J. Szantyr – Lecture No. 29 –Principles of Gas Dynamics I

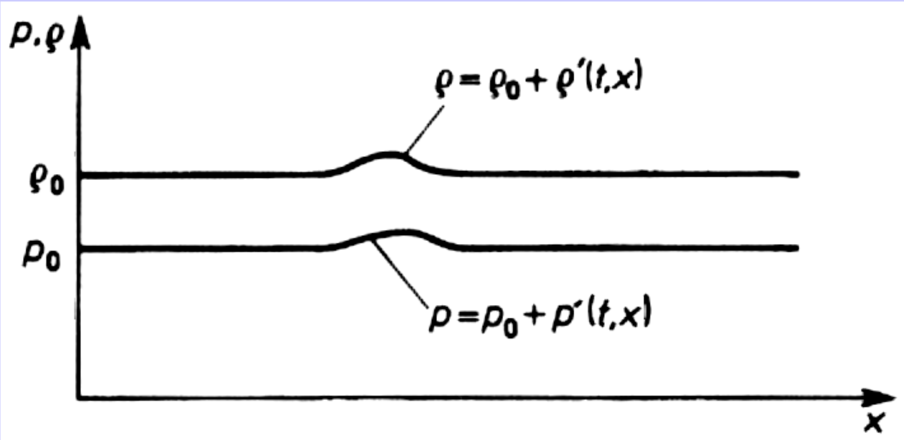
The model of compressible fluid implies that a positive change in pressure results in a positive change in density, hence:

$$\frac{\partial p}{\partial \rho} = a^2$$

In an incompressible fluid thre is: $\frac{\partial p}{\partial \rho} \to \infty$

We employ the model of an ideal and perfect gas. Out of 4 possible gas processes the adiabatic process is most frequently employed (*q=idem*), because due to the high rate of change of the gas flow parameters the absence of the heat exchange between the adjacent fluid elements is a reasonable assumption.

<u>Propagation of small disturbances in an ideal gas.</u>



We consider a one-dimensional flow:

$$\rho'(t,x) << \rho_0$$

$$p'(x,t) << p_0$$

The conservation equations for such a flow have the form:

mass conservation equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0$$

momentum conservation equation (Euler):

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial x}$$

In order to close the system the Poisson adiabatic must be included:

$$\frac{p}{\rho^{\kappa}} = const$$

where: $\kappa = \frac{c_p}{c_v}$ - Poisson adiabatic exponent

Now we have 3 equations and 3 unknowns: *p, u, ρ*

Linearization of the system of equations leads, after some transformations, to the linear wave equations:

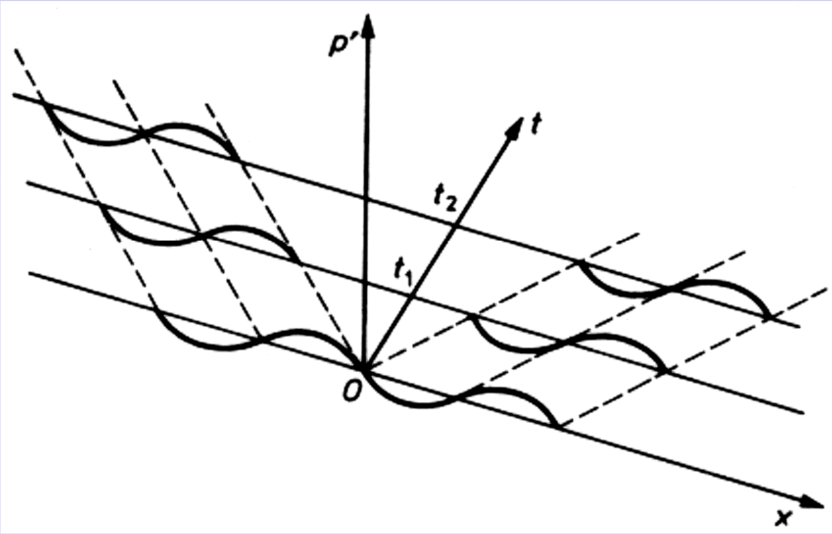
for density:

$$\frac{\partial^2 \rho'}{\partial t^2} - a_0^2 \frac{\partial^2 \rho'}{\partial x^2} = 0$$

for pressure: $$\frac{\partial^2 p'}{\partial t^2} - a_0^2 \frac{\partial^2 p'}{\partial x^2} = 0$$

for velocity: $$\frac{\partial^2 u'}{\partial t^2} - a_0^2 \frac{\partial^2 u'}{\partial t^2} = 0$$

The solutions of the wave equation e.g. for pressure have the form:



$$p'(x,t) = f(x - a_0 t)$$

it describes a wave of an initial profile , $p'(x,0) = f(x)$, propagating in the positive x direction, and:

$$p'(x,0) = g(x + a_0 t)$$

it describes a wave of an initial profile $p'(x,0) = g(x)$, propagating in the negative x direction.

