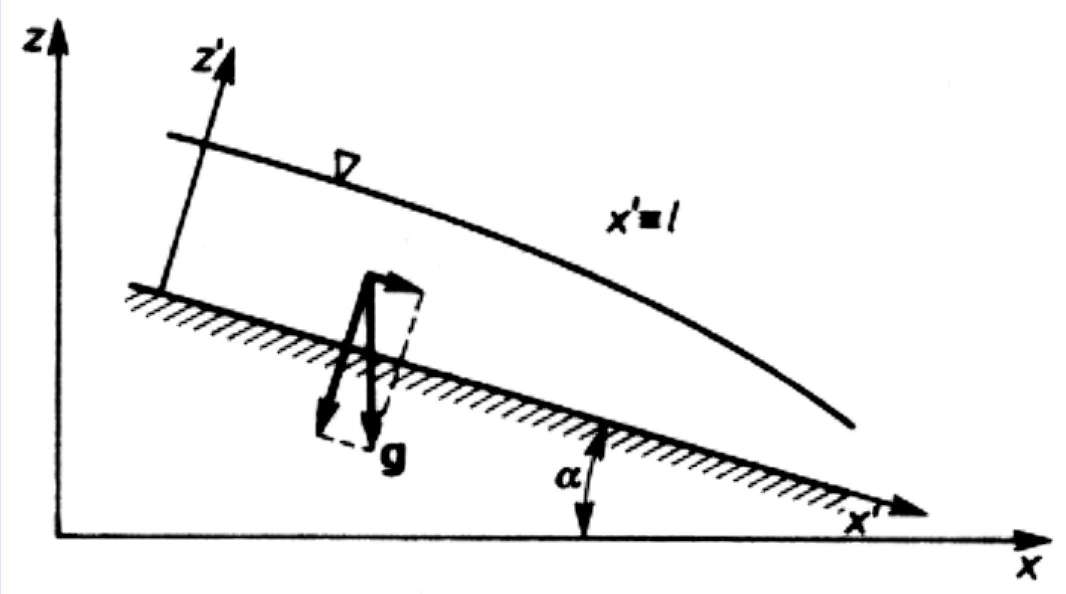
J. Szantyr – Lecture No. 27 – Flows in open channels I

The flows in open channels are in most cases enforced by the gravity force. At the begining we consider a simplified case of flow of a layer of inviscid liquid, having thickness *h* and unit width, along an inclined plane. We disregard the influence of the channel side walls. This is possible because the friction of the liquid against the channel surface is neglected. We introduce two systems of co-ordinates, the system $Ox'z'$ is linked to the inclined plane.



In such a case the momentum conservation equation has the form:

$$\frac{\partial p}{\partial z'} = -\rho g \cos\alpha$$ in direction z'

$$\rho u h \frac{du}{dx'} = \rho g h \sin\alpha - \frac{dp}{dx'} h$$ in direction x'

From the first we get: $$p = p_0 + \rho g \left(h - z'\right) \cos\alpha$$

where: p_0 - the pressure on the free surface

After substituting to the second: $$\rho q \frac{du}{dx'} = \rho g h \sin\alpha - \frac{dh}{dx'} \rho g h \cos\alpha$$

where: $q = u \cdot h \cdot 1$ - the volumetric intensity of flow in the layer

what leads to: $$-q^2 \frac{1}{h^2} \frac{dh}{dx'} = g h \sin\alpha - g h \frac{dh}{dx'} \cos\alpha$$

This may be converted to the form: $$\frac{dh}{dx'} = \frac{-\sin\alpha}{\left(\frac{q^2}{gh^3\cos\alpha} - 1\right)\cos\alpha}$$

