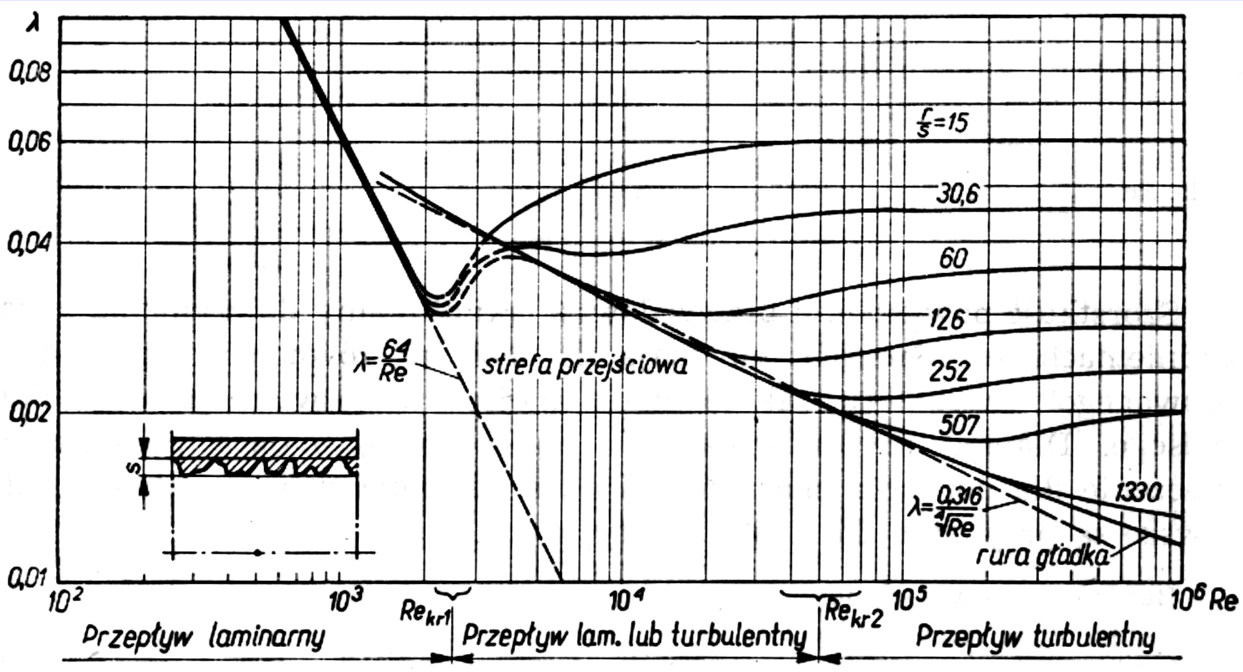
$$\Delta p = \lambda \frac{l}{d} \frac{\rho \tilde{u}^2}{2}$$

Where λ – resistance coefficient or **the coefficient of linear losses**

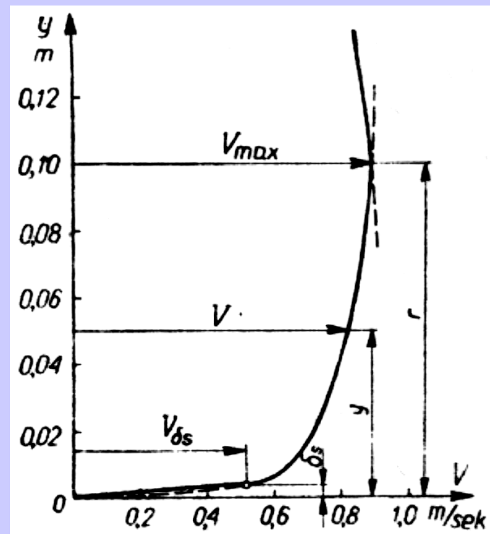
In the laminar flow: $$\lambda = \frac{64}{\mathrm{Re}}$$

In the turbulent flow through the hydrodynamically smooth channels: $$\lambda = \frac{0.3164}{\sqrt[4]{\mathrm{Re}}}$$

In a general case of rough channels: $$\lambda = \lambda\left(\mathrm{Re}, \frac{r_0}{k}\right)$$



The turbulent velocity profile in a channel



The more complicated case: a channel inclined at an angle α.