The constant profile of the propagating wave is a consequence of the assumption about <u>small</u> disturbances (i.e. linear form of equations).

It follows from the linear wave equation that the small disturbances propagate in gas with a constant velocity. As the sound waves are also small disturbances, their velocity of propagation may be interpreted as **the speed of sound:**

$$a_0 = \sqrt{\kappa \frac{p_0}{\rho_0}}$$

The local speed of sound:

$$a = \frac{\partial p}{\partial \rho} = \sqrt{\kappa \frac{p}{\rho}} = \sqrt{\kappa \cdot R \cdot T}$$

It follows from the above formula, that the speed of sound is higher in the less compressible media. In air at the sea level the speed of sound is about 340 [m/s], while in water it is about 1500 [m/s].

The similarity criterion for the high speed gas flows is the Mach number:

$$Ma = \frac{u}{a} = \frac{velocity\_of\_flow}{velocity\_of\_sound}$$

The flows may be categorized according to the value of Mach number:

- low subsonic – Ma<0.3 (compressibility effects are negligible)
- subsonic – 0.3<Ma<1.0
- transonic (in limited areas there is Ma>1.0)
- supersonic – 1.0<Ma<3.0
- hypersonic – Ma>3.0

Propagation of disturbances of a finite (large) amplitude.

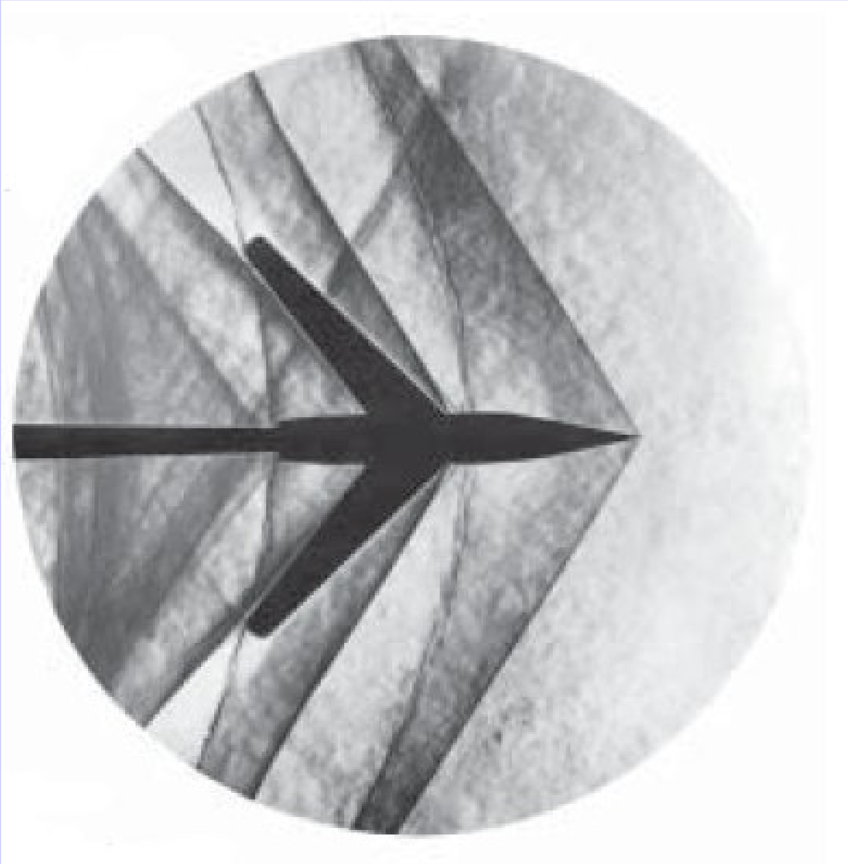
In such a case the linearization of the conservation equations is not possible. Their solution in a non-linear form is complicated and it leads to the relations for propagation velocity of the following format:

$$C_\rho = \pm \frac{2a_0}{\kappa - 1}\left[\frac{\kappa+1}{2}\left(\left(\frac{\rho}{\rho_0}\right)^{\frac{\kappa-1}{2}} - 1\right)\right]$$

