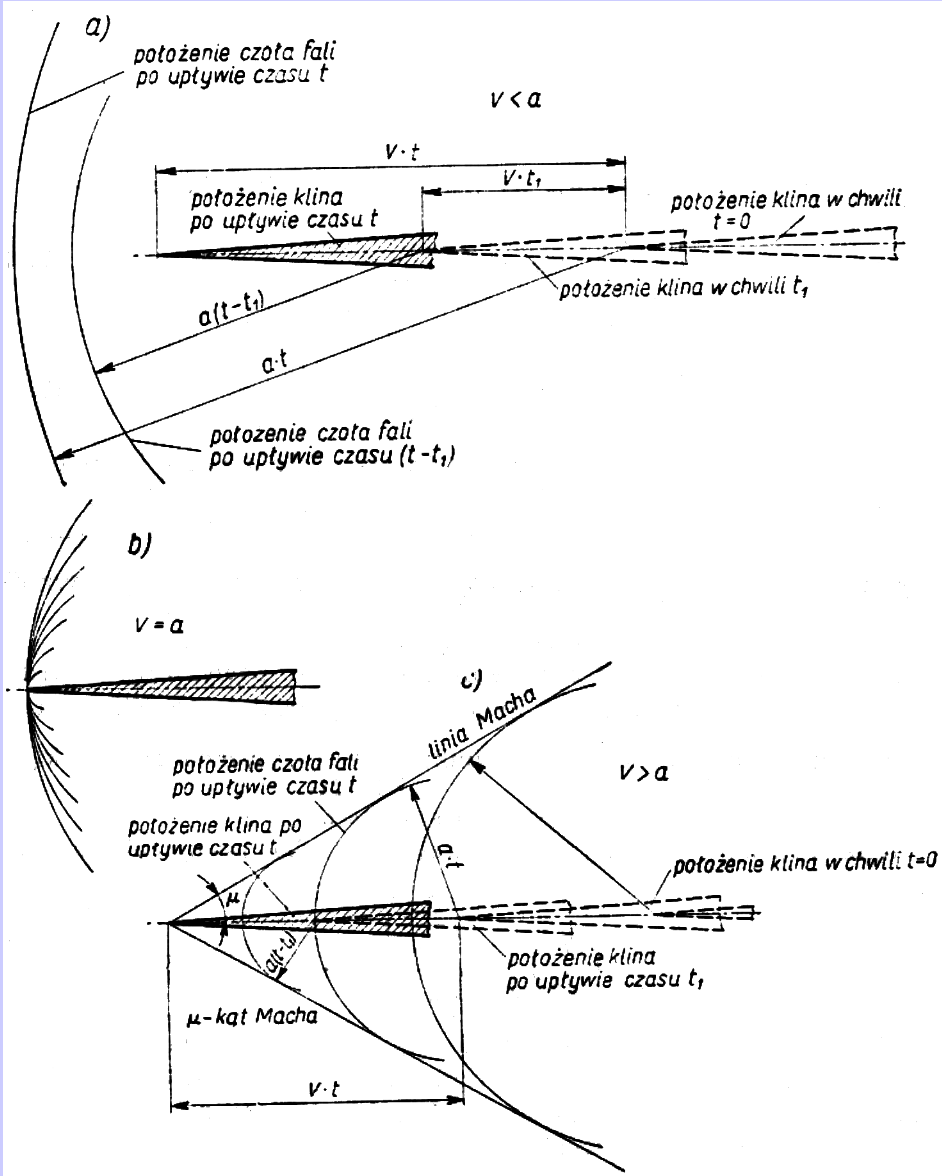
J. Szantyr – Lecture No. 30 – Principles of gas dynamics II