If we assume a stationary flow, then the momentum conservation equation takes the following form:

$$\tilde{u}\frac{\partial \tilde{u}}{\partial l} = f_l - \frac{1}{\rho}\frac{\partial p}{\partial l} - \frac{p_\tau}{\rho}\frac{C}{S} \rightarrow \frac{\partial}{\partial l}\left(\frac{\tilde{u}^2}{2} + \frac{p}{\rho} + gz\right) = -\frac{p_\tau}{\rho}\frac{C}{S}$$

Where the mass force component along *l* is substituted as:

$$f_l = g\sin\alpha = -g\frac{dz}{dl}$$

After integration between two cross-sections of the channel we obtain the **Bernoulli equation for a real flow with losses:**

$$\left(\frac{\tilde{u}_1^2}{2} + \frac{p_1}{\rho} + gz_1\right) - \left(\frac{\tilde{u}_2^2}{2} + \frac{p_2}{\rho} + gz_2\right) = \int_1^2 \frac{p_\tau}{\rho}\frac{C}{S}dl$$

or:

$$\frac{\tilde{u}_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{\tilde{u}_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + h_s = H = const$$

where h_s - the loss head

The loss head may be divided into two contributions:

-the linear loss head, related to the friction of the fluid against the walls of the straight line channel of a constant cross-section,

-the local loss head, related to the presence of valves,confusers, switches, forks etc.

The linear loss head may be calculated as: $$h_s = \frac{\tilde{u}^2}{2g}\lambda\frac{l}{d}$$

The local loss head may be calculated as: $$h_s = \varsigma\frac{\tilde{u}_1^2}{2g} = \varsigma'\frac{\tilde{u}_2^2}{2g}$$

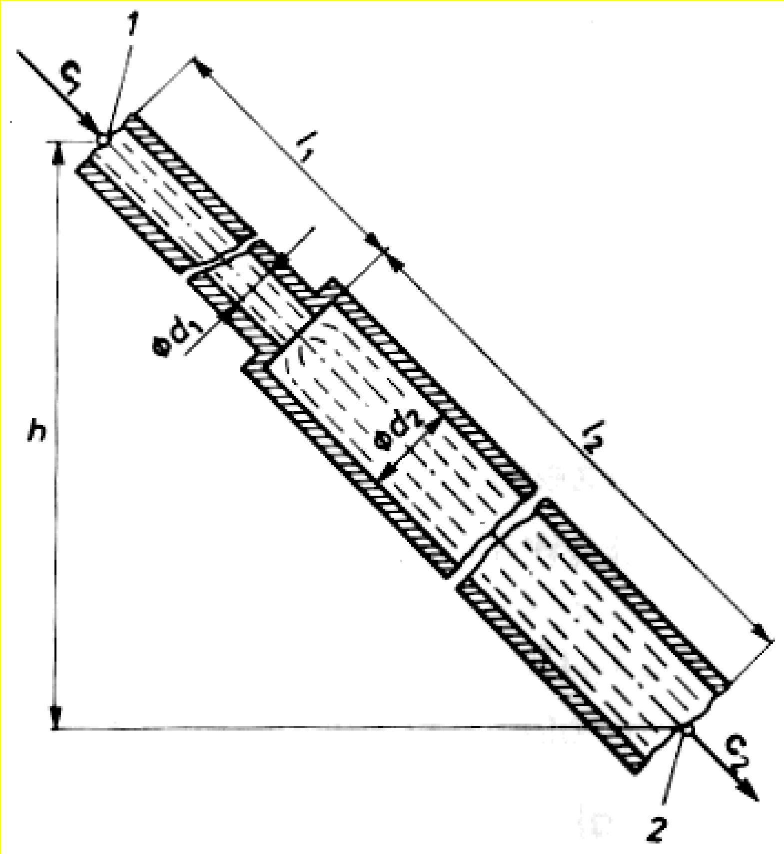
where ζ is the local loss coefficient, which may be determined in reference to the velocity before or behind the device. The coefficients ζ are determined experimentally and they may be found in the appropriate tables. Several examples of the local loss coefficients are reproduced below.

