J. Szantyr –Lecture No. 26 – Flows in closed channels 2

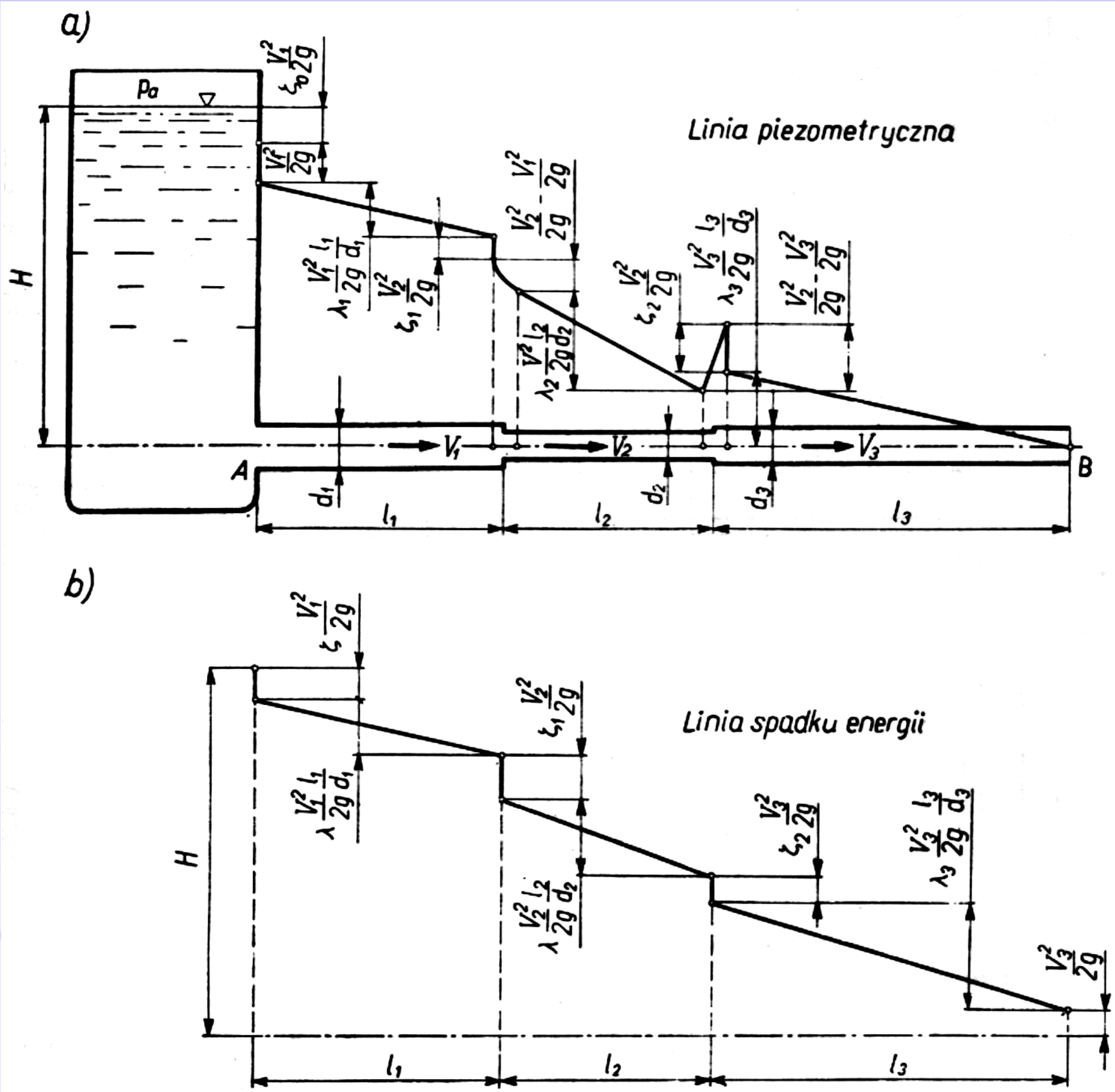
In practice we often deal with more or less complicated pipelines. If the stream of fluid is not branched, we are dealing with the straight pipeline. In the opposite case we are dealing with the pipeline grid. The grids are divided into the branched grids (when the grid does not include closed circuits) and into the circular grids (when the closed circuits are present). The hydraulic grid analysis concerns determination of all parameters in all elements of the grid. The hydraulic grid calculation is performed for a steady case when the Strouhal number is less than one:

$$Sh = \frac{t_{char}}{t_{zm}} < 1$$

t_{char} - the time of flow through a pipeline section

t_{zm} - the time of variation of the conditions at inflow to this element

Calculation of a straight pipeline



The piezometric line shows the variation of pressure along the pipeline axis. Such a pressure would be indicated by the pressure gauges at the respective points of the pipeline.

