VOF-DEM-FEM COMBINED MODEL OF THE REEF BREAKWATER COLLAPSE

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A combined model was built of three main modules based on the Volume of Fluid (VOF) method, Discrete Element Method (DEM), and Finite Element Method (FEM). It was proposed to utilize this model to simulate the deformation of the rubble mound and the sandy bed due to surface wave action. The model included the full interaction between wave motion with free surface and replaceable separate particles of the rubble mound. Momentary arrangement of the fluid, particles and resulting permeability was tracked within a domain of time and two-dimensional space by maintaining cyclic data transfer between the three method modules. A new technique of porosity adjustment was presented. The model results were compared to small-scale laboratory test results. Based on the comparison, the VOF-DEM-FEM model appeared to be a promising tool to handle the destruction process of the rubble coastal structures built on a permeable bottom.

Keywords: VOF; DEM; FEM; submerged breakwater; destructive numerical model; combined numerical model.

1. Introduction

Since numerical models were discovered and applied for hydraulic problems in the fifties, they have evolved from very basic simple phenomena into more and more
complex multi-dimensional solutions. Various equations were developed for specific purposes. The poro-elastic equations originally developed by Biot [1941] as the three-dimensional consolidation theory were later coupled with the Darcy equation by Nago and Maeno [1984]. This approach was successfully applied to calculate pore pressure wave propagation within highly saturated porous media. These equations, solved by the Finite Element Method (FEM), proved very efficient at solving many problems of saturated sandy seabed under pressure variations e.g. Nago and Maeno [1987], Maeno and Nago [1988].

Attempts were made during the seventies to apply FEM solutions for modeling the deformation of soils. However, the large expenditure of calculations based on the FEM provoked development of an alternative method. The Distinct Element Method (DEM) was originally proposed by Cundall [1971] and Cundall and Strack [1979]. The idea was based on basic physical laws governing the interaction between a pair of separate elastic elements of discrete material. The DEM modification used in the present research was elaborated by Gotoh and Sakai [1997], and Gotoh et al. [2000]. That solution has proved very efficient to simulate the flotation phenomenon of buried pipelines due to cyclic hydrodynamic load by Maeno and Magda [2004], where laboratory results obtained by Maeno et al. [1999] were adequately reproduced.

The current version of the Volume of Fluid (VOF) method named CADMAS-SURF was elaborated by Isobe et al. [1999, 2001] as an improved solution for the porous flow with a free surface. The VOF was elaborated upon as a successive solution to MAC by Welch et al. [1966], SOLA-VOF by Hirt and Nichols [1981], and NASA-VOF3D by Torrey et al. [1987].

The above mentioned three different methods proved to be very efficient for specific applications, which could be combined into an effective numerical model to handle separate aspects of a system. A double model made of the VOF and FEM was proposed by Maeno et al. [2002] to clarify the pore pressure distribution in standing wave conditions in the sandy seabed and backfill in the vicinity of a sheet pile wall. Bierawski and Maeno [2006] presented a DEM-FEM coupled model to track the outflow of backfilling sand from behind the sheet pile wall under wave action. The FEM was used to calculate the pore pressure variations inside the sandy bed, and the DEM to simulate movements of particles caused by pressure changes. Itoh et al. [2002] proposed a VOF-DEM coupled model to show the destructive impact of the bore on rubble structures built over a rigid bottom.

In recent years, the submerged detached breakwater, known as the reef breakwater, is widely used as effective shore protection works in consideration of not only the environmental impact but also the aesthetic requirements. The submerged breakwaters are mostly constructed as rubble mound structures. Subsequent collapses of such structures under wave action reveal imperfections in the current knowledge of hydraulic phenomena generated by waves. Therefore, it is very important to develop a numerical model to simulate the destruction process of rubble breakwaters with wave-seabed-structure interaction. For this purpose, this paper presents a more
advanced proposal to deal with the issue, which is a triple-module numerical model combined of all the methods mentioned above, namely the VOF, DEM and FEM. The VOF is to handle the turbulent flow with free surface within the wave zone and also throughout the porous body of the rubble breakwater. The FEM is to provide data on pressure transmission by the pore fluid of the sandy bed, while the DEM allows for deformation of the rubble body of the breakwater and the top layer of the sandy bed. All the methods separately were proved over many applications to be capable of handling their designated problems. The current threefold model was proposed to profit from the specific characteristics and advantages of each of the three methods to deal with three different aspects in existence of rubble breakwaters over permeable beds under wave action. The aspects are: the porous flow with a free surface, the pore pressure fluctuations in highly saturated beds and deformation of the rubble breakwaters due to large wave impacts.

