WAVE-INDUCED UPLIFT FORCE ON A SUBMARINE PIPELINE BURIED IN A COMPRESSIBLE SEABED

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Abstract—A two-dimensional finite-element simulation of the wave-induced hydrodynamic uplift force acting on a submarine pipeline buried in sandy seabed sediments subject to continuous loading of sinusoidal surface waves is presented. Neglecting inertia forces, a linear-elastic stress-strain relationship for the soil and Darcy's law for the flow of pore fluid are assumed. The model takes into account the compressibility of both components (i.e., pore fluid and soil skeleton) of the two-phase medium.

The results of numerical analysis are presented and discussed with respect to soil and pore fluid parameters where special attention is paid to the question of soil saturation conditions. The meaning of the results is also related to surface wave conditions. As a general conclusion, the practical, engineering recommendation is given in order to make a realistic, safe and economic estimation of the wave-induced uplift force acting on a buried submarine pipeline.

NOMENCLATURE

- \( a \) wave number
- \( B \) width of the main finite-element domain
- \( b \) depth of burial of the pipeline
- \( D \) outer diameter of the pipeline
- \( E_s \) shear modulus of soil
- \( F_z \) absolute maximum wave-induced hydrodynamic uplift force, per 1 m of pipeline length
- \( F_z^r \) relative maximum wave-induced hydrodynamic uplift force, per 1 m of pipeline length
- \( G_p \) Young's modulus of pipe-wall material
- \( G_s \) Young's modulus of soil
- \( H \) height of the main finite-element domain
- \( H_w \) waveheight
- \( h \) water depth
- \( k \) coefficient of isotropic permeability of soil
- \( L \) wavelength
- \( N_q \) indicator for the Gaussian \( N_q \)-point quadrature rule of numerical integration
- \( n_r \) number of divisions of the pipeline vicinity domain in radial direction
- \( n_s \) porosity of soil
- \( n_t \) number of divisions of the pipeline vicinity domain in tangential direction
- \( n_{ts} \) number of time-steps within one period of wave loading
- \( P_0 \) amplitude of hydrodynamic bottom pressure
- \( p \) wave-induced excess pore pressure
- \( p_a \) absolute hydrostatic pressure
- \( p_a \) atmospheric pressure
- \( p_h \) wave-induced hydrodynamic bottom pressure
- \( p_h \) hydrostatic pressure
A wave-induced excess pore pressure, cyclically generated in the vicinity of a buried submarine pipeline, constitutes one of the main factors that has to be considered in the pipeline stability analysis. The hydrodynamic uplift force acting continuously on the pipeline is comparable to the displaced water weight (Monkmeyer et al., 1983; Cheng and Liu, 1986; Magda, 1992a and b) if the pipeline is located relatively close to the seabed surface. An inadequate design can cause flotation of the pipeline, very often leading subsequently to costly failures and environmental catastrophies. Therefore, it is essential to improve the knowledge on interactions among waves, seabed, and pipeline-like marine structures.

It is a very complex and challenging task to define properly the wave-induced excess pore pressure around a submarine pipeline buried in a porous seabed. For many purposes in soil mechanics, it is permissible to uncouple the soil and fluid parts of any analysis in order to treat two simpler analyses separately. Such a treatment leads to a solution of only one partial differential equation (‘potential’ problem constituted by the Laplace’s equation) describing the pore pressure changes in the soil skeleton when both the pore fluid and soil skeleton are assumed to be incompressible (Putnam, 1949).

Most of the analytical (MacPherson, 1978; McDougal et al., 1988; Monkmeyer et al., 1983) and numerical (Lennon, 1985) solutions to the uplift force for submarine pipelines buried in seabed sediments, including the perturbation pore pressure effects, are devoted only to the case of incompressible media. The only publications concerning a compressible seabed sediment two-phase system were issued by Bobby et al. (1979) and Cheng and Liu (1986) who used the finite-element method (FEM) and the boundary integral equation method (BIEM), respectively, in their analyses. Their works, however, have some disadvantages that limit their practical applications.

In the numerical example presented by Bobby et al. (1979), for instance, the geometry of the problem is not realistic because a concrete-coated submarine pipe buried 7.5 m below the sea bottom is assumed for the analysis. In coastal engineering practice, however, pipelines located in water depth up to 60 m are buried but the cover layer has a thickness ranging from 0.5 m to 1.0 m (Dursthoff and Mazurkiewicz, 1985). Moreover, Bobby et al. (1979) said that the increase of pore pressure at the crown of the pipe was marginal. It is no wonder when the pipeline is placed far below the seabed surface. Additionally, a
pore pressure distribution around the pipeline should be shown with respect to the hydrodynamic bottom pressure which would be, in case of the cited paper, very meaningful because of very severe wave conditions chosen for the analysis. In spite of this, the analysis by Bobby et al. (1979) was performed only for one set of compressibility parameters where the pore fluid compressibility, for example, respective to only fully saturated soil conditions was assumed.

The limitations in the above mentioned solutions were avoided by Cheng and Liu (1986). There are, however, some others listed below:

- Only one value of the shear modulus of soil \( G_s = 10^4 \) kN/m² was used in their analysis; this value corresponds to loose sandy sediments properties. It will be shown in the present work that the uplift force can be even higher when the shear modulus typical for semi-dense or dense sandy sediments \( G_s = 2.6 \times 10^5 \) kN/m² is introduced into computations.

- Only one value of the wavelength \( L = 60 \) m was used in their analysis, and they indicated that the uplift force could be as large as 60% of the displaced water weight. In fact, as it will be also shown in the present work, the uplift force can be much larger if the longer waves are introduced into computations.

- The compressibility of pore fluid, corresponding to the degree of saturation in the range of \( S = 0.98 - 99.9\% \), was used in their analysis. The present work will use the degree of saturation from a wider range and indicate the optimum value of uplift force. An optimalization procedure for the pipeline uplift force can be performed with respect not only to the soil permeability [as shown by Cheng and Liu (1986)] but also by taking saturation conditions (pore fluid compressibility) into account. However, using only three values of the degree of saturation (Cheng and Liu, 1986), it is rather difficult to indicate an optimum value of the pipeline uplift force.

- There is no analysis of the phase lag phenomenon accompanying indispensably a compressible pore fluid flow through compressible porous media.