Oblique shock waves



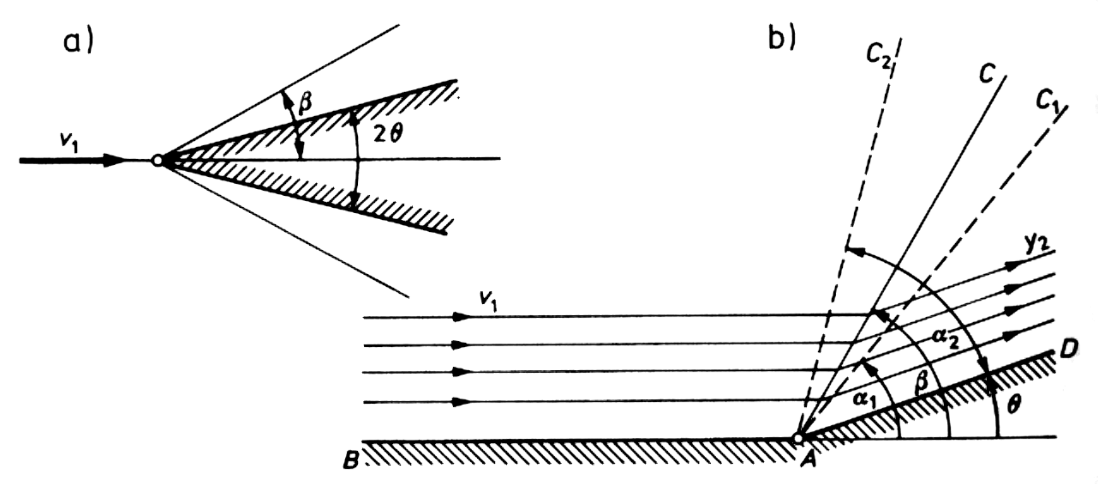
a) Subsonic flow – the sound wave overtakes the generating object

b) Transonic flow – as the velocity increases the sound waves concentrate and at v=a they form the shock wave

c) Supersonic flow – the object overtakes the generated sound waves – the shock wave front forms the so called Mach cone with an angle:

$$\mu = \arcsin\left(\frac{1}{Ma_\infty}\right)$$

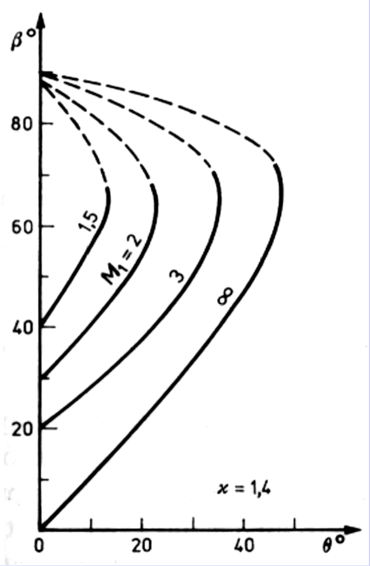
The oblique shock waves in internal flows are often generated by the geometric discontinuity of the channel walls:

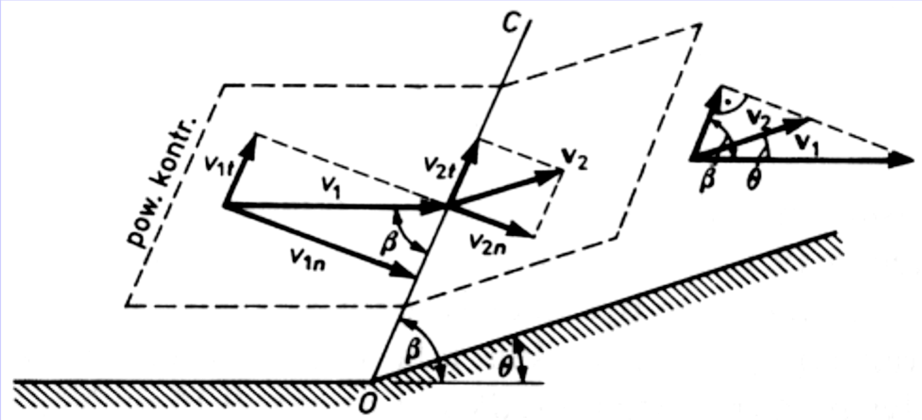


In such a case we have two flows of different directions: <u>before</u> and <u>behind</u> the shock wave – consequently two Mach angles may be determined. As this would be physically unrealistic, we have only one shock wave A-C inclined at an angle β, defined by the formula:

$$tg\theta = \frac{Ma_1^2 \sin^2\beta - 1}{1 + Ma_1^2\left(\frac{\kappa+1}{2} - \sin^2\beta\right)} ctg\beta$$

