Simple multi-laminate model for soft soils incorporating structural anisotropy and destructuration

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ABSTRACT: A multi-laminate constitutive model for soft soils incorporating structural anisotropy is presented. Stress induced anisotropy of strength, which is directly obtained when using multi-laminate type constitutive models for clays, is augmented by directionally distributed overconsolidation. The model is presented in a nonviscous version in order to simulate compressional anisotropy of soft natural soils and destructuration effects. These two important features of the natural soil behaviour are interrelated in the presented model. The performance of the model is shown for some element tests. The effects of the applied directional distribution are shown on a macro level by the resulting shapes of the yield surface on the triaxial plane. The possibilities, limitations and feasible extensions of the proposed model, are discussed.

1 INTRODUCTION

The evidence of anisotropy of soft natural soils has been shown in many high-quality laboratory studies for the stiffness and strength. The anisotropic mechanical characteristics of clayey soils originate from the arrangement of the clay fabric which has arisen during the sedimentation process. This arrangement is highly anisotropic for both natural and reconstituted clays. Most of the differences in the mechanical behaviour of the natural and reconstituted soft soils are related to the existence of inter-particle bonds that were developed during the diagenesis of natural clays. The degree of anisotropy of the small-strain stiffness is of a similar intensity for both natural and reconstituted clays, however, the degree of the strength anisotropy is observed to be higher for natural material. Another important feature in the mechanical behaviour of the naturally bonded soils is a destructuration process, observed during plastic yielding. The destructuration process of a soft soil sample is connected with significant changes of its compressibility and the strength anisotropy. An exemplary experimental evidence is given by Leroueil (1979).

The presented version of the model for soft natural clays is developed to realistically model the strength anisotropy and its evolution as well as destructuration effects especially on the so-called wet side of the critical state, where hardening takes place. Recently presented extension to the multi-laminate framework of modelling by Pietruszczak&Pande (2001) is used. It was proposed to model the inherent anisotropy of the strength by introducing the directional dependency of the material parameters. This approach was found to be straightforward and intuitive for the modelling of strength cross-anisotropy and volumetric destructuration of soft clay. Only a few additional parameters to the standard material constants are used for this task. The model is presented in the basic version - open for any extensions.
2 FORMULATION OF THE MODEL

The most important and the original features of the model are presented. A detailed description of the multi-laminate framework may be found in Pande&Sharma (1983) or Pietruszczak&Pande (1987). In the multi-laminate framework, soil is assumed to be a solid block behaving elastically (macro level), intersected by an infinite number of randomly oriented planes where plastic straining may occur (micro level). A static constraints are used. This means, that the actual macro stress tensor $\sigma$ is projected to the micro stress vectors $\vec{\sigma}^k$ on every $k$-th plane where the possible plastic strain increments are calculated. Components of $\vec{\sigma}$ are defined in the local coordinate system where the third unit vector is aligned with the normal to the $k$-th sampling plane $\vec{n}_k$. Plastic contribution from all planes are then spatially summed up to obtain the macro plastic strain increment: $d\vec{\sigma}^p = \sum_{k} d\vec{\sigma}^{pk}$. In the numerical implementation this summation from an infinite number of planes is evaluated using a numerical integration rule with a finite number of sampling planes:

- stress projection: $\vec{\sigma}_i^k = T_{ij}^{rk} \vec{\sigma}_j$, $i=1..3, j=1..6$
- plastic micro strain increment: $d\vec{e}^{pk} = d\lambda^k \frac{\partial \vec{g}^k}{\partial \vec{\sigma}_j}, i=1..3$
- plastic macro strain increment: $d\vec{e}_i^p = \sum_{k=1}^m T_{ij}^{rk} d\vec{e}_j^{pk} w_k$, $i=1..6, j=1..3$

where $w_k$ are the weights, $d\lambda^k$ and $\vec{g}^k$ are the plastic multiplier and potential function on the $k$-th plane respectively. $T^{\sigma}$, $T^{\varepsilon}$ are the stress and strain transformation matrices respectively. Note that the macro stress and strain tensors are expressed here exceptionally in the vector form: $\vec{\sigma}, \vec{\varepsilon}$. Among various integration schemes proposed by Fliege&Maier (1996), the system with 64 sampling planes is used.

2.1 Elastic part

In the multi-laminate framework the elastic part of the strain increment $d\varepsilon^e$ is calculated on the macro level. A broad class of elastic models for soils may be used, depending on the significance of the small elastic deformations in the simulated engineering problem.