Some cases of laboratory studies reported in the literature (Monkmeyer et al., 1983; McDougal et al., 1988) show evidently that there is a difference between theoretically computed values of instantaneous pore pressures and those observed in experiments. The reported differences between theoretical and experimental results could have been due to the following three main reasons, namely: (1) the potential theory used in computations does not contain all important and decisive soil and pore fluid parameters (incompressibility of pore fluid and soil skeleton is assumed); (2) boundary conditions used in computations are not realistic, especially when compared with laboratory model tests (finite thickness of the soil layer); (3) input data for calculations are not adequate with \textit{in situ} values from laboratory modelling.

Implementation of certain soil and pore fluid parameters, e.g. saturation (indirectly: compressibility of pore fluid), soil permeability and compressibility, does not lead to the Laplace equation, which makes the problem independent of these parameters, but to a much more complex system of three equations (Verruijt, 1969; Madsen, 1978; Yamamoto et al., 1978; Nago and Maeno, 1984; Okusa, 1985). The solution to this type of equation can be formed, for example, as a combination of solutions to both the Laplace equation and the diffusion equation (Okusa, 1985). Although an analytical solution is possible, using the technique of separation of variables, the complexity of the mathematical formulation as
well as the difficulties involved in solving it [it is not an easy task to handle it with special functions, i.e. the Hankel functions, the use of which is incorporated into the solution to the Helmholtz equation in the polar coordinates system (Magda, 1992a)], have provided motivation to look for another way of treating the governing problem. This time, a numerical method has been chosen to perform the analysis of the true coupled performance of a composite continuum, in which the two phases interact, and the wave-induced oscillations of pore pressure is completely dependent upon the relative stiffness of the components of the system.

Figure 1 illustrates schematically the geometry of the governing problem; a submarine pipeline of a diameter $D$ is embedded in a sandy soil up to a depth $b$ (depth of burial $b$ is measured from the seabed surface to the top of the pipe) and loaded hydrodynamically by simple two-dimensional harmonic waves. The pipeline wall is assumed to be formed of steel, thereby it is non-deformable compared with deformations in the soil skeleton.

2. GOVERNING PLANE STRAIN EQUATIONS

2.1. Seabed (two-phase medium) domain

Under plane strain conditions, the following two equations, describing elastic deformations of the soil skeleton, together with the ‘storage’ equation constitute the coupled problem and can be written as:

$$\begin{align*}
G_s \left( \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_z}{\partial z^2} \right) + \frac{G_s}{1-2\mu_s} \frac{\partial}{\partial x} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) &= \frac{\partial p}{\partial x} \\
\eta &= \frac{H}{2} \cos(\alpha x - \omega t)
\end{align*}$$

(1a)

Fig. 1. Definition sketch.
where: \( p \) is the wave-induced excess pore pressure, \( u_x \) and \( u_z \) are the horizontal and vertical displacement of soil skeleton, respectively, \( G_s \) is the shear modulus of soil, \( \mu_s \) is the Poisson’s ratio of soil, \( k \) is the coefficient of isotropic permeability of soil, \( \gamma \) is the unit weight of pore fluid, \( \beta' \) is the compressibility of pore fluid, \( n \) is the porosity of soil, \( x \) and \( z \) are the horizontal and vertical components of the Cartesian coordinates system, respectively, and \( t \) is the time.

A linear stress–strain relationship for the soil is represented by Equation (1a) and Equation (1b) which are formed from the equilibrium condition in the \( x \)- and \( z \)-directions, respectively. Equation (1c) reflects the continuity principle incorporating Darcy’s law of fluid flow through a porous medium. The inertial forces are small (Cheng and Liu, 1986) and therefore the body forces are set to zero and neglected in the present analysis.

Under realistic conditions, the pore fluid is represented by a two-phase medium where the water and air components can be distinguished. However, in order to simplify the calculation procedure, the compressibility of this two-phase medium is defined by a very convenient, from the engineering point of view, formula proposed by Verruijt (1969) and applied by many other researchers (e.g., Madsen, 1978; Yamamoto et al., 1978; Okusa, 1985). The formula, describing the pore fluid compressibility with respect to saturation conditions represented by the degree of saturation, has the following form:

\[
\beta' = \beta + \frac{1-S}{p_a} \quad \text{for} \quad 1-S \leq 1
\]  

(2)

where: \( \beta' \) is the compressibility of pore fluid, \( \beta \) is the compressibility of pure water, \( S \) is the degree of saturation, and \( p_a \) is the absolute hydrostatic pressure \((p_a=p_{at}+p_h, \text{ where } p_{at} \text{ is the atmospheric pressure, and } p_h \text{ is the hydrostatic pressure})\). Table 1 shows, for a water depth of 10 m, how strong the dependence of the pore fluid compressibility on saturation conditions is.

<table>
<thead>
<tr>
<th>Degree of saturation ( S ) [-]</th>
<th>Compressibility of pore fluid ( \beta' ) [m³/kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>4.2×10⁻⁷</td>
</tr>
<tr>
<td>0.999</td>
<td>5.4×10⁻⁶</td>
</tr>
<tr>
<td>0.99</td>
<td>5.0×10⁻⁵</td>
</tr>
<tr>
<td>0.98</td>
<td>1.0×10⁻⁴</td>
</tr>
<tr>
<td>0.95</td>
<td>2.5×10⁻⁴</td>
</tr>
<tr>
<td>0.90</td>
<td>5.0×10⁻⁴</td>
</tr>
</tbody>
</table>
2.2. Pipe-wall domain

The system of coupled equations describing the governing problem within the pipe-wall material is even simpler than in case of the soil skeleton. Being still in the range of elastic deformations and assuming that the pipeline solid wall (e.g., made of steel or concrete) creates an impermeable boundary condition, only two equilibrium equations are required. They can be easily obtained from the first two equations [i.e., Equations (1a) and (1b)] describing the governing process in the soil, after neglecting the pore fluid terms and replacing the values of \( G_s \) and \( \mu_s \), assumed for the soil skeleton, by proper values of \( G_p \) and \( \mu_p \), respectively, characteristic for the pipe-wall material. The third equation [Equation (1c)] has no practical meaning as far as the pipe-wall domain is concerned and therefore disappears from the equations system.

2.3. Boundary conditions

The boundary conditions (primary and natural) assumed for the governing problem are schematically illustrated in Fig. 2. The wave-induced hydrodynamic bottom pressure, \( p_b \), acting at the seabed surface (\( z=0 \)) was calculated using Airy’s linear wave theory:

\[
p = p_b = \gamma \frac{H_w}{2 \cosh(ah)} \cos(ax - \omega t)
\]

where \( \gamma \) is the unit weight of water, \( H_w \) is the wave height, \( a \) is the wave number (\( a=2\pi/L, \) \( L \) is the wavelength), \( \omega \) is the wave frequency (\( \omega=2\pi/T, \) \( T \) is the wave period), \( h \) is the water depth, \( x \) is the horizontal coordinate, and \( t \) is the time.