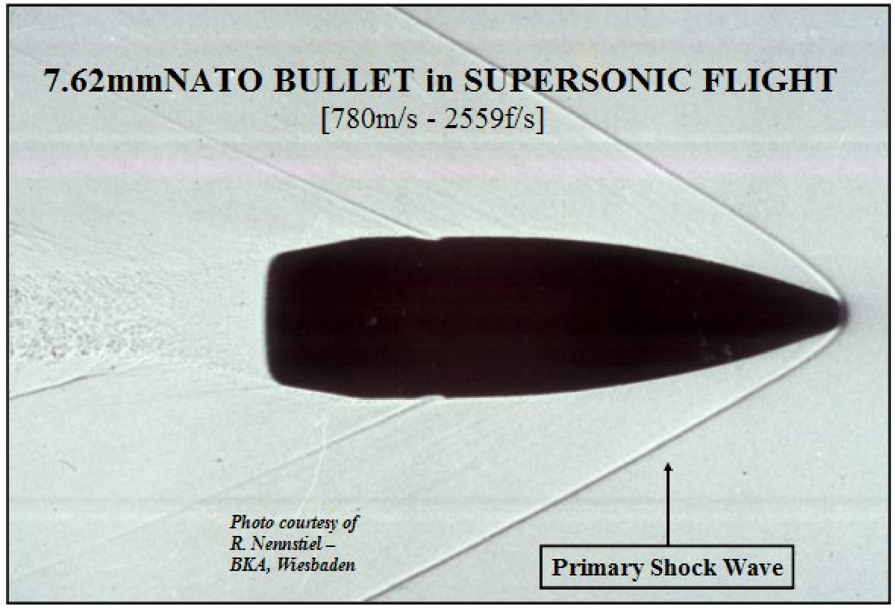
- for a density disturbance

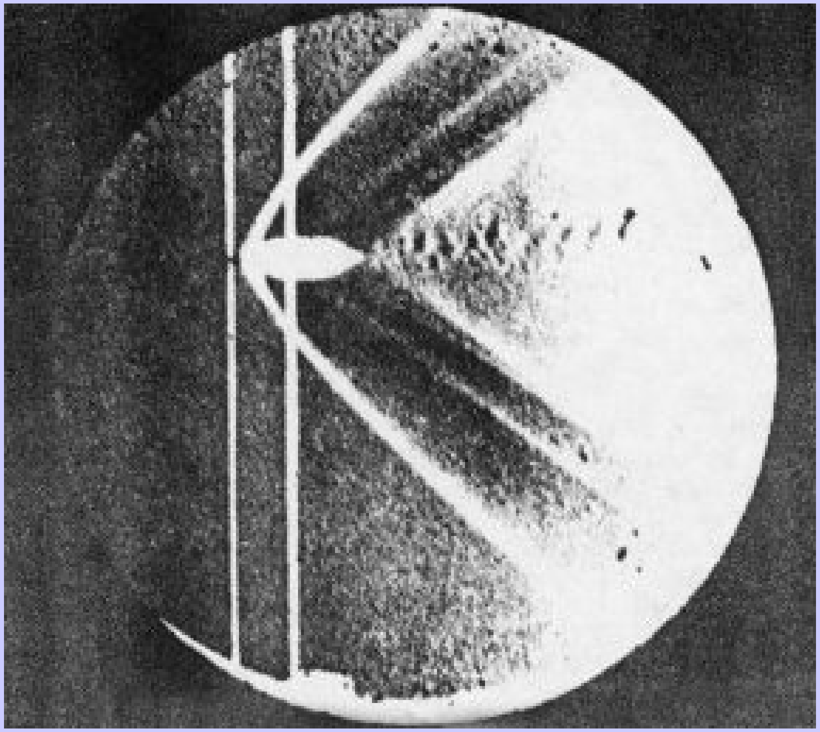
It follows from the above formula that the velocity of propagation increases with the increasing disturbance amplitude. As a result, the shape of the disturbance is deformed during propagation. This leads to the formation of a surface of an abrupt change of the gas flow parameters, or **the shock wave.**

