1. INTRODUCTION

In devices in which electric power is processed, voltage switching operations as well as current switching operations occur. They are conducted by means of components which in this case play the role of ‘electric switches’ [1, 2]. Except for the classical switches which are electromechanical relays, the function is also performed by other solid state components (e.g. transistors or thyristors) or by phenomena such as electric spark-based discharge. The operation of switching the current or voltage between two parts of an electrical circuit is accompanied by a sudden change of the signal level which occurs in a short period of time. When the switching operation is conducted by means of an ideal electric switch which is connected between an ideal source and an ideal load resistance (see Fig. 1), the occurring sudden change in the amount of the current or voltage level has in the time domain the shape of a step signal. The result of a unit step function in the frequency domain is a continuous spectrum whose envelope disappears at a speed of $-20 \text{ dB/dec}$. Since no ideal sources or electric switches, or ideal resistances exist then it shall be expected that their residual parameters have an influence on the spectrum of signals generated as a result of the switching operation. Determining the envelope of the spectrum of the signals as mentioned above enables the evaluation of the given electric switch as a potential source of broadband electromagnetic interference (EMI). The basic problem which arises while the measurements are undertaken is separation of the spectrum which is
connected with the state ‘on’ or the state ‘off’ of the switch under test. This issue of ‘separating’ the spectra is a subject of the paper. On top of it, the paper presents results of the measurements executed by the author in the discussed measurement circuit.

Fig. 1. Mechanism of sudden value changes of voltage or current rise caused by switching the electric switch. a) Simple electric circuit with ideal components and an ideal switch. b) Signal of sudden change of voltage value arising as the result of switching the electric switch on and off (the shape of the signal was drawn in; the signal arises while switching direct current sources and in case when the signal of the source changes like a sin (…) function). c) Spectrum envelope of voltage from b).

2. BASIC PROBLEMS CONNECTED WITH MEASUREMENT OF ENVELOPE OF MODULE OF SIGNALS’ SPECTRUM

An instrument that is predisposed to measure the spectral density magnitude of electric signals is a test receiver. It is strictly related to typical traits of the measurement instrument, in particular: precisely determined IF bandwidth, high dynamic selectivity, standardized time constants of measurement detectors [3, 4, 5]. However, determining the level of the envelope magnitude of a single pulse spectrum in a wide frequency range within a single measurement is not feasible even if new generation automatic test receivers are applied [6, 7]. The completion of such a task requires simultaneous tuning of the instrument to numerous different measurement frequencies. Therefore, in order to measure the envelope of a spectrum of e.g. a signal that is generated by switching an
electric switch, one should generate repeatedly and periodically this state of an electric switch being at work. However, to generate this state periodically (state of having the switch switched on), the switch must be also periodically switched off. Subsequently there is a chain of repeatedly occurring states of switching the switch on and switching it off and a superposition of spectra of both of the states. In such a situation, in order to measure the spectrum envelope resulting from the state ‘on’ the switch under test should be switched in such a way so that the state ‘off’ generates a signal whose spectrum has a level at least ten \[\text{dB}\] lower than the level of the spectrum of the signal ‘on’. Fulfilling this condition is possible when switching the switch ‘on’ takes place at value of voltages difference of zero between the terminals of the switch, and switching the switch ‘off’ takes place when no current flows through it. The level of the signal between the state ‘on’ and ‘off’ should most smoothly decrease towards the zero level and should reach zero before the final switching ‘off’. It means that the switch under test has only one active state of generation and the spectrum measured in these conditions is the spectrum of a signal generated exactly at this state. The parameters of the measuring instrument impose the necessity of periodical generation of the state of an active switch and this is the reason why the signal applied to the input of the measuring receiver is a periodical signal whose spectrum is a discrete one in comparison to a single pulse of switching whose spectrum is a constant one. However, if the repetition frequency of the measured signal has a value of a few dozens of hertz, then by the IF bandwidth of the measuring instrument \(B_i = 10 \text{ kHz}\) the measured level of the spectral density magnitude of a signal may be treated as quasi continuous [8]. The envelope of the measured spectrum has exactly the same shape as the envelope of a single pulse on the condition that the repeated pulses are identical and a pause between the previous and each next pulse is long enough so that the spectra of subsequent pulses do not overlap [9]. The measurement of the spectrum of electric signals can be conducted by means of spectrum analyzers [10, 11]. However, restrictions resulting from two basic parameters of these devices must be remembered. First, the analyzer’s input is a broadband one in comparison to a narrowband input of a test receiver. Second, the linear margin of the spectrum analyzer is much lower than the reserve of the linearity margin of the tests receiver. Also in this case periodical repetition of pulses, the spectrum of which is the subject of measurement, is necessary. Applying the sampling method of a signal and then determining its spectrum by means of discrete Fourier transformation algorithm (DTF) or fast Fourier transformation (FFT) also encounters a lot of restrictions [12, 13, 14]. First of all, they result from the necessity of meeting the Kotelnikow-Shannon condition by sampling the signals whose rise speed or slope fall time amounts to a few nanoseconds. A second restricting element is the phenomenon of spectrum ‘aliasing’ and the necessity of limitation of its impact on the measurement result.
3. APPLIED METHOD OF REDIRECTING THE SWITCH UNDER TEST