The loss of energy line shows the linear and local energy losses along the pipeline axis.

Comments to the piezometric line

The pipeline axis is assumed as the reference level of the pressure head $p_a/\rho g$ i.e. the pressure at inlet is equal to $p_a + \rho g H$

Immediately after the inlet the pressure head is reduced by:

the inlet loss $\varsigma \frac{V_1^2}{2g}$

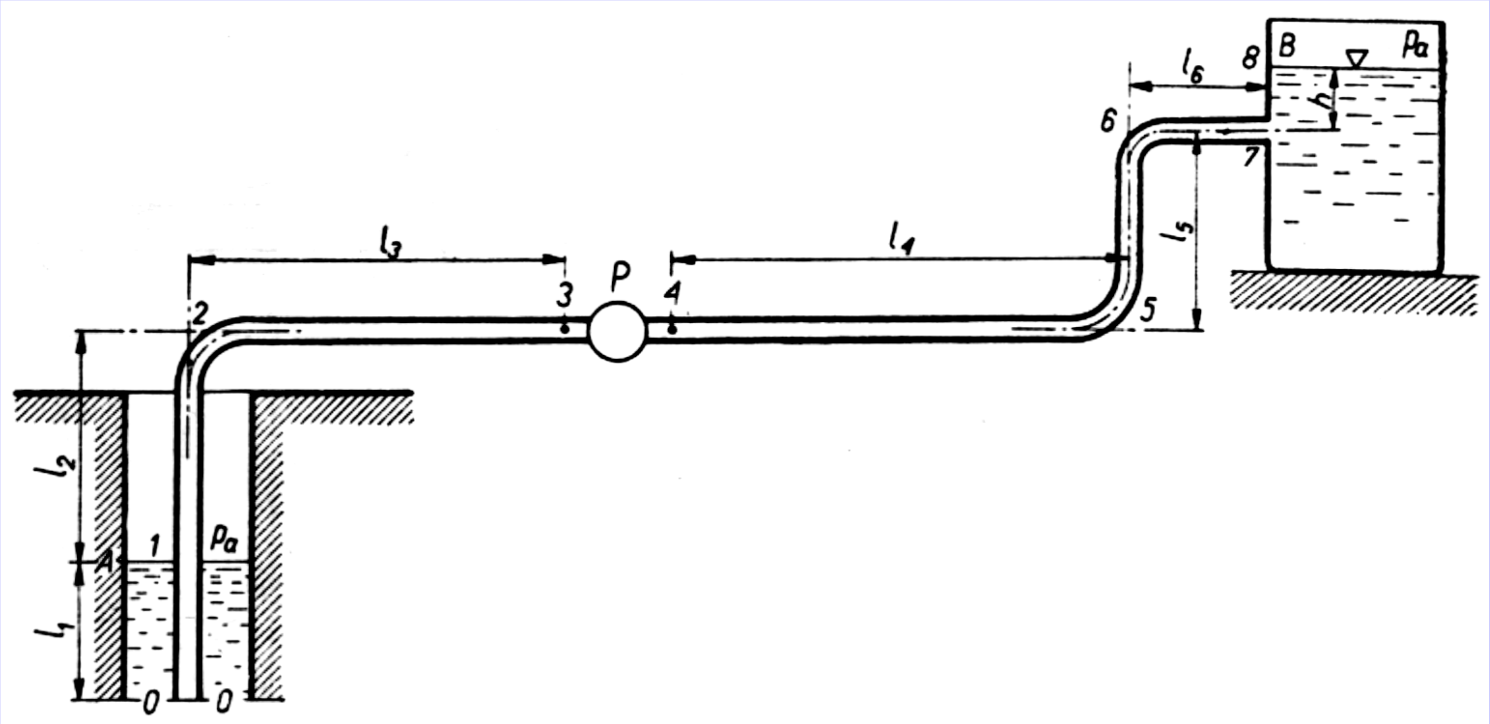
the pressure converted into kinetic energy of the flowing fluid $\frac{V_1^2}{2g}$