2.4. Solution method

The system of coupled Equations (1a)–(1c) was discretized using the Galerkin finite-element method. Due to the geometry of the problem, four-node isoparametric elements were chosen. The time-approximation was obtained using the ‘backward scheme’ which is, in a variety of problems, estimated to be ‘well-behaved’ showing no oscillations in results (Burnett, 1987).

![Fig. 2. Boundary conditions (primary and natural) assumed in the analysis of uplift force acting on a buried submarine pipeline.](image)
The pre-processing PIPELINE-MESH computer program (Magda, 1996) was used to perform a quasi-automatic FE-mesh generation that can be applied to the problem of buried submarine pipelines. In the next step, the main PIPELINE-FEM2D computer program (Magda, 1996) was used where the set of three coupled linear equations [Equations (1a)-(1c)] is solved and the wave-induced hydrodynamic uplift force, acting on a submarine pipeline buried in porous and elastic seabed sediments, constitutes the output of numerical computations. For the simplicity of presentation, the uplift force is given in its relative form, i.e.:

\[ F_z = \frac{F_z}{P_0} \quad \text{[kN/m/kPa]} \]

where \( F_z \) is the maximum relative wave-induced hydrodynamic uplift force per 1 m of pipeline length, \( F_z \) is the maximum absolute wave-induced hydrodynamic uplift force per 1 m of pipeline length, \( P_0 \) is the amplitude of hydrodynamic bottom pressure \( p_b \) [see Equation (3)]. The term ‘maximum’ denotes the highest value of the wave-induced hydrodynamic uplift force found during one wave period after a sufficient number of loading cycles required to obtain the numerical stability of computed results; the term ‘maximum’ will be omitted in the following text.

Several test computations were performed to demonstrate the model utility and check numerical accuracy influenced by the time- and space-discretization as well as the quadrature rule of numerical integration (Magda, 1996). A sufficiently acceptable local refinement of the FE mesh of the soil domain in the vicinity of the buried submarine pipeline (Fig. 3) is quite important considering the fact that the pore pressure values computed in all nodes from the pipeline outer surface constitute the main result of the computations.

The optimum discretization pattern was found where: (1) the width and height of the seabed domain \( B=L \) and \( H=L/3 \) (\( L \) is the wavelength), respectively; (2) the FE-mesh discretization pattern in the vicinity of pipeline shown in Fig. 3 (\( n_r=2 \) and \( n_t=3 \), where \( n_r \) and \( n_t \) are the divisions in radial and tangential directions, respectively); and (3) the time-step \( \Delta t=T/20 \), where \( T \) is the wave period, were used in all the computations.

Fig. 3. Sketch of the FE-mesh discretization in the vicinity of buried submarine pipeline.
2.5. Input data

The input data required for the automatic FE-mesh generation program and the main FE program consists of the following parameters: \( B \) = width of the soil domain, \( H \) = height of the soil domain, \( D \) = outer diameter of the pipeline, \( b \) = depth of burial of the pipeline (measured from the seabed surface to the top of the pipe), \( h \) = water depth, \( L \) = wavelength, \( T \) = wave period, \( n_s \) = number of time-steps within one period of wave loading, \( \mu_s \) = Poisson’s ratio of soil, \( G_s \) = shear modulus of soil, \( \beta' \) = compressibility of pore fluid, \( \mu_p \) = Poisson’s ratio of pipe-wall material, \( G_p \) = shear modulus of pipe-wall material, \( n \) = porosity of soil, \( k \) = coefficient of isotropic permeability of soil, \( \varepsilon \) = accuracy of solution (in the end-test procedure used for evaluating the results stability after each wave cycle), \( \Theta \) = coefficient in time-dependent scheme, and \( N_q \) = indicator for the Gaussian \( N_q \)-point quadrature rule of numerical integration.

In all the computations performed, the following values were assigned to some of the above mentioned parameters: \( D = 1 \) m, \( b = 1 \) m, \( h = 10 \) m, \( L = 30, 60, 90 \) or \( 120 \) m, \( T = 4.45, 7.02, 9.78 \) or \( 12.65 \) s (respectively to \( L \) and \( h \)), \( \mu_s = 0.3 \) (loose sand) or \( \mu_s = 0.2 \) (dense sand), \( \mu_p = 0.28 \), \( n = 0.4 \), \( k = 10^{-4} \) m/s (coarse sand \( k = 10^{-5} \) m/s, fine sand \( k = 10^{-5} \) m/s), \( \varepsilon = 10^{-3} \) kN/m/kPa, \( \Theta = 1 \), and \( N_q = 2 \).

Additionally, three different models of the relative compressibility of the two-phase system (pore fluid and soil skeleton) were assumed for the numerical analysis:

(a) INCOMP (pore fluid, soil skeleton, and pipe-wall incompressible)

\[
G_s = 10^{10} \text{ (} \approx \text{) kN/m}^2, \quad G_p = 10^{10} \text{ (} \approx \text{) kN/m}^2, \quad \beta' = 4.2 \times 10^{-10} \text{ (} \approx \text{) m}^2/\text{kN}.
\]

The value of coefficient of isotropic permeability of soil, \( k \), has no influence when the incompressible two-phase system is introduced.

(b) COMP-P (only pore fluid compressible)

\[
G_s = 10^{10} \text{ (} \approx \text{) kN/m}^2, \quad G_p = 10^{10} \text{ kN/m}^2,
\]

and \( \beta' = 4.2 \times 10^{-7} - 5.0 \times 10^{-4} \text{ m}^2/\text{kN} \) (\( S = 1.00 - 0.90 \)).

(c) COMP-PS (pore fluid, soil skeleton and pipe-wall compressible)

\[
G_s = 2.6 \times 10^4 \quad \text{or} \quad G_s = 2.6 \times 10^5 \text{ kN/m}^2, \quad G_p = 2.6 \times 10^6 \text{ kN/m}^2,
\]

\[
\beta' = 4.2 \times 10^{-7} - 5.0 \times 10^{-4} \text{ m}^2/\text{kN} \) (\( S = 1.00 - 0.90 \)).