Meeting the condition of switching the switch so that in each consecutive cycle of switching on and off only one of those states is active (it means: only one could generate the spectrum which is the subject of the study) was solely successful when the circuit from Fig. 2 was applied [15, 16]. The signal switched via the switch has a shape of a sine wave signal detected in a half-wave diode detector. The logic circuit enables such a synchronization of the moment of switching on or switching off with the re-switched signal that the operation of the switch takes place at the maximal value of the re-switched signal. As a result, the spectrum of the periodical signal with a frequency $f_{\text{rep.}} = 50\text{Hz}$ and a shape as shown in Fig. 1b is determined. One slope of this signal is generated at the moment when the switch works, and the second slope changes as a sin(.) function. As a result, after e.g. switching the switch (a sudden change of the signal level $u_s(t)$), its switching off takes place at the moment when the signal level is $u_s(t) = 0$ volt. In consequence there are pulses generated with a frequency $f_{\text{rep.}} = 50 \text{ Hz}$; one slope of the pulses reveals a high speed of changes $dV/dt$, and the second slope changes slowly (slow speed of changes of $dV/dt$) (see Fig. 3).

The measured spectrum of the generated signal, which is a superposition of both of the pulse’s slopes, is practically fully formed only by one steep-side slope of e.g. the switching on. It is justified by the fact that the spectrum of a slope changing like e.g. a sin(.) function or cos(.) function has a level at least a few dozen of decibels lower (see Fig. 4) and switching the switch off in the discussed case takes place when there is no signal.

![Fig. 2. Basic functional diagram enabling the measurement of the signal’s spectrum emerging as a result of switching the switch.](image-url)
Fig. 3. Images of signals tested in the time domain. The shapes are given within one period of the repetition frequency of the signal ($f_{\text{rep}} = 50$ Hz).

Fig. 4. Envelopes, appointed (results of computation) with the use of the FFT technique, of the frequency spectrum 'test-2a' mode of signals' (see Fig. 3) for different switching angles for switching on and switching off of the switch under test.
It is also confirmed by the calculation results of the envelope of the spectrum of ‘switch on’ signals and ‘switch off’ signal conducted for signals from ‘test-2’ group. They are all shown in a graph (see Fig. 5). The determined dependencies according to which the calculations were made are presented in Table 1.

Table 1. Statement of appointed dependencies which enable to calculate discrete spectral line of signal for any values of ankle of switching on or switching off of the switch under test.