Along the first section of the pipeline the pressure head falls linearly according to the linear losses: $\lambda_1 \frac{V_1^2}{2g}$

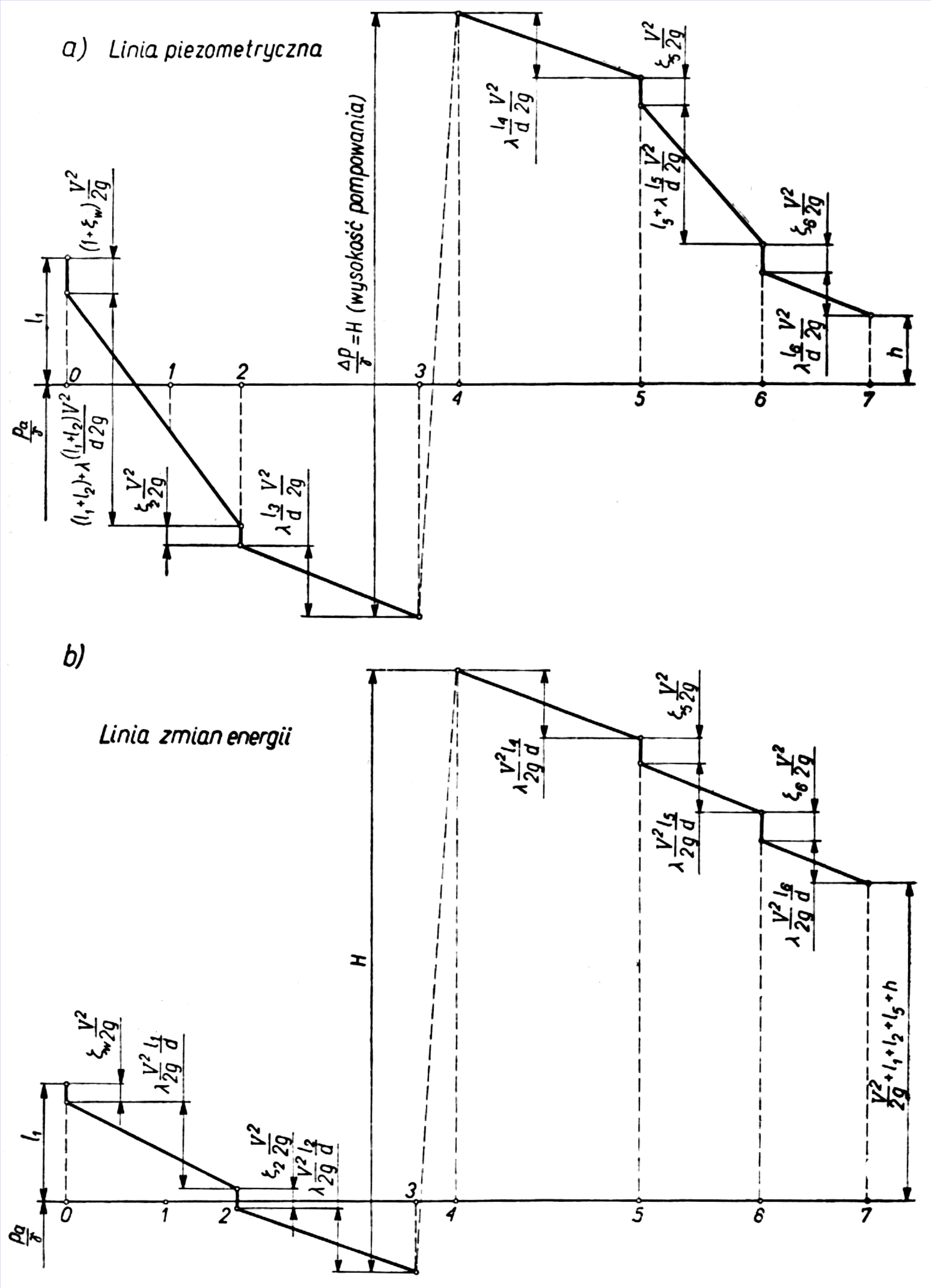
Comments to the energy loss line

This line shows only the heads of the linear and local losses, not taking into account the kinetic energy of the flowing fluid, consequently it is located above the piezometric line by the value: $\frac{V_i^2}{2g}$

Calculation of the straight pipeline with a pump



The pipeline takes water from the well A and supplies it to the tank B. The energy supplied by the pump in any instant of time is used for moving water from the level A to the level B and for overcoming the linear losses along the pipeline and the local losses. Along the section 3-4 only the net pressure increase due to the pump action (the pump head) is taken into account, without considering the losses in the pipeline sections belonging to the pump itself.