3. RESULTS OF COMPUTATIONS AND DISCUSSION

The results of numerical analysis will be discussed in the following, mainly with respect to the relative compressibility model assumed for the soil–water–pipeline interaction. The influence of other soil parameters as well as the geometry of the governing problem will be also studied in the following.

3.1. INCOMP model

The results of numerical computations with the INCOMP model are illustrated in Table 2. For comparison purposes, Table 2 contains also values computed analytically for both the infinite \( (H=\infty) \) and finite \( (H=L/3) \) thickness of the seabed layer, using the ‘image-pipe’ theory (Monkmeyer et al., 1983).
Wave-induced uplift force

Table 2. Wave-induced uplift force $F_z$ [kN/m/kPa] computed analytically and numerically for INCOMP model and different wavelengths

<table>
<thead>
<tr>
<th>Wavelength $L$ [m]</th>
<th>Analytical</th>
<th>Numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$H=\infty$</td>
<td>$H=L/3$</td>
</tr>
<tr>
<td>30</td>
<td>0.251</td>
<td>0.242</td>
</tr>
<tr>
<td>60</td>
<td>0.139</td>
<td>0.135</td>
</tr>
<tr>
<td>90</td>
<td>0.096</td>
<td>0.093</td>
</tr>
<tr>
<td>120</td>
<td>0.073</td>
<td>0.071</td>
</tr>
</tbody>
</table>

The uplift force computed numerically for the totally incompressible system is slightly higher than the analytically computed values due to the discretization errors. The differences in uplift force, computed numerically with respect to the analytical solution, range from 2.5% ($L=30$ m) to 2.8% ($L=120$ m).

Figure 4 illustrates the pore pressure distribution on the pipeline circumference at the moment when wave loading induces the maximum uplift force (results obtained for wavelength $L=30$ m are presented) in the totally incompressible two-phase system.

Fig. 4. Wave-induced pore pressure distribution on the pipeline circumference for a totally incompressible system (INCOMP model, $L=30$ m).
3.2. COMP-P model

As a next step in complexity of the problem, the compressibility of pore fluid (due to unsaturated soil conditions $S < 1.00$) was introduced, whereas the soil skeleton is still treated as incompressible. The assumed variation in the compressibility of pore fluid $\beta' = 4.2 \times 10^{-7} - 5.0 \times 10^{-4} \text{ m}^2/\text{kN}$ reflects the variation in the degree of saturation from $S=1.00$ to $S=0.90$, respectively (see Table 1). The results of computations are illustrated in Fig. 5 and presented in Table 3.

The uplift force computed numerically for $S=1.00$ (i.e., soil skeleton fully saturated with pore fluid treated as a near-compressible medium) is higher than the values obtained from the INCOMP model. The longer the wavelength the larger the difference in the uplift force is; comparing with the INCOMP model, the differences are equal to 8.9%, 39.4%, 69.2%, and 100.0%, respectively to wavelength $L=30$, 60, 90, and 120 m.

It is interesting to note that the solution obtained for the pore fluid compressibility...
Wave-induced uplift force

respective to the degree of saturation ranging between 0.90 and 1.00 shows the maximum. The maximum value of $F_z$, which seems to be independent on the wavelength (see Fig. 4), is equal to 0.625 kN/m/kPa and appears within a relatively narrow range of variation in the degree of saturation from $S=0.988$ ($L=120$ m) to $S=0.994$ ($L=30$ m). The maximum is surely influenced by the geometry of the problem (depth of burial and outer diameter of the pipe) as well as permeability conditions (coefficient of isotropic permeability) of the soil. This will be discussed in the following.

Comparing the uplift force solution $F_z^{(i)}$ from the INCOMP model with the solution $F_z^{(c)}$ from the COMP-P model, it becomes obvious that the compressible model can multiply by even a few times the solution obtained from the incompressible model (e.g., $F_z^{(i)} = 2.5F_z^{(c)}$ for $L=30$ m, and $F_z^{(i)} = 8.5F_z^{(c)}$ for $L=120$ m). The analysis shows additionally that, in the whole range of saturation conditions assumed for the analysis (i.e., $S=0.90-1.00$), the results obtained from the COMP-P model are always larger than the results from the INCOMP model; only for a relatively short wave (i.e., $L=30$ m) was it found locally ($S=0.90-0.92$) that $F_z^{(i)} < F_z^{(c)}$.

The pore pressure distribution on the pipeline circumference, for the moment of the maximum uplift force appearance, is shown in Fig. 6 (the pore fluid compressibility relates to soil saturation conditions $S=0.994$), and in Fig. 7 where the pore fluid compressibility is induced by even more severe unsaturated soil conditions ($S=0.95$).

The results obtained from the incompressible system show an ideally symmetrical picture of the pore pressure field in the proximity of the buried submarine pipeline. Moreover, the wave trough is situated exactly above the pipeline centre which indicates there is no
phase-lag in the uplift force. Upon introducing a near-incompressible component (here: pore fluid) into the two-phase seabed medium, however, damping effects start to play a significant role which is especially noticeable in the lower part of the pipeline (Fig. 6). A further reduction in the degree of saturation (i.e., $S=0.95$, see Fig. 7) magnifies the pore pressure damping effects and a significant phase-lag appears in the pore pressure 'wave' propagating into the seabed sediments, with respect to the phase of harmonic oscillations of the hydrodynamic bottom pressure.

A non-zero phase-lag in the pore pressure oscillations around the pipeline implies that the moment of occurrence of the maximum uplift force does not stay in phase with the appearance of the wave trough above the pipe centre. The values of phase lag obtained in the computations are shown in Fig. 8.

In the COMP-P model, the phase-lag varies from $\delta=0^\circ$, for saturated soil conditions $S=1.00$, up to $\delta=4\times\Delta\delta = 72^\circ$ ($L=120$ m) and $\delta=7\times\Delta\delta = 126^\circ$ ($L=30$ m), for unsaturated soil conditions $S=0.90$; $\Delta\delta=360^\circ/n_{ts}$, where $n_{ts}$ is the number of time-steps within the one whole wave period $T$. A general rule is thereby seen, namely: the longer the wavelength the smaller the phase-lag.

3.3. COMP-PS model

Assuming the two-phase seabed medium consists of two compressible components (i.e., pore fluid and soil skeleton), the problem becomes even more complicated than the first two. The results of computations, shown in Figs 9 and 10, are compared to those obtained from the COMP-P model. The following general effects can be indicated, namely:
Fig. 8. Phase-lag of the wave-induced hydrodynamic uplift force (COMP-P model).