This paper is focused on the modelling of the evolution of compressional strength anisotropy where stress dependency of the elastic stiffness is of special importance. The coupling of anisotropy and stress dependency of elastic stiffness is, however, complex and requires additional, not standard material parameters. For these reasons the isotropic hypoelastic stiffness based on Hooke's law, often used in critical state models, is applied with a constant Poisson ratio $\nu$ and a pressure dependent Young modulus defined as $E(p) = 3(1-2\nu) p / \kappa^*$, where $\kappa^*$ is the swelling index estimated from ln $p$-$\varepsilon$ compression diagrams.

2.2 Yield surface and plastic potential on the micro level

The constitutive law is isotropic within the micro sampling plane and the yield function may be expressed as a function of only the normal and shear invariants of micro stress vector:

$$\sigma_n^k = -\sigma_3^k, \quad \tau^k = \sqrt{(\sigma_1^k)^2 + (\sigma_2^k)^2}$$

where $\sigma_n^k$ and $\tau^k$ are the normal and tangential stress invariants respectively.

The simplicity of the constitutive relationship is favourable, since the performance of the multi-laminate model is validated on the macro level after spatial integration.

Multi-laminate framework in elastoplasticity is limited in its description of post-peak or softening behaviour. This fact is connected with the static constraints that are used. In structured soft natural clays, shear softening may occur, e.g. for undrained stress paths starting from stress states in the overconsolidated region. However, the shear softening has a discontinuous nature – irreversible strains localise in a narrow shear band and the surrounding material is unloaded. The modelling requires some additional techniques related to the regularisation if the material model is to be implemented into a finite element code.
Because of this limitation, the conservative solution is chosen. In the model, the shear strength in the overconsolidated area is controlled by the Mohr-Coulomb criterion and is not subject to hardening or softening.

The yield surface on the sampling plane level is shown in Figure 1. It consists of the cone and cap parts which are responsible for the shear and compressional strength respectively.

![Yield surface on the sampling plane level.](image)

The cone part uses a non-associated flow rule and the yield function and plastic potential are:

\[
f^k_{cone} = \tau^k - \sigma^k_n \mu - c = 0, \quad g^k_{cone} = \tau^k - \sigma^k_n \tan \psi
\]

where \(\mu = \tan \phi\) and \(\phi, c, \psi\) are the effective friction angle, effective cohesion and dilatancy angle respectively. The incorporation of \(c\) is optional, however, in finite element computations a small value is chosen to avoid a singular apex of the yield surface.

The cap part of the yield surface uses an associated flow rule and the yield function is defined as

\[
f^k_{cap} = (\sigma^k_n - \sigma^k_{np}) \mu^2 \left[ \frac{2\beta c}{\mu} + (1 + \beta)\sigma^k_{np} + (-1 + \beta)\sigma^k_{np} \right] + (\tau^k \beta)^2 (1 + \beta) = 0
\]

where \(\sigma^k_{np}\) is a micro preconsolidation pressure and \(\beta\) is an additional parameter that controls the steepness of the cap surface. On the macro level, \(\beta\) allows to control the asymptotic value of \(K^{nc}_0\) resulting from the simulation of an oedometer compression.

### 2.3 Directional overconsolidation

Recently, Pietruszczak&Pande (2001) have shown the improved version of the multi-laminate framework where the anisotropic distribution of mechanical properties is obtained with a help of microstructure tensor. They have proposed that strength parameters like the friction angle or cohesion may be distributed directionally. However, it was found during the present study, that for materials like soft natural clays it would be more realistic to distribute the overconsolidation ratio which is directly related to the initial bonding of the soil fabric. In order to agree with the critical state mechanics of clayey soils, strength parameters will be taken as intrinsic constants; Burland (1990).

The microstructure tensor \(a\) is defined as a measure of material fabric (spatial arrangement of clay platelets or inter-granular contacts). For practical applications, it appears to be difficult to obtain any information related to the microstructure since advanced methods of the fabric analysis are very complicated and have not been standardised. Nevertheless, for structure related directional distribution, only a deviatoric measure of the material microstructure \(\Omega\) is needed. This tensor is traceless (\(\Omega_{ii}=0\)) and symmetric. In the case of cross-anisotropy, typical for natural soft soil deposits, there are two distinct eigenvalues of \(\Omega\). In the geometrical frame of principal axes of cross-anisotropic microstructure (\(x_2\) vertical), tensor \(\Omega\) has the following components:

\[
\Omega_{ij} = 0 \quad \text{for} \quad i \neq j, \quad \Omega_{44} = \Omega_{33} = -\Omega_{22}/2, \quad \Omega_{22} = \Omega_{y}
\]