The pump head is:

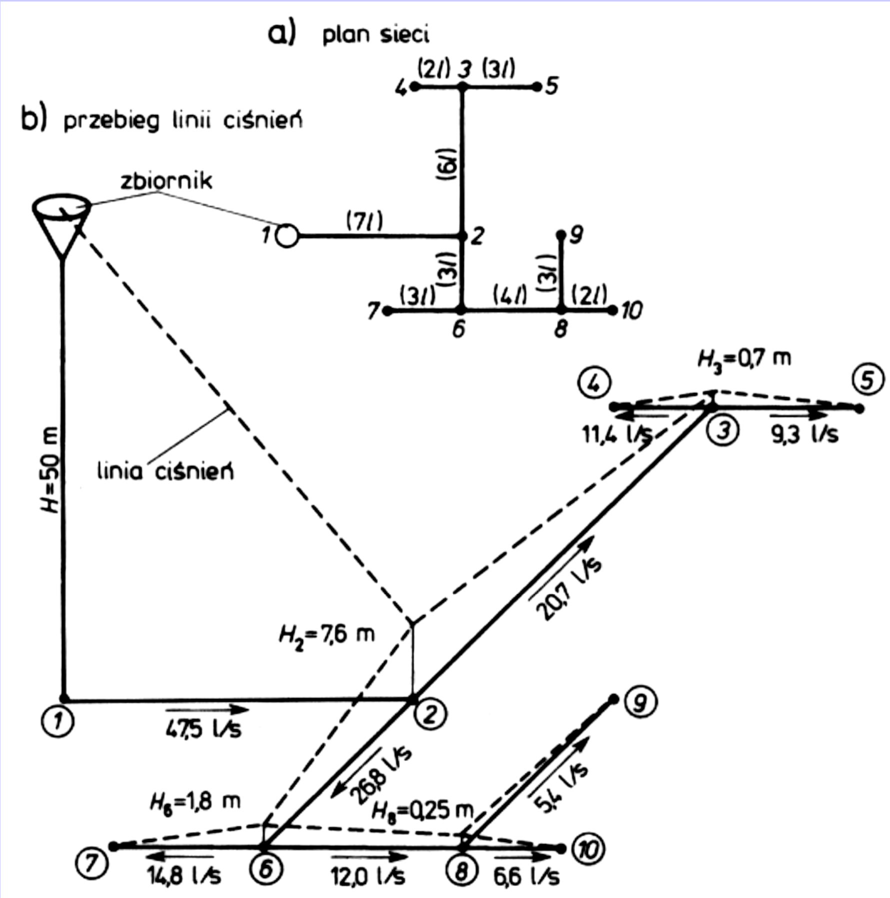
$$H = \frac{\Delta p}{\rho g}$$

The reference level is at the inlet to the pipeline. The pressure head there is equal to:

$$l_1 + \frac{p_a}{\rho g}$$

The pipeline has a constant cross-section, consequently the velocity of flow is constant along the pipeline.

Calculation of a branched grid



The gived branched grid is fed from a tank of constant overpressure H=50 [m]. In the points 4, 5, 7, 9, 10 the water flows into the atmosphere. Calculate the intensity of flow in the sections of the grid and determine the piezometric lines. Disregard the local lossess.

The grid is composed of 9 sections, 4 nodes and 6 end-points (1 inlet and 5 outlets). Unknown are 9 volumetric intensities of flow at sections and 4 overpressure values in the nodes. We can use 9 Bernoulli equations and 4 continuity equations in the nodes.