Fig. 9. Influence of the compressibility of pore fluid on the wave-induced hydrodynamic uplift force (COMP-PS model; loose sand: $G_s=2.6\times10^4$ kN/m$^2$).
Fig. 10. Influence of the compressibility of pore fluid on the wave-induced hydrodynamic uplift force (COMP-PS model; dense sand: $G_s = 2.6 \times 10^5$ kN/m$^2$).

1. for loose sandy sediments ($G_s = 2.6 \times 10^4$ kN/m$^2$), the influence of the degree of saturation is not so strong and the uplift force stays almost constant in the whole range of variety in soil saturation conditions,

2. for dense sandy sediments ($G_s = 2.6 \times 10^5$ kN/m$^2$), the influence of the degree of saturation is much stronger than in case of loose sediments,

3. a rapid increase of the uplift force was observed in the case of dense sediments when the degree of saturation falls down from $S=1.00$ to ca $S=0.98-0.99$, depending on the wavelength,

4. the longer the wavelength the higher the value of uplift force is, but the highest value is always limited by the solution obtained from the COMP-P model,

5. the two cases (i.e., loose and dense sandy sediments) show maximum uplift force for each of the wavelengths assumed in the analysis; the range of the degree of saturation where the maximum uplift force occurs is in case of loose sandy sediments slightly larger ($S=0.97-1.00$) than in case of dense sandy sediments ($S=0.98-1.00$).

Figures 11 and 12 show the pore pressure distribution on the pipeline circumference for the case of the compressible two-phase system (COMP-PS model). Both figures present the situation from the instant when the maximum uplift force occurs during a cycle of wave loading. It can be easily indicated that the phase-lag is already present for fully saturated soil conditions ($S=1.00$, see Fig. 11) which was not the case in the COMP-P model. A further decrease of the degree of saturation ($S=0.95$, see Fig. 11) amplifies the damping effects which is certified by the larger phase-lag and smaller pore pressures acting on the pipeline outer surface.

Contrary to the COMP-P model, it is clearly visible that the pipeline circumference is divided into few sections where the pore (over)pressure and pore (under)pressure act alternatively and, additionally, the picture of the pore pressure distribution on the pipeline
Wave-induced uplift force

BURIED SUBMARINE PIPELINE (2-D FE-analysis)
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Phase lag

Phase of wave-loading

(Under)pressure

(Over)pressure

Cycle No. 4 (4) Uplift force $F_z = -0.229 \text{ kN/m/kPa}$
Time-step No. 13 (20) PRESS-down (+)
Model time [s] 15.99 LIFT-up (-)

Fig. 11. Wave-induced pore pressure distribution on the pipeline circumference; COMP-PS model (loose sand: $G_i=2.6\times10^4 \text{ kN/m}^2$, pore fluid: $\beta'=4.2\times10^{-7} \text{ m}^2/\text{kN}$ for $S=1.00$, wave: $L=30 \text{ m}$).

circumference is not symmetric any more. The terms of pore (over)pressure and pore (under)pressure describe the oscillations of wave-induced hydrodynamic pore pressure above and below the hydrostatic pressure level, respectively.

It is interesting to see, however, that the compressibility of the two-phase system, amplified by the additional introduction of compressibility of the soil skeleton (loose sandy sediments $G_i=2.6\times10^4 \text{ kN/m}^2$), leads only to a relatively small, further increase of the phase-lag of uplift force, compared with the results from the COMP-P model.

The next two figures present the range of variety in the uplift force, from the minimum (lower limit 'l') to the maximum (upper limit 'u') obtained from computations for the wavelength $L=30–120 \text{ m}$ and soil saturation conditions $S=0.90–1.00$, acting on a pipeline buried in both loose (Fig. 13) and dense (Fig. 14) sandy sediments. In the case of loose sandy sediments, the upper and lower limits of the uplift force do not differ so much from each other, although this narrow range of variety in the uplift force was found by assuming the wide range of the degree of saturation. A quite different picture is formed as far as dense sandy sediments are concerned. Here, the upper and lower limits of the uplift force are spread significantly. Simultaneously, it is very symptomatic to see that the upper limit ($u$) of the uplift force, that constitutes the most important value in the stability analysis of buried submarine pipelines, approaches a value close to that obtained from the COMP-P model without any respect to the wavelength. Additionally, all the values from the upper limit ($u$) of the uplift force were obtained when soil conditions other than fully-saturated (i.e., $S <1.00$) were assumed for computations.
Fig. 12. Wave-induced pore pressure distribution on the pipeline circumference; COMP-PS model (loose sand: $G_s=2.6\times10^4$ kN/m$^2$, pore fluid: $\beta=2.504\times10^{-4}$ m$^3$/kN for $S=0.95$, wave: $L=30$ m).

Fig. 13. Comparison of the uplift force obtained from the COMP-PS model (loose sediments: $G_s=2.6\times10^4$ kN/m$^2$, shadowed area) with the INCOMP and COMP-P models.
Wave-induced uplift force

3.4. Influence of the soil permeability and Poisson’s ratio

The influence of filtration conditions on the wave-induced hydrodynamic uplift force was studied formerly by Cheng and Liu (1986) for the case of loose sandy sediments. In order to get more insight into the problem, an additional supplementary case of dense sandy sediments was investigated in the present analysis. The results of computations are shown in Fig. 15.

The computations were performed for: \( L=30 \text{ m} \), \( \mu_\ell=0.3 \), and both loose (\( \beta'=3.042\times10^{-5} \text{ m}^2/\text{kN} \) for \( S=0.994 \)) and dense (\( \beta'=4.042\times10^{-5} \text{ m}^2/\text{kN} \) for \( S=0.992 \)) sandy sediments; the degrees of saturation chosen for computations are close to those which cause the maximum wave-induced hydrodynamic uplift force.

In both cases of loose and dense sandy sediments, the optimum values of coefficient of permeability can be easily seen, indicating the existence of both the maximum and minimum value of the uplift force within the range of variety of the coefficient of permeability typical for sandy soils (i.e., \( k=10^{-2}-10^{-7} \text{ m/s} \)).

The influence of filtration conditions on the uplift force is stronger in the case of loose sandy sediments. The maximum uplift force, computed for loose sediments (\( F_z=0.677 \text{ kN/m/kPa} \)), is higher than the maximum uplift force computed for dense sediments (\( F_z=0.536 \text{ kN/m/kPa} \)). Both maxima are situated almost symmetrically with respect to the coefficient of permeability \( k=10^{-5} \text{ m/s} \) where, on the other hand, the minima (\( F_z=0.069 \text{ kN/m/kPa} \) for loose sediments, \( F_z=0.139 \text{ kN/m/kPa} \) for dense sediments) are more or less concentrated. In the range of coefficient of permeability assumed in the present study for sandy sediments, the uplift force varies from almost zero (\( F_z=0.069 \text{ kN/m/kPa} \) for loose sediments) to a relatively high value (\( F_z=0.677 \text{ kN/m/kPa} \) for dense sediments).