Procedures have been developed to maintain permanent data transfer between these methods. The pressure is maintained to be continuous at the boundary between the VOF and FEM. The accuracy of the porosity ratio is especially important for the behavior of the rubble mound because the change of the porosity influences the porous flow. Therefore, the porosity of cells in the VOF domain is updated according to relocation of the DEM particles.

It is a common engineering practice that the length, in other words the third dimension, is simply neglected in case of linear structures with invariable cross-section, exposed to wave action. Additionally, the third dimension is neglected to maintain reasonable resources and calculation time. Therefore, to provide experimental data for comparison, both the proposed numerical model and the physical model are developed in two-dimensional wave flumes.

2. Summary of the VOF-DEM-FEM Model

2.1. Outline of the flow analysis (VOF based model)

The Volume of Fluid (VOF) method was used to track the free surface [Hirt and Nichols; 1981]. To simulate the flow through a highly permeable porous rubble structure and sandy seabed, porosity was introduced into the basic equations [Isobe et al. 1999, 2001; Edward et al. 2000; Maeno et al. 2007]. The general equations are shown below: the continuity equation, the momentum equations and the transport equation of the VOF function $\mathbf{F}$. The momentum equations are a modification of Navier-Stokes equations including turbulence terms. The SMAC method is used to solve Eqs. (1) to (3). For details, refer to Isobe et al. [2001].

\[
\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0
\]
\[
\begin{align*}
\frac{n}{\partial t} \frac{\partial u}{\partial x} + \frac{\partial uu}{\partial x} + \frac{\partial uu}{\partial z} &= -n^2 \left( \frac{1}{\rho_w} \frac{\partial p}{\partial x} - g_x - R_x \right) \\
&\quad + \frac{\partial}{\partial x} \left[ n \nu_e \left( 2 \frac{\partial u}{\partial x} \right) \right] + \frac{\partial}{\partial z} \left[ n \nu_e \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial z} \right) \right] - \frac{2 \partial k}{3 \partial x} \\
\frac{n}{\partial t} \frac{\partial w}{\partial x} + \frac{\partial ww}{\partial x} + \frac{\partial ww}{\partial z} &= -n^2 \left( \frac{1}{\rho_w} \frac{\partial p}{\partial z} - g_z - R_z \right) \\
&\quad + \frac{\partial}{\partial x} \left[ n \nu_e \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial x} \right) \right] + \frac{\partial}{\partial z} \left[ n \nu_e \left( \frac{2 \partial w}{\partial z} \right) \right] - \frac{2 \partial k}{3 \partial z} \\
\frac{n}{\partial t} \frac{\partial F}{\partial x} + \frac{\partial uF}{\partial x} + \frac{\partial wF}{\partial z} &= 0
\end{align*}
\]

where \( n \) is the porosity, \( t \) is the time, \( x \) and \( z \) are the space coordinates in horizontal and vertical directions respectively, \( u \) and \( w \) are the flow rate components in \( x \) and \( z \) directions, \( g_x \) and \( g_z \) are the acceleration components, \( R_x \) and \( R_z \) are the flow drag components, \( \rho_w \) is the density of fluid, \( p \) is the pressure, \( k \) is the turbulent energy, and \( \nu_e \) is the sum of \( \nu + \nu_t \), where \( \nu \) is the molecular viscosity coefficient, and \( \nu_t \) is the kinematic eddy viscosity.

The basic equations assume a constant porosity ratio in time and space. It seems contradictory to the general idea of the presented combined model. However, the model maintains the constancy of the porosity ratio inside each calculation cell over a very short time — a single calculation step. Furthermore, the porosity ratio changes very slowly due to diminutive replacements of the DEM particles in every time step.

The turbulence energy \( k \) and energy dissipation ratio \( \varepsilon \) are determined by the following equations.

\[
\begin{align*}
\frac{n}{\partial t} \frac{\partial k}{\partial x} + \frac{\partial uk}{\partial x} + \frac{\partial wk}{\partial z} &= n \frac{\partial}{\partial x} \left[ \nu_k \left( \frac{\partial k}{\partial x} \right) \right] + n \frac{\partial}{\partial z} \left[ \nu_k \left( \frac{\partial k}{\partial z} \right) \right] + G_s - n \varepsilon - 2n^2 R_x k/u \\
\frac{n}{\partial t} \frac{\partial \varepsilon}{\partial x} + \frac{\partial u\varepsilon}{\partial x} + \frac{\partial w\varepsilon}{\partial z} &= n \frac{\partial}{\partial x} \left( \nu_e \frac{\partial \varepsilon}{\partial x} \right) + n \frac{\partial}{\partial z} \left( \nu_e \frac{\partial \varepsilon}{\partial z} \right) + C_1 \frac{\varepsilon}{k} G_z - n C_2 \frac{\varepsilon^2}{k} - 2n^2 R_z \varepsilon/w
\end{align*}
\]