We employ another formula for the linear losses in the grid sections:

$$\Delta h_{li} = l_i \frac{Q_i^2}{K_i^2}$$ where: $K = 0{,}061\left[m^3/s\right]$ the pipeline characteristics

On the basis of the Bernoulli equation we get the following:

$$\frac{H_1 - H_2}{7 \cdot l} = \frac{Q_{12}^2}{K^2} \qquad \frac{H_2 - H_3}{6 \cdot l} = \frac{Q_{23}^2}{K^2} \qquad \frac{H_2 - H_6}{3 \cdot l} = \frac{Q_{26}^2}{K^2} \qquad \frac{H_6 - H_8}{4 \cdot l} = \frac{Q_{68}^2}{K^2}$$

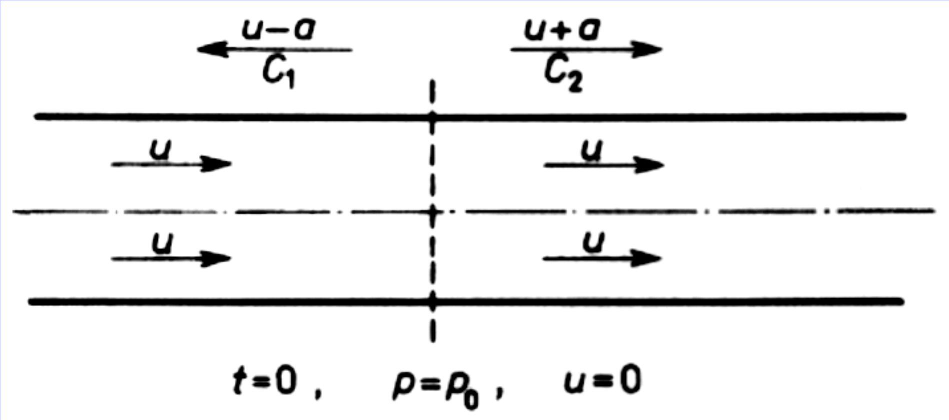
$$\frac{H_3}{2 \cdot l} = \frac{Q_{34}^2}{K^2} \qquad \frac{H_3}{3 \cdot l} = \frac{Q_{35}^2}{K^2} \qquad \frac{H_6}{3 \cdot l} = \frac{Q_{67}^2}{K^2} \qquad \frac{H_8}{3 \cdot l} = \frac{Q_{89}^2}{K^2} \qquad \frac{H_8}{2 \cdot l} = \frac{Q_{8,10}^2}{K^2}$$

On the basis of the continuity of flow equation in the nodes we get the following equations:

$$Q_{68} = Q_{89} + Q_{8,10} \qquad Q_{26} = Q_{67} + Q_{68} \qquad Q_{23} = Q_{34} + Q_{35} \qquad Q_{12} = Q_{23} + Q_{26}$$

The results of solution of this system of equations are shown in the picture.

The hydraulic shock



The hydraulic shock is a strongly dynamic phenomenon, taking place for example in sudden closing of a pipeline with the flowing liquid.

The computational analysis of the hydraulic shock should take into account the elasticity of the pipeline walls and the (usually disregarded) compressibility of the liquid. Sudden closing of the pipeline generates a wave of reduced pressure which propagates downstream and the wave of increased pressure which propagates upstream of the initial flow.

The velocity of pressure wave propagation:

$$a = \sqrt{\frac{1}{\rho_0\left(\frac{1}{E_c} + \frac{d}{\delta}\frac{1}{E_s}\right)}}$$

where: δ– the pipeline wall thickness

d – the pipeline diameter

ρ_0 - the initial liquid density

E_c - the liquid module of elasticity

E_s - the pipeline material module of elasticity

Increased pressure: $p = p_0 + \rho_0 u a$

Reduced pressure: $p = p_0 - \rho_0 u a$

The wave of reduced pressure may lead to cavitation and to erosive destruction of the pipeline walls below the gate, and the wave of increased pressure may break the pipeline above the gate valve.

Example

In the steel pipeline of the diameter d=600 [mm] and the wall thickness δ=12 [mm] the water flows with the velocity u=3.0 [m/s]. Determine the increase of pressure at sudden closing of the gate valve, if $E_s = 2{,}06 \cdot 10^5 \left[MPa\right]$ and $E_c = 0{,}2 \cdot 10^4 \left[MPa\right]$

$$a = \sqrt{\frac{1}{1000.0\left(\frac{1}{0.2 \cdot 10^{10}} + \frac{0.6}{0.012}\frac{1}{2.06 \cdot 10^{11}}\right)}} = 1160\left[m/s\right]$$

$$\Delta p = \rho_0 u a = 1000.0 \cdot 3.0 \cdot 1160.0 = 3480000\left[Pa\right] = 3.48\left[MPa\right]$$