The local loss coefficients

Type of the local loss	The loss coefficient
Inlet from a tank	$\varsigma' = 0.5$
Channel deflection by φ	$\varsigma = 0.946 \sin^2 \varphi/2 + 2.05 \sin^4 \varphi/2$
Increase of cross-section	$\varsigma = \left(1 - A_1/A_2\right)^2$ $\varsigma' = \left(A_2/A_1 - 1\right)^2$
Cock opened by 5 deg.	$\varsigma = 0.05$
Cock opened by 45 deg.	$\varsigma = 31.2$
Suction pump inlet	$\varsigma = 10.0$

Example No.1: 19600 kg of fuel of density ρ=930 kg/m3 and kinematic viscosity coefficient ν=0,000061 m**2/s flows in an hour through a pipe of variable cross-section. Determine the pressure loss in the pipe if the dimensions are:**

$$l_1 = 5[m], d_1 = 50[mm], l_2 = 10[m], d_2 = 100[mm], h = 5[m]$$



Solution

Volumetric flow intensity: $$Q = \frac{19600}{930 \cdot 3600} = 0.00585 \left[m^3/s \right]$$

Mean velocity in part 1: $$c_1 = \frac{4Q}{\pi d_1^2} = \frac{4 \cdot 0.00585}{3.14 \cdot 0.05^2} = 2.98 \left[m/s \right]$$

Mean velocity in part 2: $$c_2 = \frac{4Q}{\pi d_2^2} = \frac{4 \cdot 0.00585}{3.14 \cdot 0.1^2} = 0.745 \left[m/s \right]$$

Reynolds number in part 1: $$\mathrm{Re}_1 = \frac{c_1 d_1}{\nu} = \frac{2.98 \cdot 0.05}{0.000061} = 2443$$

Reynolds number in part 2: $$\mathrm{Re}_2 = \frac{c_2 d_2}{\nu} = \frac{0.745 \cdot 0.1}{0.000061} = 1221$$

Linear losses coefficient in part 1:

$$\lambda_1 = \frac{0.3164}{\sqrt[4]{\mathrm{Re}_1}} = \frac{0.3164}{\sqrt[4]{2443}} = 0.045$$

Linear losses coefficient in part 2: $$\lambda_2 = \frac{64}{\mathrm{Re}_2} = \frac{64}{1221} = 0.052$$

Loss of pressure height due to linear losses is equal to:

$$h_\lambda = \frac{c_1^2}{2g}\lambda_1\frac{l_1}{d_1} + \frac{c_2^2}{2g}\lambda_2\frac{l_2}{d_2} = \frac{2.98^2}{2\cdot 9.81}0.045\frac{5}{0.05} + \frac{0.745^2}{2\cdot 9.81}0.052\frac{10}{0.1} = 2.183[m]$$

Loss of pressure height due to criss-section increase is equal to:

$$h_\varsigma = \frac{\left(c_1 - c_2\right)^2}{2g} = \frac{\left(2.98 - 0.745\right)^2}{2\cdot 9.81} = 0.255[m]$$

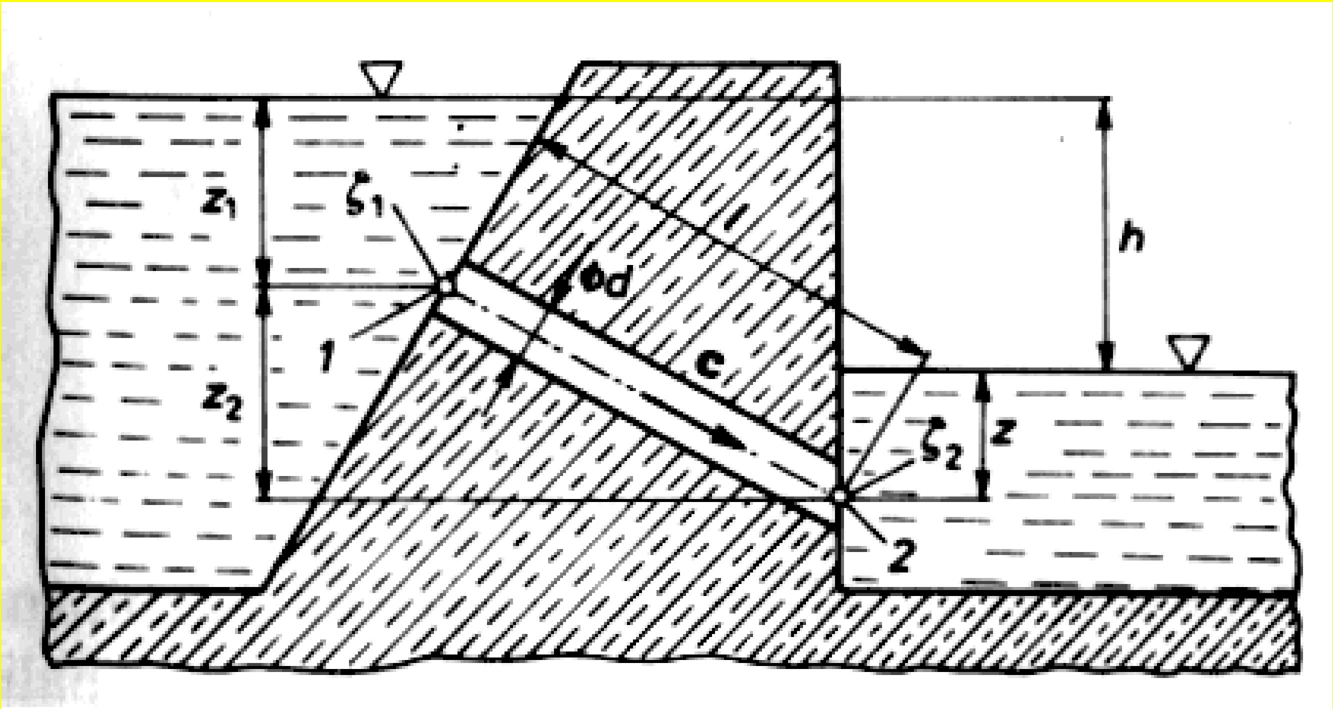
Total pressure height loss is equal to:

$$h_s = h_\lambda + h_\varsigma = 2.183 + 0.255 = 2.438[m]$$

Loss of pressure in the pipe is equal to:

$$\Delta p = \rho g\left[\frac{c_2^2 - c_1^2}{2g} + h_s - h\right] = 27245[Pa] \approx 0.272[MPa]$$

Example No. 2: A pipe of diameter *d* and length *l* connects two tanks, in which the water levels are displaced by h. Determine the volumetric intensity of flow through the pipe knowing the linear losses coefficient λ and the local losses coefficients at inflow and outflow.



Solution

Bernoulli equation for sections 1 and 2:

$$\frac{c_1^2}{2g} + z_1 + z_2 = \frac{c_2^2}{2g} + z + \sum h_s$$

In steady flow we have: $c_1 = c_2 = c$

What leads to: $z_1 + z_2 - z = \sum h_s$

As there is: $z_1 + z_2 - z = h$ and $\sum h_s = \frac{c^2}{2g}\left(\lambda \frac{l}{d} + \varsigma_1 + \varsigma_2\right)$

We get:

$$h = \frac{c^2}{2g}\left(\lambda \frac{l}{d} + \varsigma_1 + \varsigma_2\right)$$

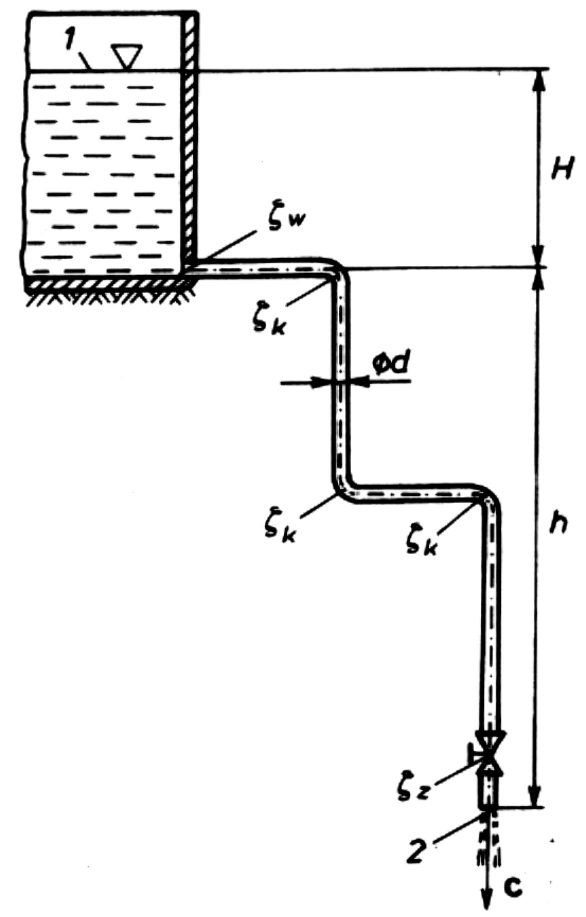
From that we determine the mean flow velocity:

$$c = \sqrt{\frac{2gh}{\lambda \frac{l}{d} + \varsigma_1 + \varsigma_2}}$$

Then we calculate the volumetric flow intensity through the pipe:

$$Q = \frac{\pi d^2}{4}\sqrt{\frac{2gh}{\lambda \frac{l}{d} + \varsigma_1 + \varsigma_2}}$$

Example No. 3



Water flows from an open tank through a pipe of length l=200 [m] and diameter d=100 [mm]. What should be the water level H in the tank to ensure the volumetric intensity of outflow from the pipe equal to Q=40 [l/s]?