\[
\begin{align*}
\nu_t &= C_\mu \left( \frac{k^2}{\varepsilon} \right), \quad \nu_k = \nu + \nu_t / \sigma_k, \quad \nu_e = \nu + \nu_t / \sigma_e, \\
G_s &= \nu_t \{ 2(\partial u/\partial x)^2 + 2(\partial w/\partial z) + (\partial w/\partial x + \partial u/\partial z)^2 \}
\end{align*}
\]

The constant empirical values \( C_\mu = 0.09, \quad C_1 = 1.44, \quad C_2 = 1.92, \quad \sigma_k = 1.0, \) and \( \sigma_e = 1.3 \) are applied following Launder and Spalding [1974], Isobe et al. [1999].

The flow drag components calculated over each cell: \( R_x \) and \( R_z \), are used to model the turbulent porous flows following Edward et al. [2000].
resistance to flow, given by Ward [1964], consists of resistance related to the laminar and turbulent flows as shown in Eqs. (7) and (8). Applicability of the approach was confirmed by Michioku et al. [2005].

\[ R_x = u \left( \nu + \frac{C}{\sqrt{K}} \sqrt{u^2 + w^2} \right) \]  
\[ R_z = w \left( \nu + \frac{C}{\sqrt{K}} \sqrt{u^2 + w^2} \right) \] 

Here, \( C \) is a coefficient related to the turbulent flow, and \( K \) is the intrinsic permeability.

In the present studies, to obtain values for parameters \( C \) and \( K \), the resistance law by Ergun [1952] was used, as presented in Eqs. (9) and (10).

\[ R_x = u \left( k_1 \frac{(1-n)^2}{n^3} \frac{\nu}{D_{sp}^2} + k_2 \frac{1-n}{n^3} \frac{1}{D_{sp}} \sqrt{u^2 + w^2} \right) \]  
\[ R_z = w \left( k_1 \frac{(1-n)^2}{n^3} \frac{\nu}{D_{sp}^2} + k_2 \frac{1-n}{n^3} \frac{1}{D_{sp}} \sqrt{u^2 + w^2} \right) \]

In the above equations, \( D_{sp} \) stands for a function of the specific surface area \( S_\nu = \frac{S_p}{V_p} \), where \( S_p \) is the total surface area of the particles in the porous region, and \( V_p \) is the total volume to the particles. In case of spherical particles, the specific surface area function becomes \( D_{sp} = \frac{6}{S_\nu} \), and corresponds to the particle diameter. Coefficients \( k_1 \) and \( k_2 \) are the factors of the particle shape. Combination of Eqs. (7) through (10) yields:

\[ K = n^3 D_{sp}^2 /[k_1 (1-n)^2], \quad C = k_2 / \sqrt{k_1 n^3} \] 

Ergun [1952] experimentally obtained values of \( k_1 = 150 \) and \( k_2 = 1.75 \) for uniform spheres and sand. In this research, the formula by Ergun [1952] has been proved to be applicable to moving particles, whilst \( k_1 = 200 \) and \( k_2 = 2.8 \) are used following Maeno et al. [2007].

2.2. Outline of the DEM

The DEM solution by Gotoh and Sakai [1997] and Gotoh et al. [2000] is used to model the submerged breakwater and the top layer of the sandy seabed. The analytical area of the DEM is set larger than the observed range of deformations in the laboratory experiment. In this method, each particle is assumed to be a rigid body. The interaction between pairs of particles of coordinates \((x_i, z_i)\) is modeled with Kelvin-Voight system consisting a spring and a viscous dashpot. Motion of the particles is caused by the seepage force components \( F_x \) and \( F_z \) calculated by the
other modules. The governing equations of the DEM can be written as follows:

\[(M_i + M'_i)\ddot{x}_i = \sum_j \left(-f_n \cos \alpha_{ij} + f_s \sin \alpha_{ij}\right) + F_x\]  

(12)

\[(M_i + M'_i)\ddot{z}_i = \sum_j \left(-f_n \sin \alpha_{ij} + f_s \cos \alpha_{ij}\right) - (M_i - \rho_w V_i)g + F_z\]  

(13)

\[I_i \ddot{\theta}_i = \frac{d_i}{2} \cdot \sum_j (f_s) j\]  