<table>
<thead>
<tr>
<th>Shape of signal u(t)</th>
<th>Fourier series</th>
<th>remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;switch on&quot;</td>
<td>$$a_k = \frac{A}{2\pi} \left[\cos((1+k)\pi - \cos((1+k)2\pi \frac{x}{T} + \frac{x}{T} (1-k))\right]$$</td>
<td>$k=1$</td>
</tr>
<tr>
<td></td>
<td>$$b_k = \frac{A}{2\pi} \left[\sin((1-k)\pi - \sin((1-k)2\pi \frac{x}{T} + \frac{x}{T} (1-k))\right]$$</td>
<td>$k=1$</td>
</tr>
<tr>
<td></td>
<td>$$a_k = \frac{A}{2\pi} \left[\sin^2 \frac{x}{T}\right], b_k = \frac{A}{2\pi} \left[\frac{\pi - 2\pi \frac{x}{T} + \sin 4x \frac{x}{T}}{2}\right], a_k = a_k(k=0)$$</td>
<td>$k=1$</td>
</tr>
<tr>
<td>&quot;switch off&quot;</td>
<td>$$a_k = \frac{A}{2\pi} \left[\cos((1+k)\pi + \frac{x}{T} + \frac{x}{T} (1-k))\right]$$</td>
<td>$k=1$</td>
</tr>
<tr>
<td></td>
<td>$$b_k = \frac{A}{2\pi} \left[\sin((1-k)\pi + \frac{x}{T} + \frac{x}{T} (1-k))\right]$$</td>
<td>$k=1$</td>
</tr>
<tr>
<td></td>
<td>$$a_k = \frac{A}{2\pi} \left[\sin^2 \frac{x}{T}\right], b_k = \frac{A}{2\pi} \left[\frac{\pi + 2\pi \frac{x}{T} + \sin 4x \frac{x}{T}}{2}\right], a_k = a_k(k=0)$$</td>
<td>$k=1$</td>
</tr>
</tbody>
</table>

Fourier distribution transformation

$$\text{Re}[F(j\omega)] = A \frac{2\pi}{\omega} \left[\cos \gamma \cdot \cos(k\gamma) + k \cdot \sin \gamma \cdot \sin(k\gamma) + \cos k\gamma\right]$$

$$\text{Im}[F(j\omega)] = A \frac{2\pi}{\omega} \left[k \cdot \sin \gamma \cdot \cos(k\gamma) - \sin \gamma \cdot \cos \gamma \cdot \sin k\gamma\right]$$

It is also possible to apply in the circuit in Fig. 2 signals of a different shape than a sine wave and of frequency much higher than $f_{\text{rep.}} = 50$ Hz.
4. SWITCH PROPERTIES – AN IDEAL SWITCH VERSUS A REAL SWITCH

A signal that can be described by means of a unit step function (Heaviside’s function) is the result of switching an ideal switch. The spectrum which is an image of a signal in the frequency domain is determined by calculating the Fourier integral, and it the complex spectrum of a function $F(j\omega)$ received as a result is presented most often in the shape of a complex function module $|F(j\omega)|$ and an argument of complex function $F(j\omega)$. Due to practical reasons a greater significance is attributed to the magnitude of spectral density of a signal since this parameter is directly measured by test receivers. In case of re-switching of an ideal switch there is a rapid change of voltage or of the current value, the measured modules of spectral density change in the following way: $|F_u(j\omega)| = U/\omega$ in case of the step function of voltage value and in the following way: $|F_i(j\omega)| = I/\omega$ in case of a current-step function. The graph of a function of these modules in the frequency domain is a straight line with a slope of $-20 \text{ dB/dec.}$ (see Fig. 2c). The real switch strays with its parameters from the parameters of an ideal switch. It causes in the time domain a change of the signal’s slope shape and a change in speed of the change of voltage or of current. In consequence also the spectrum of this pulse is changed in the frequency domain. The direction and the size of these changes is an effect of simultaneous operation of numerous factors such as material properties of the real switch, its residual parameters and a whole lot of other factors dependent also on frequencies, parameters of circuits between which the switch under test is installed [2, 17–19]. It should be expected that ‘modification’ of the spectrum of the pulse in a general case will also reduce the value of the spectrum magnitude.

![Appointed envelopes of frequency spectrum mode of signals tested from Fig. 3](image)
and it restriction in the high frequency range in which this spectrum occurs. Next, the following cases are also possible: selective gain or attenuation of the spectrum parts by resonance phenomena which occur in electric network’s impedance [23] in which the real switch works.