Figure 16 illustrates the variation of the wave-induced hydrodynamic uplift force due
Fig. 15. Influence of filtration conditions on the wave-induced hydrodynamic uplift force [pore fluid: $\beta'=3.042\times10^{-5}$ m$^2$/kN for $S=0.994$ (loose sand), $\beta'=4.042\times10^{-5}$ m$^2$/kN for $S=0.992$ (dense sand)]; $L=30$ m.

Fig. 16. Influence of the Poisson ratio on the wave-induced hydrodynamic uplift force [pore fluid: $\beta'=3.042\times10^{-5}$ m$^2$/kN for $S=0.994$ (loose sand), $\beta'=4.042\times10^{-5}$ m$^2$/kN for $S=0.992$ (dense sand)]; $L=30$ m.
to different values of the Poisson ratio. In both cases of loose and dense sediments, the dependence is not so strong and near-linear when the Poisson ratio is in the range of \( \mu_s = 0.2 - 0.3 \), which is almost always assumed for linear-elastic analyses. The uplift forces computed for \( \mu_s = 0.2 \) are 12.9% (loose sediments) and 3.3% (dense sediments) smaller than the respective values computed for \( \mu_s = 0.3 \).

Simultaneously, loose sediments bring smaller values of the uplift force (\( F_z = 0.2 \) kN/m/kPa) than dense sediments (\( F_z = 0.5 \) kN/m/kPa). The higher values of Poisson’s ratio (i.e., \( 0.3 < \mu_s < 0.5 \)) generate much stronger non-linear dependence only in the case of loose sediments. The uplift forces computed for \( \mu_s = 0.499 \) are 158.8% (loose sediments) and 17.4% (dense sediments) higher than the respective values computed for \( \mu_s = 0.3 \). This finding certifies the need to assume a certain variability of \( \mu_s \) in all procedures where the elastoplastic analysis is going to be performed.

It is also interesting to note that the uplift force computed for a Poisson ratio very close to \( \mu_s = 0.5 \) converges to the same value (\( F_z = 0.615 \) kN/m/kPa) without any respect to the state of sandy sediments, i.e. to the compressibility of the soil skeleton.

It has to be stressed that in all the computations performed in the present analysis the Poisson ratio was kept constant (\( \mu_s = 0.3 \)) during the cyclic wave loading. In more sophisticated problems, the computational procedure has to take into account a variability in the Poisson ratio depending on the number of loading cycles and the description of behaviour of the soil skeleton (e.g., from linear-elastic to plastic). In such cases it is very realistic that the Poisson ratio varies and it can happen that \( \mu_s > 0.3 \).

3.5. Influence of the ‘compressibility–permeability’ term

Up to now, all the presented computations were carried out assuming the coefficient of isotropic permeability of soil \( k = 10^{-4} \) m/s. The maximum uplift force \( F_z \) was found for a certain soil saturation condition defined by the optimum degree of saturation \( S_{op} \). On the other hand, the optimum degree of saturation depends on the geometry of the governing problem (Fig. 17).

Due to some practical requirements for the proper generation of the finite-element mesh in the proximity of the buried submarine pipeline, the minimum value of \( b/D \) was set to 0.5. The data shown in Fig. 17 was found during the numerical analysis carried out for \( 0.5 \leq b/D \leq 3.0 \), and then extrapolated (with polynomial of the second order) in order to obtain \( S_{op} \) for \( b/D = 0 \).

In order to cover the practical range of soil filtration conditions typical for sandy soils, the analysis should be repeated for other cases of soil filtration, i.e.: \( k = 10^{-2} - 10^{-4} \) m/s (clean sand and sand gravel mixture) as well as \( k = 10^{-4} - 10^{-7} \) m/s (very fine sands, silts and clay-silt laminats [Craig, 1992]). Such an analysis can be performed numerically, as it was shown before for \( k = 10^{-4} \) m/s, or—more convenient and faster—analytically based on the numerical calculations already performed.

It is very interesting to note that, using the COMP-P model of soil–water interaction, the maximum value of wave-induced hydrodynamic uplift force \( F_z \) is directly correlated with a certain optimum value of the ‘compressibility–permeability’ term \( 1/(kE) = \beta/k \). For example, it was found that \( F_z = 0.625 \) kN/m/kPa (for \( D = 1.0 \) m and \( b/D = 0.5 \)) when the optimum value of the ‘compressibility–permeability’ term:
The optimum degree of saturation \( S_{opt} \) inducing the wave-induced hydrodynamic uplift force \( F_z \) to be maximum (soil saturation conditions: \( k=10^{-4} \) m/s).

\[
\left( \frac{\beta'}{k} \right)_{opt} = 6.042 \times 10^{-1} \text{ (m}^2/\text{kN})/(\text{m/s})
\]  

(4)

It means that all practical possible combinations of \( \beta' \)- and \( k \)-values, fulfilling the condition of Equation (4), will impel the wave-induced hydrodynamic uplift force to be maximum. Analysing the whole practical range of sandy soil permeabilities, one can predict soil saturation conditions at which the maximum uplift force will happen (e.g., Table 4, for: \( D=1.00 \) m and \( b/D=0.5 \)).

Table 4 shows the following tendency, namely: the higher the coefficient of soil permeability \( k \), the smaller is the optimum degree of saturation \( S_{opt} \) needed to create the maximum value of wave-induced hydrodynamic uplift force \( F_z \), and vice versa.