(14)

where the subscripts \(i\) and \(j\) indicate specific particles, \(f_n\) and \(f_s\) are the normal and tangential forces acting at the contact point of two particles, \(M_i\) is the particle mass, \(\alpha_{ij}\) is the contact angle between contacting particles, \(V_i\) is the particle volume, \(I_i\) is the momentum of inertia, \(\theta_i\) is the rotation angle of the particle, \(d_i\) is the grain diameter, \(\rho_w\) is the density of water, and \(M'_i\) is the added mass. The added mass \(M'_i\) is calculated using the following expression \(M'_i = \rho_w C_M V_i\), where the added mass coefficient \(C_M = 0.5\) is used. \(F_x\) and \(F_z\) are the external forces transmitted from the fluid to each DEM particle. In the present research they are composed of the flow induced drag force \(F_D\) and the pressure gradient induced force \(F_S\).

\[F_x = F_{Dx} + F_{Sx}, \quad F_z = F_{Dz} + F_{Sz}\]  

(15)

The external force \(F_D\) is calculated using expressions as follows:

\[F_{Dx} = \frac{1}{2} \rho_w C_D A_{pi} \sqrt{(U - u_{pi})^2 + (W - w_{pi})^2(U - u_{pi})}\]  

(16)

\[F_{Dz} = \frac{1}{2} \rho_w C_D A_{pi} \sqrt{(U - u_{pi})^2 + (W - w_{pi})^2(W - w_{pi})}\]  

(17)

\[C_D = 0.4 + \frac{24\nu}{\sqrt{(U - u_{pi})^2 + (W - w_{pi})^2} d_i}\]  

(18)

where \(U(U = u/n)\) and \(W(W = w/n)\) are the velocities in \(x\) and \(z\) directions respectively, \(u_{pi}\) and \(w_{pi}\) are the instantaneous particle velocities in \(x\) and \(z\) directions respectively, \(A_{pi}\) is the cross-section area of the particle, and \(C_D\) is the drag coefficient.

The seepage velocities \(U\) and \(W\) are calculated by the VOF module for centers of cells. Therefore, in the case of a particle located partly in the two adjacent cells, its average value is taken. The external force component \(F_S\) originates from the water pressure gradient force acting on particle volume \(V_s\), and can be expressed as below

\[F_{Sx} = \rho_w g V_s \frac{\partial h}{\partial x}, \quad F_{Sz} = \rho_w g V_s \frac{\partial h}{\partial z}\]  

(19)

where \(h\) is the piezometric water head.
The pressure gradients are calculated using the VOF or FEM results, depending on which of the domains the particle center is located at the time. With respect to overlapped region, the pressure gradients are calculated using the FEM results, which was proved to be effective for the sandy bed modeling.

2.3. The summary of FEM (sandy bed)

With respect to sandy bed analysis the basic equations by Maeno and Nago [1988] are used. They are based on Biot’s consolidation equations. In the equations, compressibility of the seabed due to the existence of a small amount of air is considered. Applicability of the equations was verified by Mase et al. [1994] and Maeno et al. [2002].

\[
\frac{\partial}{\partial x} \left( c_{11} \frac{\partial u_x}{\partial x} + c_{12} \frac{\partial u_z}{\partial z} \right) + c_{33} \frac{\partial}{\partial z} \left( \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right) = \rho_w g \frac{\partial h}{\partial x} \tag{20}
\]

\[
\frac{c_{33}}{\partial x} \left( \frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x} \right) + \frac{\partial}{\partial z} \left( c_{12} \frac{\partial u_x}{\partial x} + c_{22} \frac{\partial u_z}{\partial z} \right) = \rho_w g \frac{\partial h}{\partial z} \tag{21}
\]

\[
\rho_w g \left( \beta \lambda_w + \frac{\lambda_a}{\rho} \right) \frac{\partial h}{\partial t} + \frac{\partial}{\partial t} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \right) = k_b \left( \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} \right) \tag{22}
\]

where \( u_x \) and \( u_z \) are the displacements of the sand stratum in \( x \) and \( z \) directions respectively, \( \rho_w \) is the density of water, \( g \) is the gravity acceleration, \( h \) is the pore water pressure head, \( \lambda_w \) is the water content, \( \lambda_a \) is the air content \( k_b \) is the permeability coefficient of sand, \( \beta \) is the compressibility of pore water, \( E \) is the Young modulus of the soil skeleton, \( G \) is the shear modulus of the soil skeleton, and \( \nu_s \) is the Poisson ratio of the soil skeleton.