Note that only one parameter identifying the spatial bias of cross-anisotropic microstructure (\(\Omega_{z}\)) is needed which may be estimated on the basis of mechanical behaviour observed in laboratory tests.
The initial distribution of the preconsolidation pressure on every $k$-th sampling plane can be expressed by the following equation:

\[
\begin{align*}
\sigma_{0k}^\text{eq, } b = 0.75 \\
\sigma_{0k}^\text{eq, } b = 1.0 \\
\sigma_{0k}^\text{eq, } b = 3.0 \\
\sigma_{0k}^\text{eq, } b = 2.0 \\
\sigma_{0k}^\text{eq, } b = 1.0 \\
\sigma_{0k}^\text{eq, } b = 0.5 \\
\sigma_{0k}^\text{eq, } b = 0.25 \\
\sigma_{0k}^\text{eq, } b = 0.75 \\
\sigma_{0k}^\text{eq, } b = 1.0 \\
\sigma_{0k}^\text{eq, } b = 3.0 \\
\end{align*}
\]

where $b_0$ is an average or isotropic bonding parameter. Now, the stress induced strength anisotropy (related to the initial stress state $\sigma_0^0$) is amplified and adjusted by the component which is connected with the microstructural bonding. Initialisation of the directional overconsolidation in the presented model is shown schematically, with the help of polar plots, in Figure 2.

2.4 Hardening law

Incorporation of the destructuration process is achieved through the evolution equation for a state variable representing bonding on every sampling plane $b^k$. The modelling framework proposed by Gens&Nova (1993) is used. In this framework hardening or softening is influenced by two competing components. The first of which is related to the evolution of the so-called unbonded yield surface, usually hardening, whereas the second is connected with the shrinking of the yield locus due to bond degradation.

The evolution equation for $b^k$, which controls the bonded yield locus, is defined as

\[
b^k = b_0^k \exp \left( -a |\varepsilon_{np}^{pk} | \right)
\]

where $a$ is an additional parameter describing the reduction of bonding with increasing accumulated normal plastic strain $\varepsilon_{np}^{pk}$.

The standard law from the Cam-clay model for normally consolidated soils, is employed for the unbonded component of hardening. The final hardening law for the preconsolidation pressure on the $k$-th sampling plane may be written in the incremental form as
\[
\sigma_{np}' = \left[ \frac{1}{\lambda^* - \kappa^*} \right] \left[ -ab_0^k \exp\left(-a_{s}^{p^k}\right) + b_0^k \exp\left(-a_{s}^{g^k}\right) \right] dE_n^p
\]

where \( \lambda^* \) is a compression index estimated from \( p-\varepsilon \) compression diagrams.

Depending on the chosen values of parameters, both volumetric hardening or softening may result. Volumetric softening is indeed realistic for some heavily structured soils but may be difficult to control in the implementation algorithm. By inspecting the hardening law, it can be derived that for fixed values of the parameters \( \lambda^* \), \( \kappa^* \) and \( b_0^k \), the condition of no volumetric softening implies

\[
a \leq a_{\text{max}}^k = \frac{1 + b_0^k}{b_0^k (\lambda^* - \kappa^*)}
\]

It was found that instead of adopting \( a \) as the model parameter it is better to impose the ratio \( a_r = a/a_{\text{max}}^k \). This means that the value of \( a \) will also be subjected to the directional distribution and consequently the superscript \( k \) will be used: \( a^k = a_r a_{\text{max}}^k \).

3 ELEMENT TESTS

The presented multi-laminate model for soft soils was implemented into the finite element code Plaxis within the user defined soil model facility, Brinkgreve (2002). Performance of the model is shown for three types of element tests. Namely oedometer compression, triaxial undrained compression/extension and triaxial radial stress path tests. For more detailed element studies and validations as well as implementation aspects the reader is refered to Cudny&Vermeer (2003). The following reference set of the material parameters will be used:

\[
\begin{align*}
\sigma_{11}^0 &= -45 \text{ kPa}, & \sigma_{22}^0 &= -22.5 \text{ kPa}, & \sigma_{33}^0 &= -30 \text{ kPa}, & \Rightarrow p_0 &= 30 \text{ kPa}
\end{align*}
\]

3.1 Oedometer compression

The evidence of the destructuration phenomenon is well recognisable during the constant rate of strain oedometer compression tests. The intensity of the destructuration is proportional to the value of the parameter \( b_0 \) (see Figure 3). In the laboratory, during the oedometer compression test, the stress ratio after the destructuration phase is asymptotically approaching an unique value of \( K_{0}^{nc} \). In the model, this can be controlled by the parameter \( \beta \) like it is presented in Figure 3.

3.2 Undrained triaxial compression and extension

Undrained triaxial compression and extension tests were simulated for different levels of destructuration. The results of these simulations are shown in Figure 4. Before undrained triaxial compression or extension phases, the material has been anisotropically consolidated along the \( K_{0}^{nc} \)-line to the five stress states (a-e). Tests (a) start from the \( K_{0}^{nc} \)-stress state where no sampling planes are mobilised and hence the behaviour is typical for the lightly overconsolidated clays excluding a softening phase. Tests (b) start from the point where the rate of the bonding reduction is very high and the bulk stiffness is
almost equal to zero. This contractive tendency provides a fast increase of the excess pore water pressure and the resulting stress paths for compression and extension form a characteristic sharp nose on the \( p-q \) plane. This phenomenon is still apparent for the tests (c) where the intensity of destructuration is still high but it disappears gradually for the higher consolidation stresses (d-e).