Aircraft generating shock waves

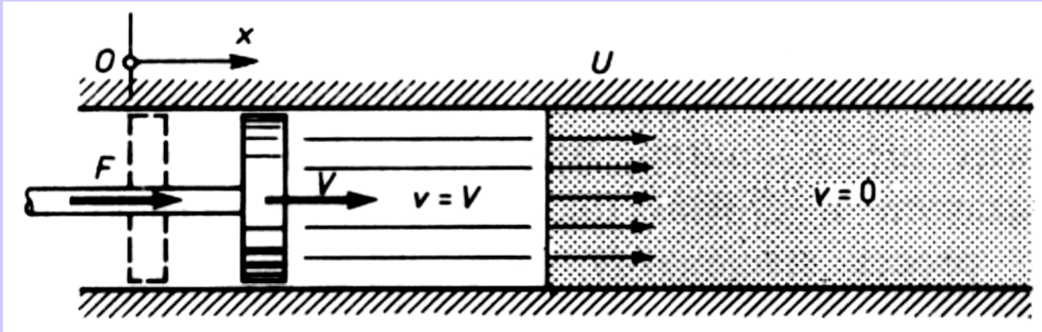


Examples of visualisation of the shock waves.To the right the original photograph made by Mach in the second half of the XIX century.





The mechanism of the shock wave generation

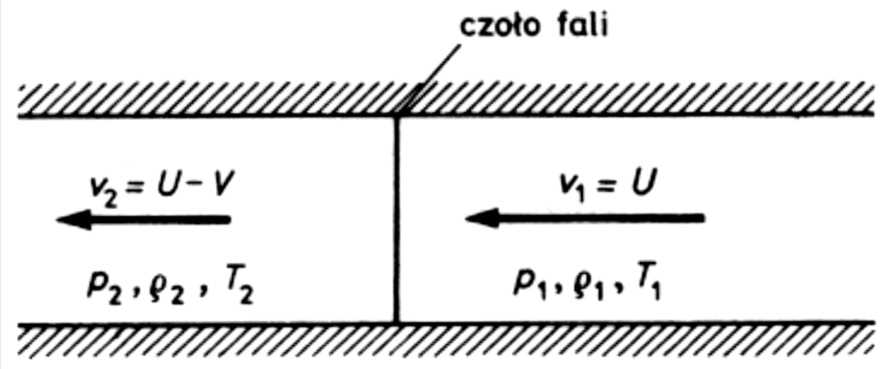


A sudden motion of the piston in the cylinder generates a local disturbance (increase in density) of a gas, which propagates with velocity *U*. This disturbance may be treated as a sequence of waves, passing one after another through the gas of parameters already changed by the previous wave. The adiabatic increase of density results in the increase of temperature and the local speed of sound. This means that each subsequent wave propagates faster and it catches up with the previous one and overlays it. Finally, a thin surface of discontinuity, i.e. an abrupt change in the gas parameters is created, called

the perpendicular shock wave.

(perpendicular to the direction of velocity)

An elementary theory of the perpendicular shock wave



The conservation equations for the perpendicular shock wave may be written in the form:

mass conservation equation: $$\rho_1 v_1 = \rho_2 v_2$$

momentum conservation equation: $$p_1 + \rho_1 v_1 = p_2 + \rho_2 v_2$$

energy conservation equation: $$c_p T_1 + \frac{v_1^2}{2} = c_p T_2 + \frac{v_2^2}{2}$$

equation of state: $$\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2} = R$$

The solution of the following system of equations leads to the relations:

$$\frac{p_2 - p_1}{p_1} = \frac{2\kappa}{\kappa + 1}\left(Ma_1^2 - 1\right)$$

$$\frac{T_2 - T_1}{T_1} = \frac{2\kappa}{\left(\kappa + 1\right)^2 Ma_1^2}\left(Ma_1^2 - 1\right)\left(1 + \kappa Ma_1^2\right)$$

$$\frac{\rho_2 - \rho_1}{\rho_1} = \left(Ma_1^2 - 1\right)\left(1 + \frac{\kappa - 1}{2} Ma_1^2\right)^{-1}$$