2.4. The modeling methodology

In the present research, the submerged breakwater is considered following Bierawski and Maeno [2002], where the same structure was examined under the non-destructive conditions. The physical model is developed in a 16 m long, two-dimensional wave flume as shown in Fig. 1. The breakwater is built over a sandy bed of the highly saturated Toyoura standard sand placed in a pit of 3 m in length and 0.6 m in width. The median diameter of the sand is \( d_{50} \approx 2.5 \times 10^{-4} \) m, while the specific gravity is 2.65, the porosity coefficient is \( n \approx 0.4 \), and the permeability coefficient is \( k_b = 1.2 \times 10^{-4} \) m/s. The breakwater has slopes of inclination 1:2. Its crest is 0.3 m in height and 0.3 m in length. Stones used to build it are of \( d_{50} \approx 0.015 \) m in diameter. The resulting porosity coefficient is \( n \approx 0.39 \) and the permeability coefficient is 0.25 m/s.
Figure 2 presents arrangement of the VOF, DEM and FEM domains. The VOF method can handle both non-porous and porous flows, therefore its domain spreads over the wave field, the breakwater and the top layer of the sandy bed of 0.1 m in height (within the limits given by the dashed line in Fig. 2). The FEM is used to the remaining part of the sandy bed and the overlapped area (see the dash-dotted line in Fig. 2), where despite of low flow rates, the influence of the pressure dissipation effect shall not be neglected. Based on the information from the VOF and FEM, the pressure gradients are expressed in terms of the forces and taken into account to the force balance of each DEM particle. As the flow rates within the VOF domain are of much larger magnitude, the drag force proportional to the velocity of fluid, is also taken into account.

Along the upper boundary of the FEM domain, the continuity of the pressure and velocity fields is ensured by using the pressure calculated by the VOF as a boundary condition to the FEM, and in turn the flow rate by FEM as a boundary condition to the VOF. In other words the water exchange process between the sandy bed and the wave field is included into the numerical model.

Two kinds of DEM particles with two different diameters are initially arranged to shape the breakwater mound as well as the top layer of the sandy bed (see the oblique hatched area in Fig. 2). Such an arrangement is aimed to allow for large displacements of the rubble breakwater body and also of the top layer of the sandy bed, as observed in the laboratory tests. On the other hand, the other part of the sandy bed is excluded from the DEM domain to make significant savings of
calculation time, as negligible small displacements are expected there. Besides the elastic inter-particle interactions and the action of moving fluid by the VOF and FEM, the DEM particles movements are powered by external forces arising from the hydrostatic lift and the gravity force.

2.5. Analytical assumptions

2.5.1. Parameters used for calculations

As the model is two-dimensional, the cylindrical shape is adopted for the DEM particles as a matter of the best suitability. A single layer of the particles is like embedded between two virtual walls, in which the only influence is to limit the motion to the cutting plane. The breakwater body is built of particles of natural scale, namely of 0.015 m in diameter, while the top layer of the sandy bed of 0.10 m in height is built of cylinders 0.005 m in diameter. The diameter of the Toyoura Standard sand is $2.5 \times 10^{-4}$ m, and the reason to increase it in the model is again to save memory resources and calculation time. Scale effects of such replacement were discussed in Bierawski and Maeno [2006]. Although the long-term deformation such as sinking of the rubble into the sandy bed is not reproduced, the rubble failure process of the top layers is modeled adequately.

In the DEM module the calculation time step is assumed $\Delta t = 1.0 \times 10^{-4}$ s. In order to save some calculation time, the VOF and FEM calculations are triggered every second cycle of the DEM calculations, in other words, the time step for the VOF and FEM is 2 times longer than that of the DEM, namely $\Delta t = 2.0 \times 10^{-4}$ s. During initial packing, excessive vibrations of particles occurred. Therefore, to dump the excessive vibrations of particles, the effect of cohesion was considered. That is, the following criteria due to a very small force $f_{nc}$ were introduced.

$$f_{nc} = -0.002k nd$$

If $f_{nc} \leq f_{ns} < 0$ then $f_s = 0$

If $f_{ns} < f_{nc}$ then $f_n = 0$, $f_s = 0$

where $k_n$ is the spring constant, $d$ is the diameter of the particle, $f_n = f_{ns} + f_{nd}$, $f_{ns}$ is the normal force due to the spring, and $f_{nd}$ is the normal force due to the dashpot.