Generally, it seems to be easier to define properly the coefficient of soil permeability \( k \) than the degree of saturation \( S \) using one of the common measuring techniques. Knowing the soil permeability conditions (e.g., \( k \leq 10^{-3} \) m/s), the optimum degree of saturation can be computed (\( S_{opt} \geq 0.879 \), see Table 4), respective to these soil permeability con-

<table>
<thead>
<tr>
<th>Coeff. of permeability ( k ) [m/s]</th>
<th>Degree of saturation ( S_{opt} ) [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{-1} )</td>
<td>0.879</td>
</tr>
<tr>
<td>( 10^{-4} )</td>
<td>0.988</td>
</tr>
<tr>
<td>( 10^{-5} )</td>
<td>0.999</td>
</tr>
<tr>
<td>( 10^{-6} )</td>
<td>1.000</td>
</tr>
</tbody>
</table>
ditions. Although some difficulties (insufficient measuring techniques, non-homogeneity of the soil–water two-phase medium, etc.) exist in performing the measurements of soil saturation conditions to the high accuracy required by mathematical models of the wave-induced soil–water interaction, one can surely find the reason to decide that the computed value of optimum degree of saturation is too small and very unlikely to happen at a certain site. This way of thinking would lead to a new limitation of possible values of the degree of saturation (e.g., $S_{opt} \geq 0.95$—from the engineering practice point of view, this can be treated as almost fully saturated soil conditions).

For such a newly established value of the degree of saturation, the numerical analysis has to be repeated in order to find a new value of the maximum wave-induced hydrodynamic uplift force. The computations show that $F_u = 0.499 \text{kN/m/kPa}$ for $S = S_{opt} = 0.95$. It means that the smaller value of maximum wave-induced hydrodynamic uplift force can be used in the further analysis of submarine pipeline stability, but this value is only 20% reduced compared to the value initially found without any artificial restriction to the range of soil saturation conditions.

This finding undoubtedly indicates that there is no practical need for additional play with the range of degree of saturation limited by $S_{opt}$. Only a relatively small benefit of decreasing the value of maximum wave-induced hydrodynamic uplift force can be reached but, simultaneously, the risk of an incorrect decision concerning the estimation of soil saturation conditions might be too high.

3.6. Influence of anchoring a submarine pipeline in seabed sediments

Presenting the boundary conditions applicable to the governing problem, it was shown that all the nodes in the pipeline-wall sub-domain can be, at a certain level of simplification, constrained by assuming $u_x = u_z = 0$ (see Fig. 2). The comparison of two cases, with constrained and unconstrained pipeline-nodes, is presented in Table 4 for $L=30$ and 120 m, and three different compressibility models.

The imposition of these two primary boundary conditions, in the case of dense sediments and shorter wavelength ($L=30$ m), makes the uplift force ca 1.4 times higher than the value obtained for more realistic BCs (i.e., $u_x \neq 0$ and $u_z \neq 0$). If loose sediments are considered the uplift force increases dramatically when the constrained BCs are applied to all nodes from the pipeline-wall elements. The uplift force acting on a submarine pipeline buried and fixed (the constrained condition can be achieved, among other ways, by anchoring the pipeline in seabed sediments) in loose sandy sediments can even be a few times higher (here: 7.6!) than the uplift force computed for the case of a buried pipeline ‘free-movable’ in loose sandy sediments. Introducing longer waves ($L=120$ m) into computations, a further increase in the uplift force was found; and thus, the uplift force computed with the constrained BCs was ca 1.8 (dense sediments) and 10.6 (loose sediments) times higher than in the case of ‘free-movable’ pipeline.

In the global stability analysis, however, these extremely high values of uplift force will simultaneously be compensated, at least to a certain extent, by additional resistant forces due to anchoring of the pipeline in seabed sediments.

The question arises, however, if the soil from the pipeline vicinity can be treated as loose, semi-dense, or dense. The answer will certainly be different depending on the man-induced or natural soil and water/wave conditions existing in the vicinity of the buried submarine pipeline (construction or operational phase, normal or severe wave conditions).
Table 5. Wave-induced uplift force \( F_i \) [kN/m/kPa] for unconstrained and constrained nodes of the pipeline domain, and for different compressibility models and wavelengths

<table>
<thead>
<tr>
<th>Compressibility model of seabed</th>
<th>Type of boundary condition</th>
<th>Unconstrained ((u_x \neq 0 \text{ and } u_y \neq 0))</th>
<th>Constrained ((u_x = u_y = 0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>COMP-P ( S=0.994 )</td>
<td></td>
<td>0.617</td>
<td>0.617</td>
</tr>
<tr>
<td>COMP-PS ( S=0.992 )</td>
<td></td>
<td>0.523</td>
<td>0.727</td>
</tr>
<tr>
<td>( G_x=2.6 \times 10^5 \text{ kN/m}^2 ) (dense)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COMP-PS ( S=0.994 )</td>
<td></td>
<td>0.233</td>
<td>1.774</td>
</tr>
<tr>
<td>( G_x=2.6 \times 10^4 \text{ kN/m}^2 ) (loose)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COMP-P ( S=0.988 )</td>
<td></td>
<td>0.626</td>
<td>0.626</td>
</tr>
<tr>
<td>COMP-PS ( S=0.98 )</td>
<td></td>
<td>0.554</td>
<td>1.003</td>
</tr>
<tr>
<td>( G_x=2.6 \times 10^5 \text{ kN/m}^2 ) (dense)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>COMP-PS ( S=0.97 )</td>
<td></td>
<td>0.320</td>
<td>3.390</td>
</tr>
<tr>
<td>( G_x=2.6 \times 10^4 \text{ kN/m}^2 ) (loose)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Wavelength: \( L=30 \text{ m} \)

Wavelength: \( L=120 \text{ m} \)

This very important problem can only be analysed and solved as a result of proper laboratory large-scale modelling and, if possible, measurements in situ.

3.7. Influence of the geometry

The above presented results confirm the need to perform a wide parameter study in order to find the maximum uplift force. The parameter study should concern not only soil and pore fluid physical properties but also the different geometry of the problem where the depth of burial and outer pipeline diameter seem to play a decisive role.

The main data used in the computations consist of: wavelength \( L=90 \text{ m} \), coefficient of soil permeability \( k=0.0001 \text{ m/s} \), three different pipeline outer diameters \( D_1=0.50 \text{ m} \), \( D_2=0.75 \text{ m} \) and \( D_3=1.00 \text{ m} \), and four relative (with respect to the pipeline outer diameter) depths of burial \( b/D=0.5, 1, 2 \) and \( 3 \). Taking into account a relatively large pipeline outer diameter (e.g., \( D=1.00 \text{ m} \)), the engineering practice concerned with the depth of burial \( b=0.5–1.0 \text{ m} \) (Dursthoff and Mazurkiewicz, 1985) implies that \( b/D \geq 0.5 \). The results of the computations are shown in Fig. 18.