![Figure 3. Influence of the parameters \( b_0 \) and \( \beta \) on the simulated behaviour in the oedometer compression.](image)

When the shear strength is reached on some sampling planes there are still a number of planes where micro stresses remain on the cap yield surface with a strong tendency to contract. The effects are, hence, similar to the application of negative value for the dilatancy angle and softening-like behaviour is observed.

![Figure 4. Simulations of the undrained triaxial compression and extension tests for different levels of destructuration along the \( K_0 \)-line.](image)

3.3 **Macro yield surface from radial stress path tests**

Radial stress path tests in triaxial apparatus are usually performed to obtain the shape and size of the yield surface for a given loading rate. In the multi-laminate models it is not possible to obtain the macro yield surface explicitly as yielding on the macro level is not distinct and relates to the integrated behaviour from the micro level. The yielding develops progressively and the same techniques for capturing the yield point have to be applied for simulation and experimental results. The commonly used methods of the location of volumetric yield point by line extrapolations are not perfect, nevertheless, it is the only means to study the development of the macro yield surface resulting from the multi-laminate model.
The obtained macro yield surfaces on the $p$-$q$ plane are presented in Figure 5. In order to obtain the presented graphs, initial conditions in the multi-laminate model have been imposed or induced by loading at the begin. Then, the radial stress path tests were simulated starting from the small isotropic stress state ($p_0=5$ kPa). Twelve radial stress paths have been applied for every macro yield locus on the $p$-$q$ plane: two directions parallel to the compression ($M_c$) and extension ($M_e$) shear failure lines, $\eta=q/p=1.2$ and $\eta=-0.857$ respectively; ten radial directions from $\eta=-0.8$ to $\eta=1.0$ with an interval of 0.2.

The left-hand graph in Figure 5 presents the evolution of the yield locus for different levels of destructuration. Destructuration was induced by loading along the $K_{0nc}$-line similarly as in the consolidation phase for the undrained triaxial simulations (Figure 5). The shape of the intact yield surface transforms with increasing levels of destructuration to the shape characteristic for the stress induced anisotropy. The rate of this process is related to the degree of bonding as well as the value of structural cross-anisotropy parameter $\Omega_v$. The right hand side graph displays the influence of the parameter $\Omega_v$ on the shape of the intact yield surface. Asymmetry about the $p$-axis as well as steepness of the yield surface is proportional to the value of the parameter $\Omega_v$.

4 CONCLUSIONS

The multi-laminate model for soft soils incorporating bonding anisotropy and destructuration was presented in the version which is potentially simple, using the framework of rate independent plasticity. The basic idea applied in the formulation of the model was the directional distribution of the overconsolidation or bonding over the sampling planes in the multi-laminate framework and to connect this distribution with the microstructure of the soil. Microstructure is understood here as the arrangement of the clay fabric and interparticle bonds having influence on the strength anisotropy. The results of element simulations with this model are shown to give qualitatively realistic results according to the known experimental evidence. The model may be useful especially for the geotechnical problems where compression of natural soft clays has an important contribution in the general behaviour.

The model has been formulated in such a way that implementation into the standard finite element code is possible without any special extensions related to the regularisation techniques, i.e.
without any kind of strain softening incorporated in the model definition. Important extensions like introduction of the rate-dependency and elastic anisotropy may be applied, however, with a cost of additional material parameters.

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REFERENCES


APPENDIX - NOTATION

\[ \varepsilon_v \] volumetric strain \( \varepsilon_v = \varepsilon_{kk} \), compression negative
\[ \sigma, \varepsilon \] effective macro stress tensor and strain tensor; compression negative,
\[ \sigma, \varepsilon \] tensors, written using the vector notation (6x1),
effective micro stress vector and strain vector on the k-th sampling plane (3x1);
components are expressed in the local coordinate system (\( \hat{n}_k \) – vertical, third unit vector)
\[ \sigma_v \] vertical component of effective stress, compression positive (ex. in oedometer tests)
preconsolidation and equivalent pressure on the k-th sampling plane,
\[ p_c \] effective consolidation pressure applied to obtain a destructured state of material,
\[ p \] effective mean pressure \( p = -1/3 \sigma_{kk} \),
\[ q \] deviatoric stress; in the figures calculated as \( q = \sigma_1 - \sigma_3 \) for axisymmetric (triaxial) conditions,
\[ (..)^0 \] initial value of (..).