The friction coefficient of the particles applied for the different materials: the rubble and the sand is 1.0 (45 deg internal friction angle), and 0.577 (30 deg internal friction angle) respectively. The normal and tangential spring constants $k_n$, $k_s$, and the normal and tangential damping coefficients $\eta_n$, $\eta_s$ of the viscous dashpot of the DEM particles can be calculated automatically using the following equations. (Gotoh and Sakai [1997] and Gotoh et al. [2000])

$$k_n = \frac{2\pi^2 M}{(\alpha_m \Delta t)^2}, \quad k_s = \frac{k_n}{2(1 + \nu_s)}$$
\[ \eta_n = 2\alpha_{cn}\sqrt{Mk_n}, \quad \eta_s = \frac{\eta_n}{\sqrt{2(1 + \nu_s)}} \]  

(25)

where \( M \) is the mass of particle, \( \alpha_{tn} = 20.0 \) and \( \alpha_{cn} = 0.3 \) were used with reference to Gotoh et al. [2000].

The wave is generated by a numerical wave source in water of \( h = 0.45 \) m in depth according to Stokes theory of 5th order with the height of \( H = 0.2 \) m, and the period of \( T = 2.0 \) s.

2.5.2. Initial packing

The pre-calculations of the DEM were performed assuming the gravity force only. In this model, the particles are added gradually, and settle down freely to build the structure. Such packing method is used to get random locations of the DEM particles, instead of the so-called hexagonal arrangement, which is supposed to generate an artificial shear resistance. After a number of trials, the initial packing used for the main calculations was obtained, as shown in Fig. 3.

2.5.3. Porosity coefficient adjustment

In the VOF method, the effect of flow through a porous medium was included by the porosity coefficient in the governing Eqs. (2) and (3). As the rubble is supposed to undergo displacements, it turned out necessary to develop a method to ensure a correct value for the porosity coefficient at every calculation cycle. Additionally, as shown in Eqs. (7), (8) and (11), the porosity coefficient influences the drag force, which is generated while the fluid flows through the porous body. For these reasons, the porosity coefficient is found to be a parameter having strong effect on the porous flow. In the present model application, significant changes of porosity are expected on the surface of the breakwater, and therefore the parameter has to be updated according to movements of the rubble. First approach to that problem was made by Bierawski [2004], where the porous area range was updated according to the top layer of the DEM particles. Simple calculation of the void area in the two-dimensional domain, based on the arrangement of the DEM cylinders is underestimated compared to the three-dimensional matrix of spheres. In this research, a new effective method is proposed to handle changes in the porosity related to the movement of the DEM particles even in the two-dimensional domain.

![Fig. 3. Initial DEM particle arrangement — result of packing.](image-url)
In view of the DEM, the voids are insignificant and simply neglected. However, the void ratio can be estimated by a measure of the gaps arising from the instantaneous particle arrangements. Obviously the amount of gaps between the uniform cylindrical particles will be different from that obtained for a three-dimensional array of spheres of the same diameter. To allow for a better correlation between the two-dimensional void ratio and the porosity coefficient measured in laboratory tests, a correction is needed. Although the DEM particles are solid, in the present research, we propose to introduce a porosity coefficient $n_0$ for the DEM particles themselves. The coefficient $n_0$ is a kind of shape coefficient, aimed to include the shape effects of the sphere to the cylinder. The shape coefficient should be considered as a measure of the imaginary porosity of the cylinders. Figure 4 shows the idea of the approach. The porosity coefficient is evaluated based on an assumption that the porosity measured in the laboratory tests $n_{\text{exp}}$ equals to the total porous area of the gaps between particles $A_f$ and the area of the imaginary voids related to the DEM particle porosity $n_0$. Such relation, expressed mathematically, brings the following equation.

$$n_{\text{exp}} = \frac{A_f + A_s n_0}{A_f + A_s}$$  \hspace{1cm} (26)

where, $A_s$ is the area of the 2-D DEM particles in a VOF cell, $A_f$ stands for the fluid filled area in a cell, $A_c = A_s + A_f$ is the area of the VOF cell.

Simple operations on equation (26) lead to an expression for the porosity coefficient:

$$n_0 = \frac{A_c n_{\text{exp}} - A_f}{A_c - A_f}$$  \hspace{1cm} (27)

The coefficient $n_0$ is calculated in separate rectangular VOF cells where applicable, only once during the program run, namely, after the packing stage is completed. The laboratory obtained values for $n_{\text{exp}}$ were for the sandy bed 0.40, while for the breakwater body 0.39. As for the main calculations, where the DEM particles move
according to the surface waves (the VOF output), the porosity for each VOF cell is estimated at every calculation cycle. The cell porosity $n$ is calculated using the fixed DEM particle porosity $n_0$ and the adequate porous areas that exist inside the cells at the current calculation time step. Equation (28) is adopted for this purpose, and becomes as below:

$$n = \frac{A_f + A_s n_0}{A_f + A_s}$$  \hspace{1cm} (28)