Very valuable for further findings is a transformation of the results presented in Fig. 18 into the results illustrated in Fig. 19 by introducing the pipeline outer equivalent diameter \( D_e=1.00 \text{ m} \); thereby all the results respective to the three diameters: \( D_1, D_2 \) and \( D_3 \) are multiplied by factors \( 2, 4/3 \) and \( 1 \), respectively. A surprising but very convenient linear relationship between the wave-induced uplift force and the pipeline outer diameter has been indicated. This finding will surely be very helpful in deriving the final engineering formula for the wave-induced hydrodynamic uplift force.
Wave-induced uplift force

Fig. 18. Influence of the geometry of the governing problem (i.e., pipeline outer diameter $D$ and relative depth of burial $b/D$) on the wave-induced hydrodynamic uplift force.

Fig. 19. Transformation of the results of the wave-induced hydrodynamic uplift force (see Fig. 17) into the pipeline equivalent-diameter ($D_e=1.0$ m).
3.8. Influence of wave conditions—uplift force (absolute) computation

Considering a simple harmonic wave loading in the present analysis, wave conditions can be represented by the wavelength and the wave height. The meaning of these two parameters, which also characterizes the geometry of the governing problem, will be discussed now in order to express the wave-induced hydrodynamic uplift force in absolute values.

A proximity of the upper limit of uplift force from the COMP-PS model and the uplift force found as a solution from the COMP-P model, together with a relatively high uncertainty in precisely defining the saturation conditions existing in seabed sediments in the coastal zone, led to the decision to set the uplift force values obtained from COMP-P model as the upper-limit values to be used in the stability analysis of the hydrodynamic uplift force acting on a submarine pipeline buried in sandy sediments.

Using two wave breaking conditions: the wave steepness \[ H_o/L = 0.143 \tanh \left(ah\right) \] and the water depth to wave height ratio (theoretically: \( H_o/h = 0.78 \), practically: \( H_o/h = 0.6 \)), the maximum possible wave height was computed with respect to the relation between the water depth and the wavelength, and the maximum wave-induced hydrodynamic bottom pressure was found (Fig. 20). Having obtained the maximum wave-induced hydrodynamic bottom pressure and the relative uplift force (from the COMP-P model), the final result for the upper limit of the absolute wave-induced hydrodynamic uplift force acting on a buried submarine pipeline was obtained.

It appears that for the assumed geometry of the problem \((h=10 \text{ m})\), the wavelength \(L=30 \text{ m}\) does not induce any significant hydrodynamic bottom pressure. Therefore, the results for shorter waves should only be treated as illustrative for studying the behaviour of the relative uplift force. As far as the absolute value of the uplift force \( (F_z = F_z \times P_o) \) is concerned, longer wavelengths have to be taken into account.

Fig. 20. Influence of the wavelength on the wave-induced hydrodynamic uplift force acting on a buried submarine pipeline (COMP-P model).
The results presented in Fig. 20 show evidently that the uplift force (absolute value) increases with the wavelength. Comparing both the hydrodynamic uplift force, $F_z$, and the hydrostatic uplift force, $F_h$, a non-linear relation between ratio $F_z/F_h$ and the wavelength can be easily indicated. Thus, $F_z/F_h=0.4, 1.5, 1.9$ and $2.1$ for $L=30, 60, 90,$ and $120$ m, respectively. These results were obtained from the computations with $\mu_r=0.3$ and $k=10^{-4}$ m/s.

3.9. Practical formula

Taking the results shown in Fig. 19 and using the least-squares curve fitting method, the relationship between the maximum wave-induced hydrodynamic uplift force $F_z$, geometry of the governing problem (pipeline outer diameter $D$ and depth of burial $b$), and wave conditions represented by the hydrodynamic bottom pressure $P_0$, was approximated with a polynomial of the second order. After the polynomial coefficients had been found, the final formula was created:

$$F_z = P_0 \times D \left[ 0.8 - 0.4 \left( \frac{b}{D} \right) + 0.07 \left( \frac{b}{D} \right)^2 \right]$$  (5)

The values of the uplift force computed for $0.5 \leq b/D \leq 3.0$ were used to derive Equation (5). It now becomes possible to predict the highest maximum value of the wave-induced hydrodynamic uplift force which is supposed to occur when the pipeline adjoins the seabed bottom; using Equation (5), it is very easy to find that $F_z=0.8$ kN/m/kPa for $b=0$ m and $D=1.00$ m.

The formula for the wave-induced hydrodynamic uplift force, acting on a submarine pipeline buried in permeable seabed sediments, was found based on the COMP-P model of the relative compressibility of the two-phase seabed medium. The formula, which is thought to serve as a good design tool for the engineer, requires only the definition of the geometry of the governing problem, represented by the following parameters:

$h$ = water depth,
$H_w$ = wave height,
$L$ = wavelength,
$D$ = pipeline outer diameter,
$b$ = depth of burial,

where the knowledge of the first three parameters is necessary for computations of the pore-fluid compressibility $\beta'$ and the hydrodynamic bottom pressure $P_0$.

4. CONCLUSIONS

The above presented analysis of the wave-induced hydrodynamic uplift force acting on a submarine pipeline buried in sandy sediments has indicated the need to treat both the pore fluid and soil skeleton as compressible media which, on the other hand, stays in accordance with realistic characteristics of the submarine environment.

The method of analysis of the wave-induced hydrodynamic uplift force seems to be well-applicable to modern engineering practice. The optimization procedure in the submarine pipeline design is of great value in view of the difficulties and uncertainties in defining the soil and pore-fluid parameters in situ i.e., coefficient of soil permeability and degree of saturation directly influencing the pore-fluid compressibility.
It was found that the solution to the uplift force, obtained from the COMP-P model (pore fluid is assumed to be compressible whereas the soil skeleton is treated as incompressible) constitutes the practical solution that can be incorporated into the stability analysis of submarine pipelines buried in sandy seabed sediments. The very simple, from the engineering point of view, formula was presented as a recommendation given in order to make a realistic, safe and economic estimation of the wave-induced uplift force acting on a buried submarine pipeline.

It is very difficult, time-demanding, and expensive to conduct laboratory studies with soil and pore fluid parameters, decisive in the governing problem, precisely defined. However, the combination of a lack of such kinds of investigations, together with the meaningful differences in the wave-induced uplift force, acting on a submarine pipeline buried in sandy seabed sediments, indicated by the most realistic COMP-PS model (pore fluid and soil skeleton compressible) between the solutions obtained for loose and dense states of seabed sediments, should serve as motivation either for such investigations in the coming future or for improvement in the optimization analysis of the wave-induced hydrodynamic uplift force. The present paper contributes to the latter.

REFERENCES