Given: h=2 [m] $\upsilon = 1 \cdot 10^{-6} \left[m^2/s \right]$

$\varsigma_w = 0.5$ inlet from the tank

$\varsigma_k = 0.2$ elbow

$\varsigma_z = 5.0$ exit valve

Solution

The mean outflow velocity is equal to:

$$\tilde{c} = \frac{4Q}{\pi d^2} = \frac{4 \cdot 0.04}{3.14 \cdot (0.1)^2} = 5.1[m/s]$$

The Reynolds is equal to:

$$\mathrm{Re} = \frac{\tilde{c}\, d}{\upsilon} = \frac{5.1 \cdot 0.1}{1 \cdot 10^{-6}} = 510000$$

The flow in the pipe is turbulent, i.e. the friction coefficient is equal to:

$$\lambda = \frac{0.3164}{\sqrt[4]{\mathrm{Re}}} = \frac{0.3164}{\sqrt[4]{510000}} = 0.012$$

The level H is determined from the Bernoulli equation:

$$\frac{\tilde{c}_1^{\,2}}{2g} + \frac{p_b}{\rho g} + h + H = \frac{\tilde{c}_2^{\,2}}{2g} + \frac{p_b}{\rho g} + \sum h_s$$

where: $c_1 = 0$ $\qquad c_2 = \tilde{c}$

The total loss head is equal to:

$$\sum h_s = \frac{\tilde{c}^2}{2g}\left(\varsigma_w + 3\cdot\varsigma_k + \varsigma_z + \lambda\frac{l}{d}\right)$$

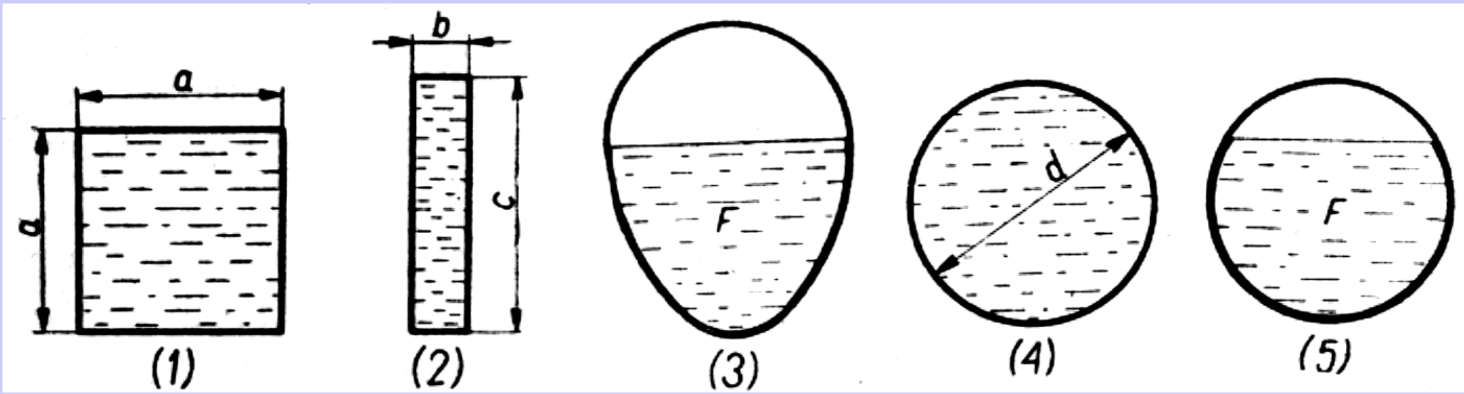
Consequently, the necessary water level H in the tank is equal to:

$$H = \frac{\tilde{c}^2}{2g}\left(1 + \varsigma_w + 3\cdot\varsigma_k + \varsigma_w + \lambda\frac{l}{d}\right)$$

After substituting numerical data we get:

$$H = \frac{(5.1)^2}{2\cdot 9.81}\left(1 + 0.5 + 3\cdot 0.2 + 5 + 0.012\frac{200}{0.1}\right) = 39.2[m]$$

The case of non-circular or partly filled channels



In the case of non-circular or partly filled channels the important parameter is so called hydraulic radius, or the ratio of the fluid stream cross-section area to the wetted circumference:

$$r_h = \frac{F}{L_z}$$

In such cases the Reynolds number is calculated using the formula:

$$\mathrm{Re} = \frac{u \cdot 4 r_h}{\upsilon}$$