confront the diagram→





The velocity distribution in an oblique shock wave may be presented by the normal and tangential components. The following equations may be written:

mass conservation equation $\rho_1 v_{1n} = \rho_2 v_{n2}$

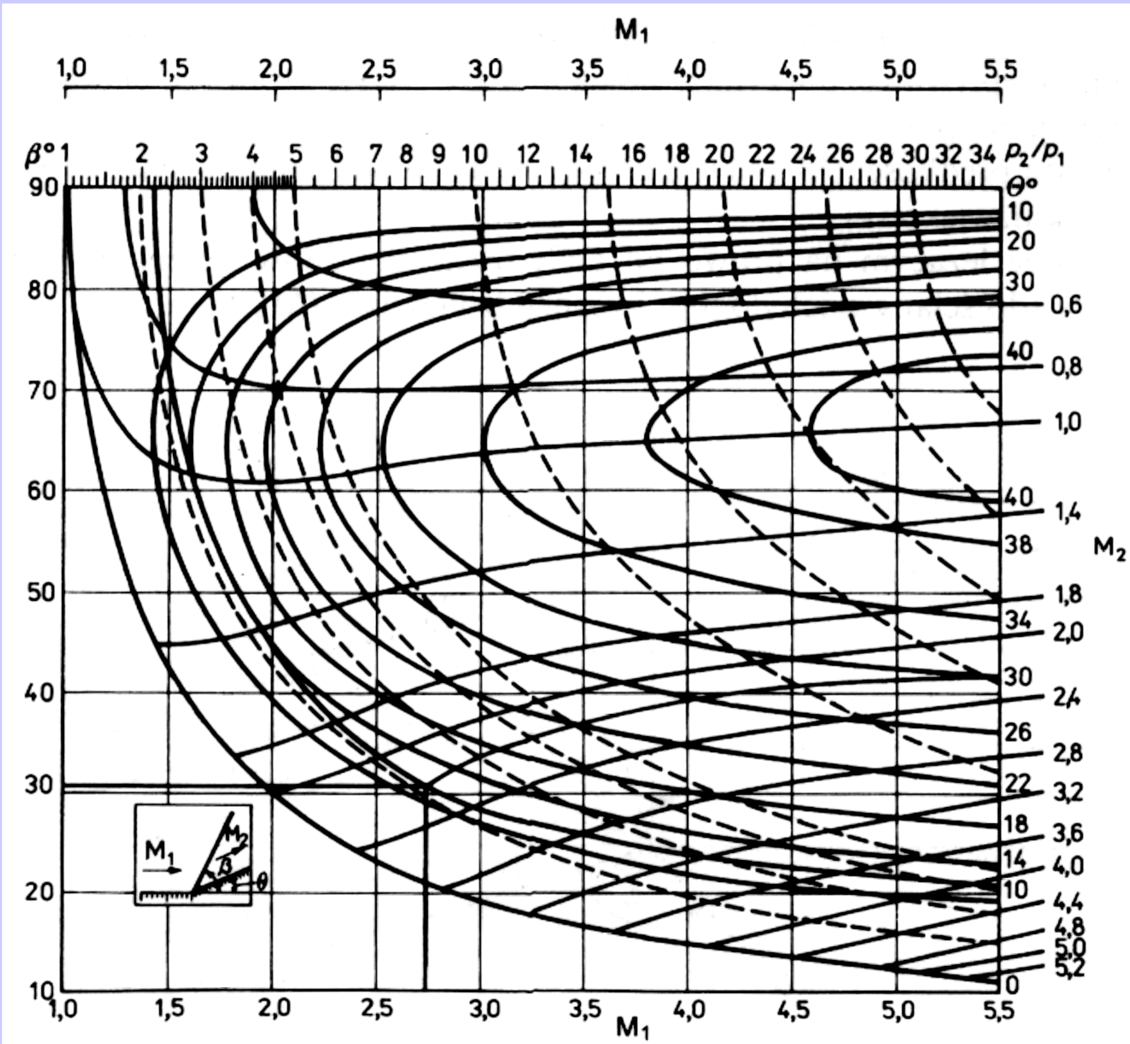
momentum conservation equation perpendicular to OC: $p_1 + \rho_1 v_{1n}^2 = p_2 + \rho_2 v_{2n}^2$

momentum conservation equation parallel to OC: $\rho_1 v_{1n} v_{1t} = \rho_2 v_{2n} v_{2t}$

energy conservation equation: $c_p T_1 + \dfrac{\rho_1 v_{1n}^2}{2} = c_p T_2 + \dfrac{\rho_2 v_{2n}^2}{2}$

These equations are identical to those for the perpendicular shock wave, as long as the velocity components normal to the shock wave are used.