5. VACUUM RELAY WITH MERCURY WETTED CONTACTS AS A ELECTRIC SWITCH

For the electromechanical real switch, in contrast to the ideal switch, the following are typical: finite values (different than zero or indefinite ones) of the residual parameters such as transfer resistance of the ‘on’ and ‘off’ contacts, stray capacity, residual inductance (see Fig. 6a) as well as finite times of switching the contacts on and switching them off.

![Fig. 6. Real vacuum relay switch with mercury wetted contacts. a) Equivalent circuit diagram of the real switch. The parameters of residual values: \( C_{12}, C_{10}, C_{20}, R_{\text{on}}, R_{\text{st}} \) according to [22], \( L_{d1}, L_{S1}, L_{S2}, L_{d2} \) estimated according to [20, 23], \( R_{\text{on}}, R_{\text{off}} \) estimated with assumption of capacitance loss \( C_{10} \) and \( C_{20} \) at the level \( \varepsilon \approx 1 \cdot 10^{-3} \), b) Basic functional diagram of the switch circuit adapted to the analysis (the switch installed between the ideal source of voltage and the resistive load \( R_{\text{load}} = 50 \text{ W} \).]
A real electric switch having equivalent circuit diagram as above was installed in the measurement head (see Fig. 2) in order to measure the spectral density module of a signal emerging as a result of switching the real switch. The circuit in which the measured signal is generated (see Fig. 6b) consists of the source of the test signal, the resistance load and the switch under test. Due to application of a dedicated measuring head [16], the components of the measurement circuit (source, load and connections) reveal negligible residual parameters within the range 50 Hz ÷ 300 MHz [24]. Therefore the only component in the measurement circuit from Fig. 6b, being different in terms of its parameters from the ideal components, in the measurement range as specified above is the electric switch under test. It enables to isolate the measured signal and to pass over ‘modifications’ of the measured spectrum by electrical network impedance of the measurement circuit.

6. BIPOLAR TRANSISTOR AS A SEMICONDUCTOR ELECTRIC SWITCH

The structure of the measurement head enables also testing of semiconductor devices which work as electric switches. In order to test the head, instead of the electric switch we place e.g. a bipolar transistor or a thyristor. The re-switched signal, applied to the switch under test, is the same as in case of the vacuum relay switch. An additional problem is proper polarization of the semiconductor switch in order to re-switch it. In the case of bipolar transistor it requires the introduction between its emitter and its base of an additional source which polarizes the junction and is synchronously re-switched (see Fig. 7). Providing the synchronous re-switching for the switching-on or switching-off operation of the switch under test enables the measurement of the signal spectrum where the signal emerges as a results of switching the switch.

Fig. 7. Basic connection diagram of the measurement circuit of semiconductor switch. (simplified diagram of the circuit in which the signal spectrum is measured; the signal resulting from switching the transistor switch).
However, in this case attributing a simple equivalent circuit to such a switch encounters a lot of difficulties. They result from, among others, dynamically changing parameters during the work time of a switch; the parameters are connected e.g. with the dislocation of the spatial electrical charges in the transistor's structure. A wider discussion on the topic is not within the core subject of this paper.

7. EXPERIMENTAL TEST RESULTS

In a measurement system whose basic functional diagram is given in point 3, a lot of measurements were conducted concerning the spectral density magnitude of the state ‘on’ and the state ‘off’ of the vacuum relay switches as well as the semiconductor switches. Part of them, are presented in Figs. 8, 9 and 10.

Fig. 8. Measurement results related to the envelope of density module of the signals arising from the vacuum relay switch ‘switching on’ and ‘switching off’ with mercury wetted contacts. The measurement executed in the circuit as in Fig. 2 and Fig. 6b for three different ‘A’ amplitude values of the re-switched voltage (value of switching angle x/T = const = 0.25).
Fig. 9. Measurement results related to the envelope of spectrum density module of the signal arising from the switch ‘switching on’ and ‘switching off’ of a BSS 79 transistor for three different ‘A’ amplitude values of the re-switched voltage (value of switching angle x/T = const = 0.25). For better clarity there are no measurement points in the graph.