The presented technique of porosity adjustment is developed to improve the accuracy of the model over both the rubble breakwater and the sandy bed, where the porosity coefficients are fluctuating due to movements of the DEM particles. The benefit of the technique is demonstrated in Figs. 5(a) and 5(b). The figures show the porosity coefficients before and after the adjustment respectively. Although the porosity is significantly underestimated before the adjustment, the average adjusted porosity of the breakwater mound is around 0.39. The adjustment significantly improves conformity with the comparative experimental data. Inside cells containing the interface between the rubble and wave field the porosity values are adequately higher.

![Example porosity distribution](image-url)
3. Results and Discussion

Figure 6(a) presents the state of the submerged breakwater after 120 seconds of the destructive laboratory experiment run. The figure presents the final shape of the breakwater. The destructive process is the most intensive at the beginning, gradually slowing down with time even though the wave conditions are kept constant. The breakwater crest is washed out at first. The acting waves push some of the rubble landwards. The most destructive, however, is the return current, which gives the final shape to the offshore slope of the breakwater. Figure 6(b) presents the result obtained also after 120 seconds of the calculations by the VOF-DEM-FEM model.

![Fig. 6. Comparison of deformation observed in the laboratory and numerical tests. (a) Experimental result (after $t = 120$ s). (b) Numerical result ($t = 120$ s).](image)

<table>
<thead>
<tr>
<th>No. of particle</th>
<th>$\Delta x$ (cm)</th>
<th>$\Delta z$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-16.85</td>
<td>-9.28</td>
</tr>
<tr>
<td>2</td>
<td>-12.85</td>
<td>-7.80</td>
</tr>
<tr>
<td>3</td>
<td>-14.78</td>
<td>-4.52</td>
</tr>
<tr>
<td>4</td>
<td>-1.33</td>
<td>-0.80</td>
</tr>
<tr>
<td>5</td>
<td>0.01</td>
<td>-0.07</td>
</tr>
<tr>
<td>6</td>
<td>-0.89</td>
<td>-0.70</td>
</tr>
<tr>
<td>7</td>
<td>-0.02</td>
<td>-0.03</td>
</tr>
<tr>
<td>8</td>
<td>0.01</td>
<td>-0.08</td>
</tr>
<tr>
<td>9</td>
<td>-0.06</td>
<td>-0.04</td>
</tr>
<tr>
<td>10</td>
<td>0.01</td>
<td>-0.05</td>
</tr>
<tr>
<td>11</td>
<td>0.04</td>
<td>-0.09</td>
</tr>
</tbody>
</table>
developed in this research. The destruction of the offshore slope of the breakwater by the return current is reproduced fairly well. However, the shore side slope remains almost unchanged. The reason for such a small difference is probably caused by underestimation of the hydrodynamic force on the shore side slope of the breakwater due to the wave and seepage flow. Figure 7 shows the movement of representative particles around the offshore side of submerged breakwater before and after the experiment. Table 1 shows the displacement of these particles. Particles at the shoulder parts Nos. 1, 2 and 3 move dynamically due to the wave action. On the other hand, particles Nos. 4 to 10 do not move so much. Comparison of the photos between $t = 0$ and 120 s shows that the rubbles also do not move so much at these areas. Although the movements of rubble elements were not tracked in the experiment, the general tendency of the particle movement is similar to the observed results.

In order to verify the model performance, laboratory and numerical results are compared: fluctuations of the free surface in front and behind the breakwater, pressure fluctuations in selected points inside the sandy bed and the rubble mound. The selected points are located as shown in Fig. 8. The considerations on precision of the model at reproduction of the surface fluctuations are conducted at two locations, namely over the offshore and shore side breakwater toes, at wave meter 1 and 2, or
in other words for approaching and transformed waves. The experimental and numerical results of water level variations from the still water level for a second arrival wave are shown in Figs. 9(a) and 9(b) respectively. And Table 2 shows the comparison of wave height \( H_1, H_2 \) and travel time \( T \). Experimentally obtained times of
Table 2. Comparison of wave height $H_1$, $H_2$ and travel time $T$.