In an oblique shock wave <u>only the normal velocity component falls below the speed of sound when passing the wave front</u> – **the total flow velocity behind the wave may be still supersonic.**

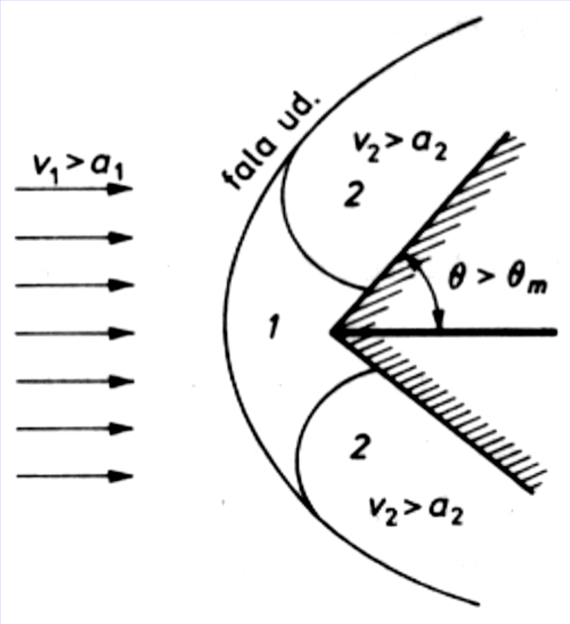


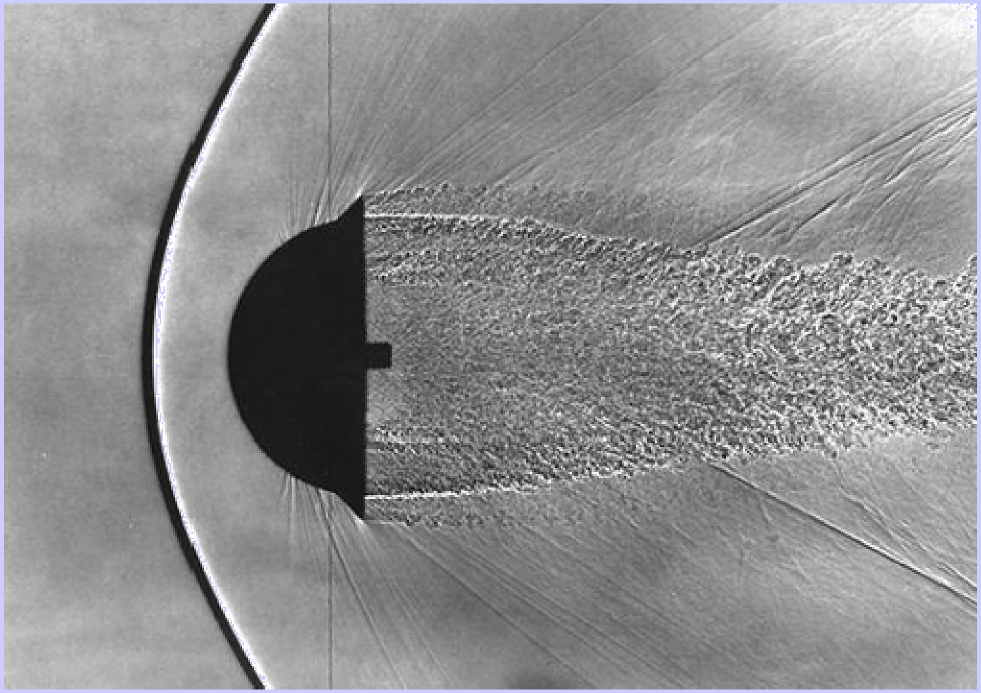
The relations between the wall inclination, shock wave inclination, and Mach numbers before (1) and behind (2) the shock wave are shown in the diagram. For example for θ=12 degrees and