It may be seen that there is a singularity at the critical value of h equal to:

$$h_{kr} = \sqrt[3]{\frac{q^2}{g\cos\alpha}}$$

The expression in the denominator may be written:

$$\frac{q^2}{gh^3\cos\alpha} = \frac{u^2h^2}{gh^3\cos\alpha} = \left(\frac{u}{\sqrt{gh\cos\alpha}}\right)^2 = Fr^2$$

or:

$$\frac{dh}{dx'} = \frac{-\sin\alpha}{(Fr^2 - 1)\cos\alpha} = \frac{I}{(1 - Fr^2)\cos\alpha}$$

where I – the geometric head

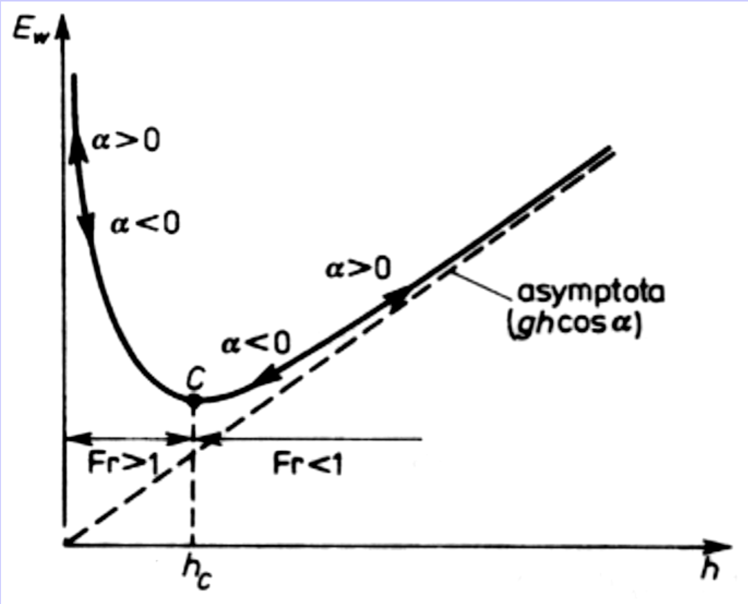
For $Fr=1$ we have the critical velocity: $u_{kr} = \sqrt{gh_{kr}\cos\alpha}$

All possible cases of flow of the liquid layer may be divided into subcritical (Fr<1) – or **the calm flow** and supercritical (Fr>1) – or **the rapid flow**.

Depending on the flow case the thickness of the liquid layer changes along the plane in different ways:

Bottom inclination	Calm flow $Fr < 1$	Rapid flow $Fr > 1$
Positive $\alpha > 0$	Increase of h $\frac{dh}{dx'} > 0$	Fall of h $\frac{dh}{dx'} < 0$
Negative $\alpha < 0$	Fall of h $\frac{dh}{dx'} < 0$	Increase of h $\frac{dh}{dx'} > 0$

The analysis of the energy equation for the case of the unit width liquid layer flow along the inclined plane leads to the following relation for the specific energy of the liquid:



$$E_w = \frac{u^2}{2} + gh\cos\alpha = \frac{q^2}{2h^2} + gh\cos\alpha$$

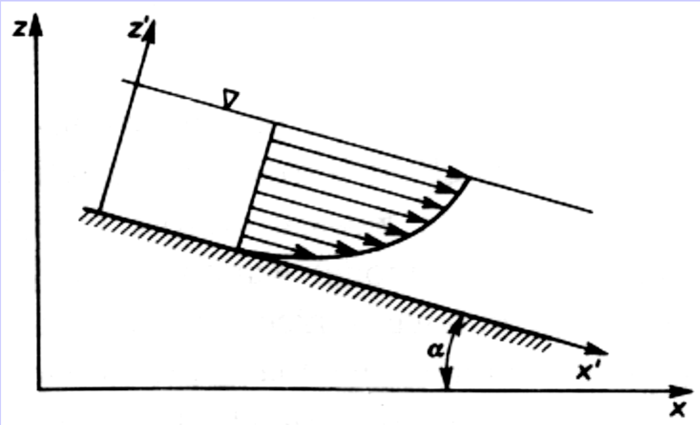
The condition of the specific energy conservation:

$$\frac{dE_w}{dx'} = g\sin\alpha$$

The form of this relation implies that for the positive inclination the specific energy always increases, and for the negative inclination – always decreases.