Fig. 10. Measurement results related to the envelope of spectrum density module of the signal ‘switching on’ and ‘switching off’ for three different transistor switches, for one value of ‘A’ amplitude of the re-switched signal (value of switching angle x/T = const = 0.25). For better clarity there are no measurement points in the graph.
8. BRIEF ANALYSIS OF THE MEASUREMENT RESULTS AND THE APPLIED METHOD. REMARKS AND CONCLUSIONS

From the spectral analysis conducted for test signals (see Table 1), breakdowns of its results (see Fig. 4) and numerical calculations which take advantage of the FFT algorithm, the following could be deduced:

- The applied method of electric switch operation enables reaching the appointed goal of measurement.
- The methods used for analysis (Fourier series, distribution method) show convergence of the results which is burdened with an uncertainty not higher than 0.5% within the whole range of the analysis (Fig. 11).
- Divergence between the results of the FFT algorithm method and the results of the Fourier series analysis does not exceed the value of 0.5% within the whole analysis scope (Fig. 12).
- In the frequency domain of the signal the spectrum lines dominate; these spectrum lines are products of an ‘active’ state of the switching the switch under test (Fig. 4).
- Spectrum lines which are products of the second ‘non-active’ state of switching the switch indicate within the whole measurement scope a level lower by a value not less than 20–30 dB/μV. The smallest difference occurs in the lower measurement range and enlarges its value in the range of higher measurement frequencies (Figs. 4 and 5).
- The useful measurement range of the applied system (Fig. 2) in case of vacuum relay switches falls within the range of 10 kHz ≤ f ≤ 300 MHz. In measurements of semiconductor switches, due to the application of an additional driving source the measurement range receives some limitation on the upper side and amounts to 10 kHz ≤ f ≤ 200 MHz ÷ 250 MHz.

As a result of analyzing the measurement data (Figs. 8, 9 and 10) obtained while carrying out the experiment in the measurement system whose basic functional diagram is presented in the Fig. 2, the following could be stated:

- The results of the experimental measurements confirm the legitimacy of the conclusions drawn from theoretical analysis of the issue related to the measurement of spectrum of signals of the real switches being switched on and switched off.
- There is a vivid dependence between the shape of the envelope module of the signal ‘off’ and ‘on’ spectrum on the signal level switched by a switch and the kind of switch under test (semiconductor switch or an electromechanical one).
- The electromechanical switch which is the subject of this experiment revealed the properties of an ‘ideal’ switch in a much wider range of frequencies than semiconductor switches under test.
- The spectrum of ‘switch on’ signal of the switches under test indicated properties of an ‘ideal’ switch being ‘switched on’ in a much wider range of frequencies than the spectrum of the ‘switch off’ signal.
Fig. 11. A similarity graph related to values of magnitude of second ($h_2$) and third harmonic ($h_3$) of the signal ‘on’ and ‘off’ in function of a standardized angle “$\alpha$” and “$\delta$” (see Table 1). The harmonics were appointed from the Fourier series and distribution method.

Fig. 12. A divergence graph, in function of frequency, of harmonic values of the ‘switch on’ signal determined with the use of the FFT algorithm and Fourier series.
• Real electric switches create broadband electromagnetic interference whose source are sudden changes of voltage or current which appear as a result of switching the switches.
• The uncertainty of the determination of the spectral magnitude envelope by measurement, which results from the applied instruments, in this experiment does not exceed the level of $\pm 0.2 \text{ [dB/} \mu\text{V]}$.
• The estimated influence of the measurement head and in particular of its residual parameters, on the level of the measured spectrum magnitude was not higher than $\pm 0.3 \text{ [dB/} \mu\text{V]}$ in the measurement scope of $10 \text{ kHz} \leq f \leq 200 \text{ MHz}$ [24].
• The dynamics of the measurement range of the levels of spectral density magnitude falls within the following level scope of $-20 \text{ [dB/} \mu\text{V}] \leq B \leq +100 \text{ [dB/} \mu\text{V}]$.

The conclusion may include the thesis that the proposed measurement system enables the measurement of signals which emerge during switching of the switches in an electric circuit and further research is possible; such research may aim to determine the parameters of real electromechanical switches as sources of broadband electromagnetic interference.

REFERENCES

22. CP Clare Corporation – catalog data.