$$Ma_1 = 2.8$$

we get: $$Ma_2 = 2.2$$

$$\beta = 31^0 \qquad p_2/p_1 = 2.2$$





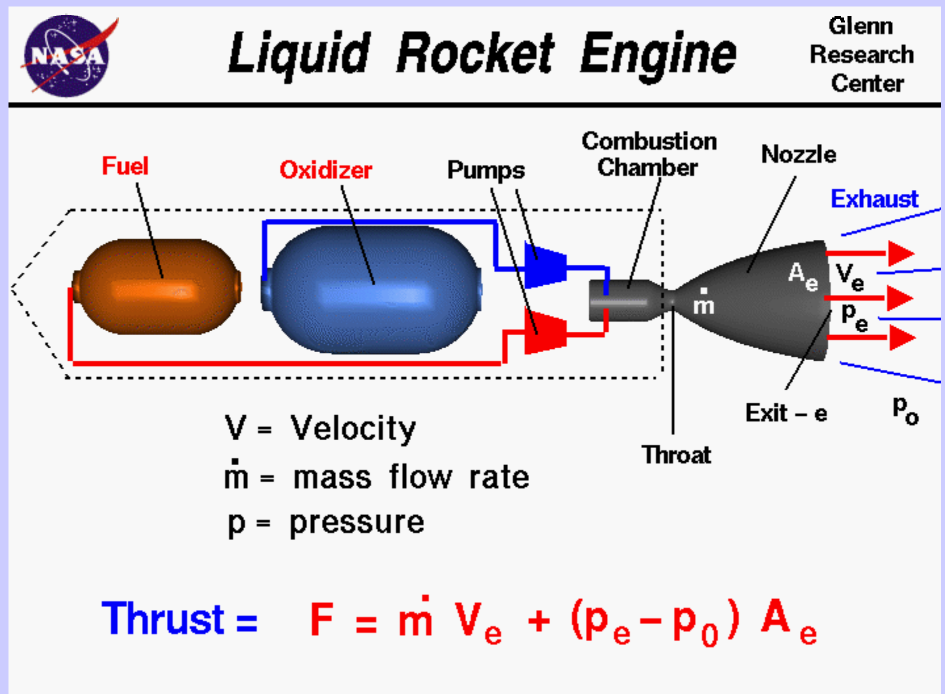
(NASA)

When the nose angle of the object generating the shock wave is large, the wave starting at the nose may not be generated. In such a case the so called displaced shock wave occurs. In this wave in zone 1 we have the subsonic flow and before the wave and in zone 2 behind the wave – the supersonic flow.

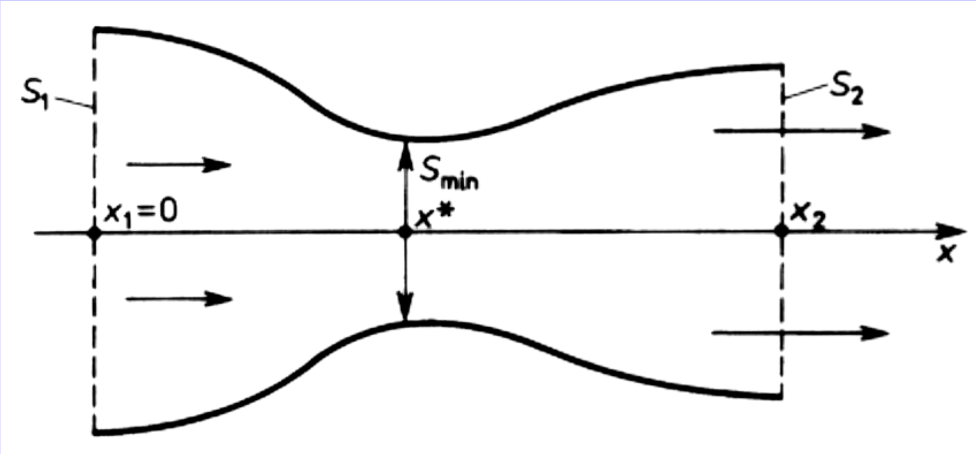
The limiting value of the nose angle depends on the Mach number and it varies around the value of 70 degrees.