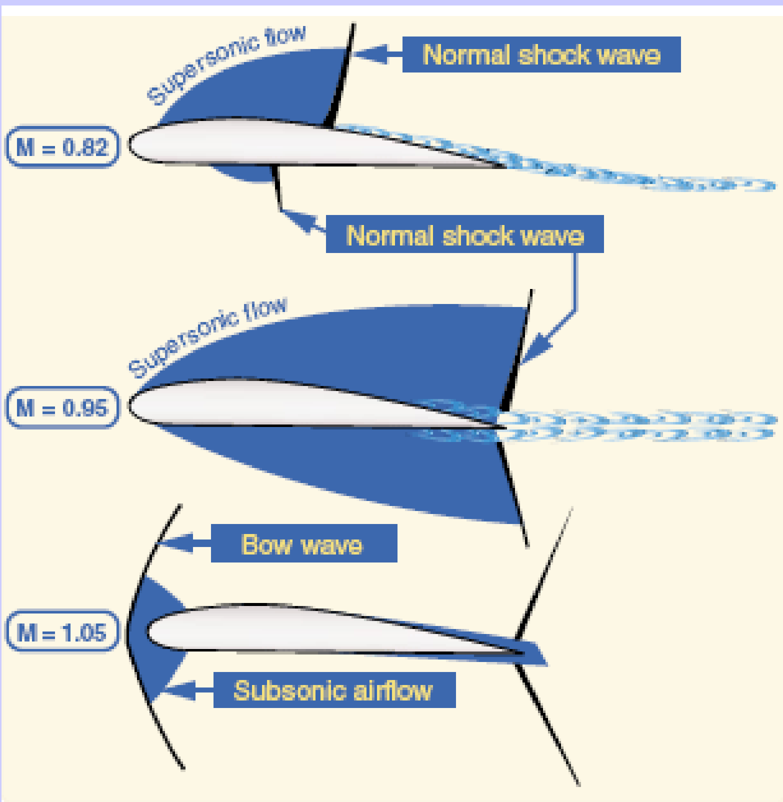
It follows from these relations that:

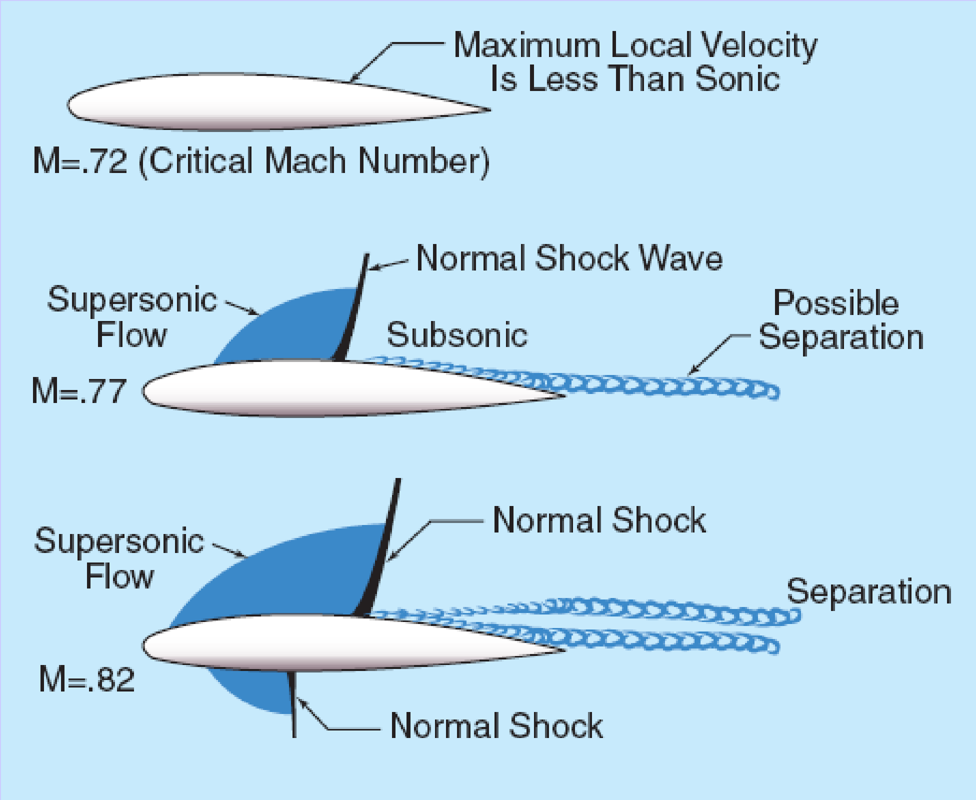
- the shock wave is generated when Ma>1.0

- after passing the perpendicular shock wave the velocity decreases and pressure, density, temperature and entropy of the gas increase

- behind the perpendicular shock wave there is always Ma<1.0

In transonic flows it often happens that locally on the object the flow is supersonic and therefore capable of generating local shock waves, despite the fact that the general velocity of the object motion relative to air is subsonic – see for example the cases illustrated below..





Examples of local shock waves indicating locally supersonic flow on aircraft flying at high angles of attack but with nominally subsonic speed.