<table>
<thead>
<tr>
<th>Case</th>
<th>Wave crest (cm)</th>
<th>Wave trough (cm)</th>
<th>Wave height (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>13.8</td>
<td>−7.6</td>
<td>$H_1 = 21.4$</td>
</tr>
<tr>
<td></td>
<td>13.3</td>
<td>−3.0</td>
<td>$H_2 = 16.3$</td>
</tr>
<tr>
<td>Analysis</td>
<td>11.1</td>
<td>−5.8</td>
<td>$H_1 = 16.9$</td>
</tr>
<tr>
<td></td>
<td>11.2</td>
<td>−3.1</td>
<td>$H_2 = 14.3$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Travel time</th>
<th>Peak time VM1 (s)</th>
<th>Peak time VM2 (s)</th>
<th>Travel time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>0.24</td>
<td>1.09</td>
<td>$T = 0.85$</td>
</tr>
<tr>
<td>Analysis</td>
<td>0.28</td>
<td>1.09</td>
<td>$T = 0.81$</td>
</tr>
</tbody>
</table>

the wave crest are well described at both offshore and shore sides of the submerged breakwater. As for the wave crest configuration, some diffusion of the water level can be seen for the numerical result at offshore side WM1. On the other hand, crest shape was well described at onshore side WM2. Numerically obtained wave height for both sides was a little bit underestimated. However, comparison of these figures proves the very ability of the model to reproduce the wave motion.

Fig. 10. Pore water pressure variation, VOF part (Pt.1).

Fig. 11. Pore water pressure variation, FEM part (Pt.2).
Figures 10 and 11 show the pore water pressure variation at Pt.1 and Pt.2 respectively. And Table 3 shows the comparison of the pressure amplitude $A$. Phase of pressure propagation well describes the experimental results at both points. The amplitudes of the pressure for both points become a little bit larger as compared with the experimental results. However, the pressure attenuation rate for the experiment is $4.4/7.8 = 0.56$ and that for the analysis is $4.7/8.3 = 0.57$. Considering these results, calculated fluctuations of the pore water pressure inside the breakwater body (Pt. 1, Fig. 10), as well as inside the sandy bed (Pt. 2, Fig. 11), show also good agreement with the experimental results.

Figures 12(a) and 12(b) show the water and pore water pressure distribution around and inside the breakwater rubble under the circumstances of the wave trough.

<table>
<thead>
<tr>
<th>Case</th>
<th>Crest pressure (cm)</th>
<th>Trough pressure (cm)</th>
<th>Amplitude $A$ (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment (Pt.1)</td>
<td>8.2</td>
<td>−7.3</td>
<td>7.8</td>
</tr>
<tr>
<td>Analysis (Pt.1)</td>
<td>8.4</td>
<td>−8.1</td>
<td>8.3</td>
</tr>
<tr>
<td>Experiment (Pt.2)</td>
<td>5.4</td>
<td>−3.4</td>
<td>4.4</td>
</tr>
<tr>
<td>Analysis (Pt.2)</td>
<td>5.7</td>
<td>−3.6</td>
<td>4.7</td>
</tr>
</tbody>
</table>

Fig. 12. Pressure field at the submerged breakwater. (a) Under the wave trough $t = 9.50\ s$. (b) Under the wave crest $t = 10.20\ s$. 
and crest respectively. The pore pressure wave propagation through the reshaped breakwater body, including the effects of porosity change, is successfully reproduced by the proposed VOF-DEM-FEM model. The opposite pressure gradients are shown in front and behind the breakwater. Figure 12(a) shows a relative pore pressure excess inside the breakwater under the wave trough, disrupting the slopes, while Fig. 12(b) shows a pore pressure deficiency which presses down the rubble.

Figures 13(a) and 13(b) are supplementary to Figs. 12(a) and 12(b), to show the fluid velocity field under wave trough and crest conditions. In these figures it is clearly visible that the influence of the breakwater deflection to the wave field is modeled adequately. In other words, the method works well to control the porosity coefficient of the VOF cells due to tests on the DEM particles present. At the place where the DEM particles are accumulated, the flow rates can be easily recognized as of the porous flow. On the other hand, at the place of destroyed crown the flow is obviously free. It can be also recognized, that the flow rates right by the offshore edge of the breakwater are very high, and their destructive effects to the rubble is multiplied by the high pore pressure gradients.

4. Conclusions
The proposed VOF-DEM-FEM model, in which the porosity adjustment was newly introduced, was proved to be able to handle deformations of the rubble breakwater due to surface wave impacts. It was possible to reflect the interdependent changes in
the breakwater shape (by the DEM) and in the wave field (by the VOF). The model provided some qualitative information on the hydraulic phenomena in the vicinity of the rubble breakwater during its collapse stage. The developed VOF-DEM-FEM model to simulate wave-seabed-structure hydrodynamic interaction seems to be a promising tool to handle destruction processes of porous coastal structures and it facilitates design appropriate for them.

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References