Flow through the de Laval nozzle

The de Laval nozzle is a device enabling acceleration of the gas flow to supersonic velocities. Besides other applications it forms the part of rocket engines.



One-dimensional steady flow of a compressible fluid



The nozzle is composed of the confusor (the convergent part), the throat (the smallest cross-section) and the diffusor (the divergent part).

mass conservation equation: $$\rho(x)\cdot u(x)\cdot S(x)=const$$

after differentiation: $$u\cdot S\cdot\frac{d\rho}{dx}+\rho\cdot S\cdot\frac{du}{dx}+\rho\cdot u\cdot\frac{dS}{dx}=0$$

after dividing by ρuS: $$\frac{1}{\rho}\frac{d\rho}{dx}+\frac{1}{u}\frac{du}{dx}+\frac{1}{S}\frac{dS}{dx}=0$$

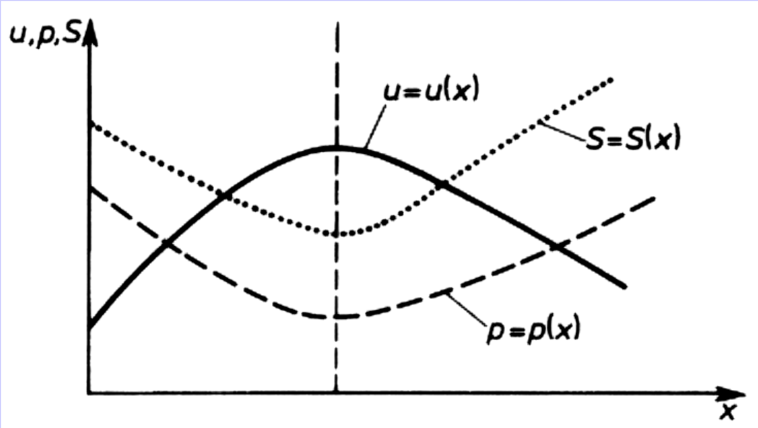
moreover, we have: $\dfrac{dp}{d\rho}=a^2$ or: $\dfrac{d\rho}{dx}=\dfrac{d\rho}{dp}\dfrac{dp}{dx}=\dfrac{1}{a^2}\dfrac{dp}{dx}$

The pressure gradient may be determined from the one-dimensional Euler equation for steady flows:

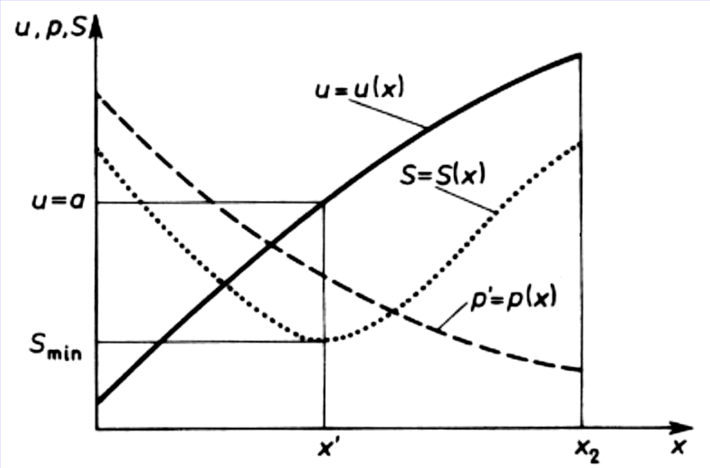
$$u\frac{du}{dx} = -\frac{1}{\rho}\frac{dp}{dx}$$

hence: $$\frac{d\rho}{dx} = -\frac{\rho u}{a^2}\frac{du}{dx} \rightarrow \left(Ma^2 - 1\right)\frac{du}{dx} = \frac{u}{S}\frac{dS}{dx} \rightarrow \left(Ma^2 - 1\right)\frac{du}{u} = \frac{dS}{S}$$

It follows that the character of the gas flow in the de Laval nozzle depends on the value of the Mach number:



The subsonic flow – the velocity is inversely proportional to the variation of the nozzle cross-section area.