The specific energy of the flowing liquid reaches its minimum value for the critical thickness of the liquid layer corresponding to Fr=1

Laminar flow of the real (viscous) liquid



In such a case it is possible to obtain the analytical solution of the momentum conservation equation, which has the following form:

$$0 = g\sin\alpha + \upsilon\frac{\partial^2 u}{\partial z'^2}$$

Boundary conditions: $u = 0$ for: $z' = 0$

$\partial u/\partial z' = 0$ for: $z' = h$

Solution leads to the following relations:

The velocity profile: $$u(z') = -\frac{1}{2}\frac{g}{\upsilon}\sin\alpha\cdot z'(2\cdot h - z')$$

The mean velocity: $$\tilde{u} = \frac{1}{h}\int_0^h u(z')dz'\frac{1}{3}\frac{gh^2\sin\alpha}{\upsilon}$$

The maximum velocity: $$u_{\max} = \frac{1}{2}\frac{gh^2\sin\alpha}{\upsilon} > \tilde{u}$$

Conclusion: The velocity of flow is proportional to the square of the liquid layer thickness, or: the flow velocity in an open channel increases with the increasing degree of filling the channel.

The validity of the solution for the laminar flow is limited to the range of Reynolds numbers below about 2000, hence:

$$\frac{u_{\max} \cdot h}{\upsilon} = \frac{1}{2}\frac{gh^3 \sin\alpha}{\upsilon^2} < 2000 \rightarrow h < \frac{0.74 \cdot 10^{-3}}{\sqrt[3]{\sin\alpha}}$$

It follows from the above formula that the laminar flow in the layer flowing along the vertical wall may take place for the layer thickness less than 0,74 [mm], and on the almost horizontal wall (inclined at an angle of 1 degree) for the layer thickness less than 2,85 [mm]. As a rule in real channels the flows are turbulent with the fully developed velocity profiles.

Turbulent flow of the real (viscous) liquid

In the case of the fully developed turbulent flow in the channel of constant inclination the flow parameters do not change along the channel. The potential energy of the liquid is converted into the internal energy (heat) of the liquid due to the action of the friction forces against the channel walls. There is no conversion of the potential energy into the kinetic energy of the flowing liquid.

$$0 = \rho g S \sin\alpha - p_\tau C$$

$$e_2 - e_1 = \Delta e = \frac{p_\tau \cdot l}{\rho \cdot R_H}$$

where: l – the length of the channel section between 1 and 2

p_τ - the viscous stresses on the channel walls

$R_H = S/C$ - the hydraulic radius of the channel

In such a situation there exists the relation between the geometric head (which is equal to the hydraulic head) and the viscous stresses:

$$I = I_H = \frac{p_\tau}{\rho g R_H}$$

Determination of the friction drag in the open channels

From the analysis of the flow in the channel having rough walls the following approximate relation for the mean flow velocity may be derived:

$$\tilde{u} = C' \sqrt{I \cdot R_H}$$

where: $C' \left[\sqrt{m} / s \right]$ - the dimensional correlation coefficient, determined for example from the following formula:

$$C' = \frac{1}{n} R_H^{1/6}$$

n=0.009 for smooth surfaces (e.g. ceramic tiles)

n=0.012 for clean pipes and smoothed concrete

n=0.014 for concrete walls

n=0.018 for natural channels

n=0.04 for badly maintained natural channels

Example

The volumetric intensity of flow in the concrete (n=0.014) channel of rectangular cross-section of width B=4.0 [m] is equal to Q=5.0 [m**3/s]. Determine the critical flow parameters for this channel.

The condition for critical flow has the form:

$$\frac{q}{g h_{kr}^{3}\cos\alpha}-1=0 \rightarrow \frac{A^{3}}{B}=\frac{Q^{2}}{g} \rightarrow h_{kr}=\sqrt[3]{\frac{Q^{2}}{B^{2}g}}=\sqrt[3]{\frac{5.0^{2}}{4.0^{2}\cdot 9.81}}=0.54[m]$$

where A – the flow cross-section area: $A=B\cdot h_{kr}$

The critical velocity: $u_{kr}=\frac{Q}{B\cdot h_{kr}}=\frac{5.0}{4.0\cdot 0.54}=2.31[m/s]$

The critical hydraulic head:

$$u_{kr}=C'\sqrt{I_{kr}\cdot R_{H}}=\frac{1}{n}R_{H}^{1/6}\sqrt{I_{kr}\cdot R_{H}} \rightarrow I_{kr}=\frac{u_{kr}^{2}n^{2}}{\left(R_{H}\right)^{4/3}}=0.0033$$