The supersonic flow – the velocity increases with the increase of the nozzle cross-section area.

At <u>subsonic velocity</u> the increase of nozzle cross-section leads to decrease in velocity and the decrease of cross-section to increase in velocity – conversely at <u>supersonic velocity</u>. **Conclusion:** the de Laval nozzle enables acceleration of the gas flow to supersonic velocity <u>on the condition of reaching the speed of sound in the nozzle throat.</u>

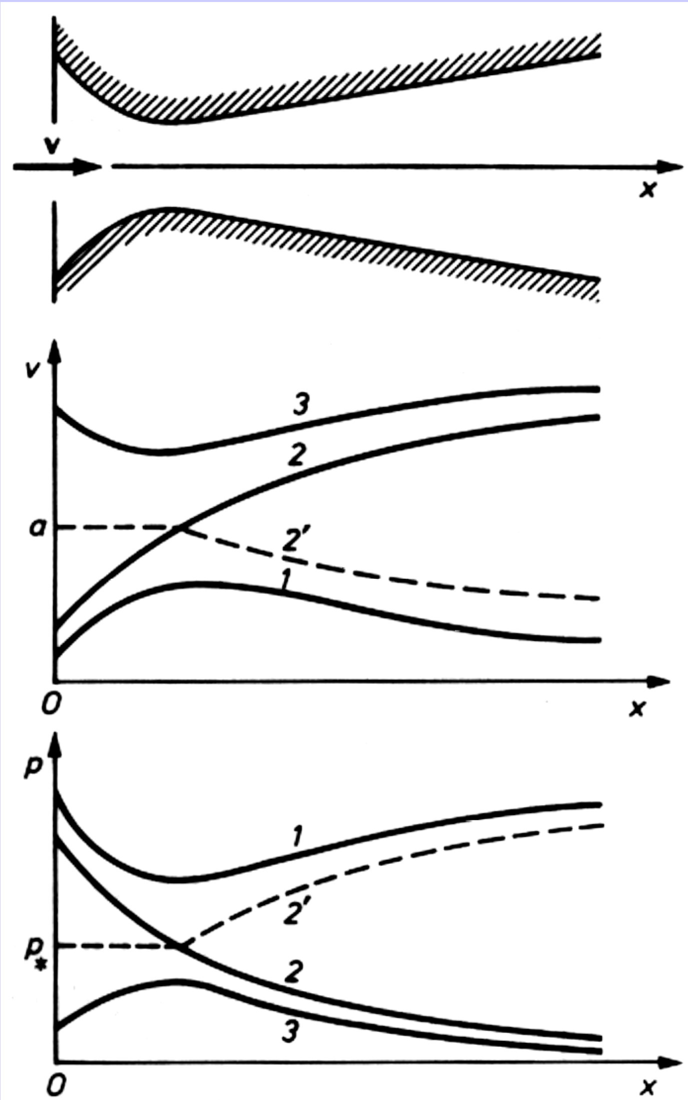
There exists the maximum possible mass flow intensity through the de Laval nozzle, defined by the following expression:

$$m_{\max} = \sqrt{\kappa \cdot p_0 \cdot \rho_0} \left(\frac{2}{\kappa + 1} \right)^{\frac{\kappa+1}{2(\kappa-1)}}$$

The maximum mass flow intensity corresponds to reaching of the speed of sound and of the critical gas pressure in the nozzle throat:

$$p_* = \rho_0 \left(\frac{2}{\kappa + 1} \right)^{\frac{\kappa}{\kappa+1}} \qquad \text{where:} \quad \kappa = \frac{c_p}{c_v}$$

The possible case of flow through the de Laval nozzle



1 – the subsonic flow – infinite number of cases may be realized depending on the value of the outlet pressure (or so called counterpressure).

2 – subsonic flow in the confusor, speed of sound in the throat, in the diffusor either sub- or supersonic flow depending on the counterpressure.

3 – gas flows into the nozzle with already supesonic velocity, this velocity is reduced in the confusor, but remains supersonic in the throat. In the diffusor the flow accelerates further, consequently we have supersonic flow in the entire nozzle.